Lecture 9
Force distance curves I

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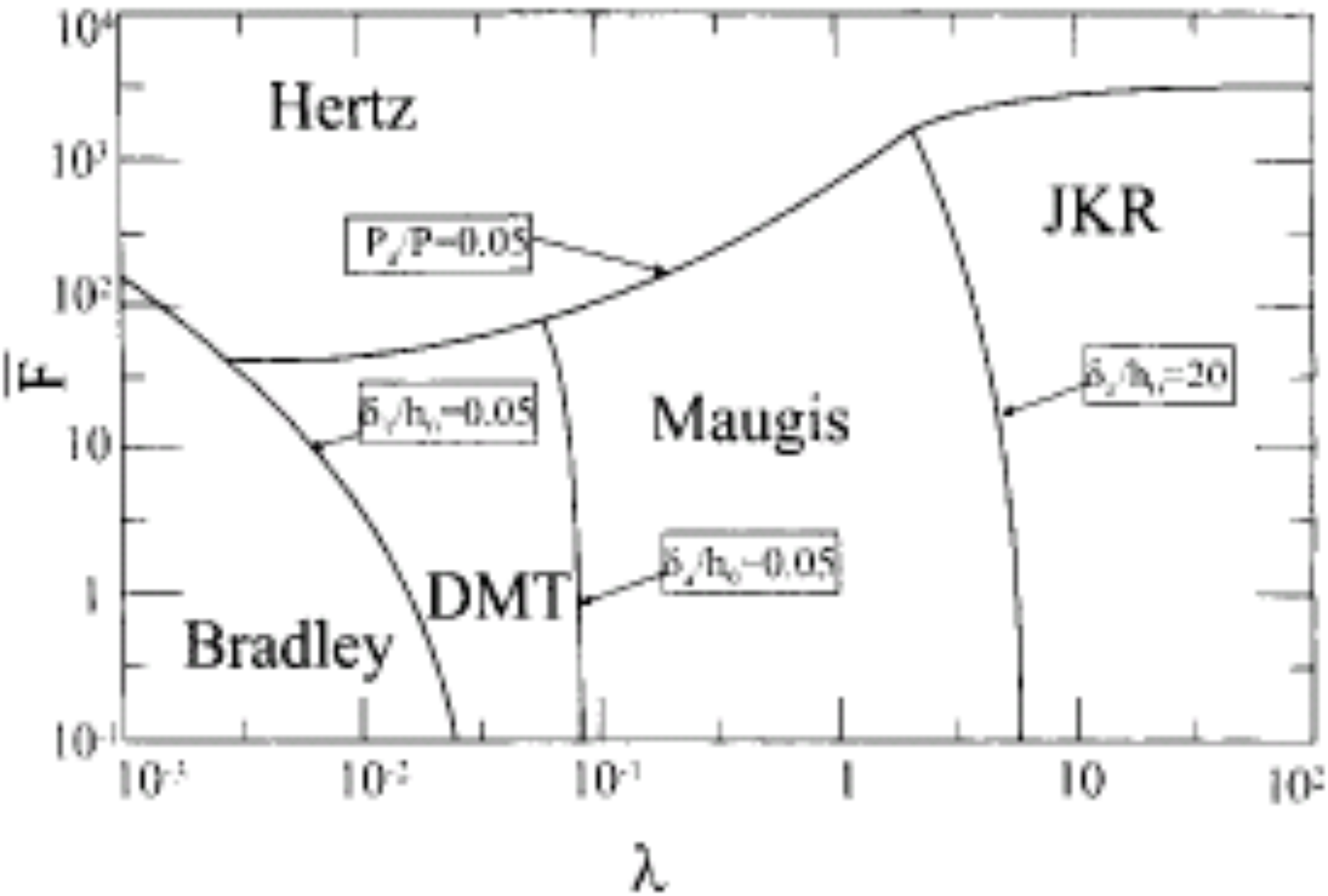
Validity of different models

$$\bar{F} = \left(\frac{F}{\pi W_{132} R_{tip}}\right)^{1/3}$$



$$\lambda = \frac{2.06}{z_0}\left(\frac{R_{tip} W_{132}^2}{\pi E_{tot}^2}\right)^{1/3} \qquad \frac{1}{E_{tot}} = \frac{3}{4}\left(\frac{1-\nu_s^2}{E_s} + \frac{1-\nu_t^2}{E_t}\right)$$

z_0 = *equilibrium separation or typical atomic distance*

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Comments on these theories

- JKR predicts infinite stress at edge of contact circle.
- In the limit of small adhesion JKR -> DMT
- Most equations of JKR and Hertz and DMT have been tested experimentally on molecularly smooth surfaces and found to apply extremely well
- Most practical limitation for AFM is that no tip is a perfect smooth sphere, small asperities make a big difference.
- Hertz, DMT describe conservative interaction forces, but in JKR, the interaction itself is non-conservative (why?) …for a force to be considered conservative it has to be describable as a gradient of potential energy.

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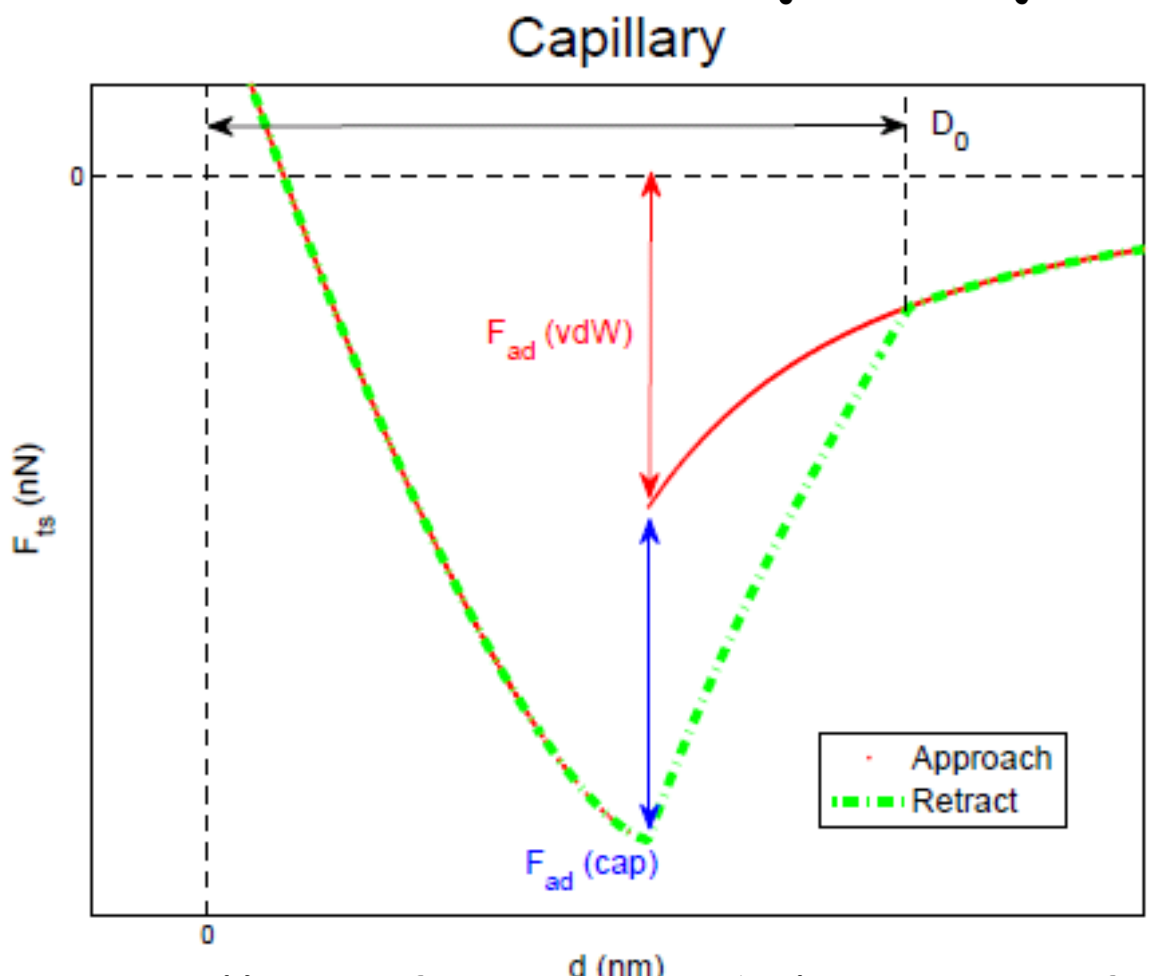
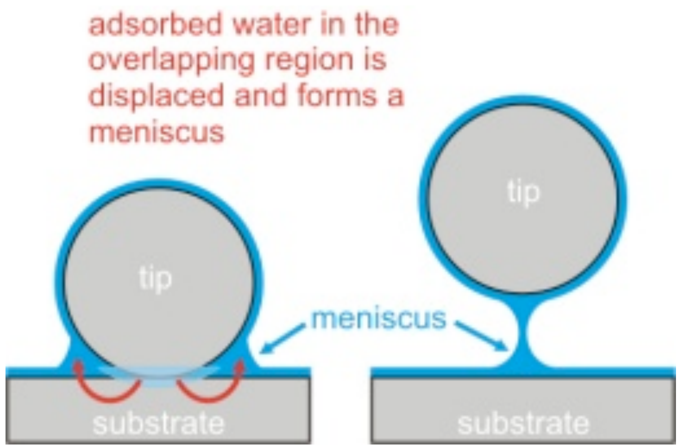
Effect of capillary condenstion





- Capillary force models range from simple to complex
- Strong dependence on humidity

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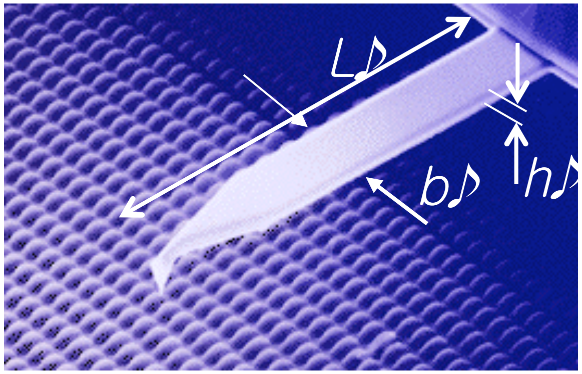
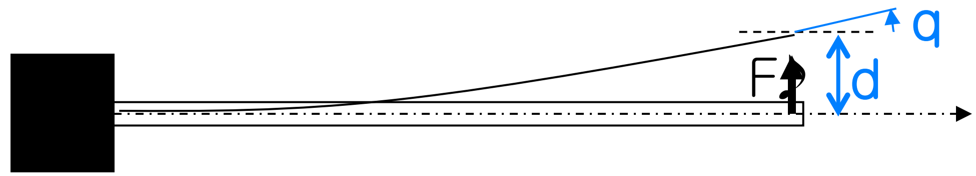
The microcantilever – the force sensor



www.olympus.co.jp♪



- From elementary beam theory, if E=Young's modulus, I =$bh^3/12$ then
- d=w(L)=F $L^3/(3EI)$, and q=dw(L)/dx=$FL^2/(2EI)$
- Deflection and slope linearly proportional to force sense d at the tip
- k=$3EI/L^3$ is called the bending stiffness of the cantileve r

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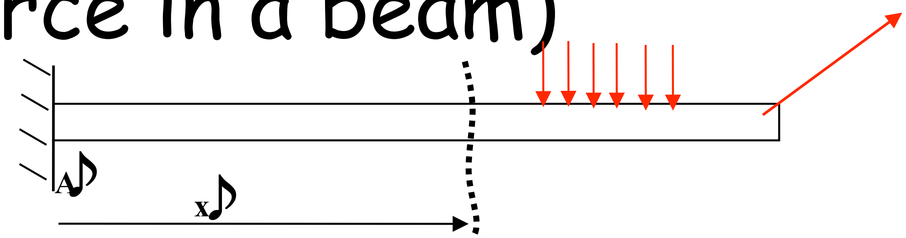
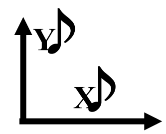
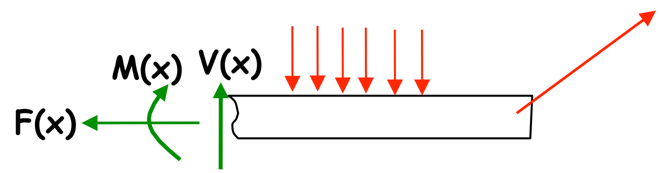
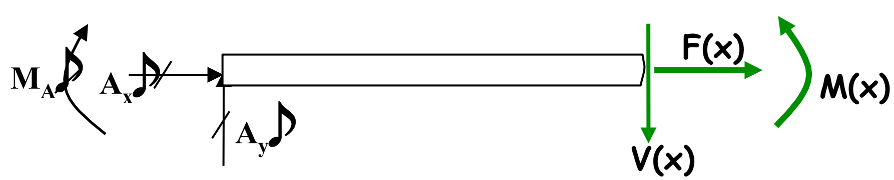
Classical beam theory

- Understanding internal resultants (shear force, bending moment and axial force in a beam)





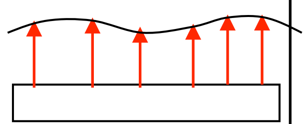
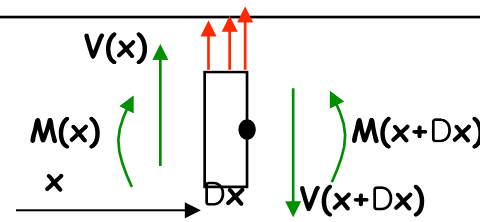




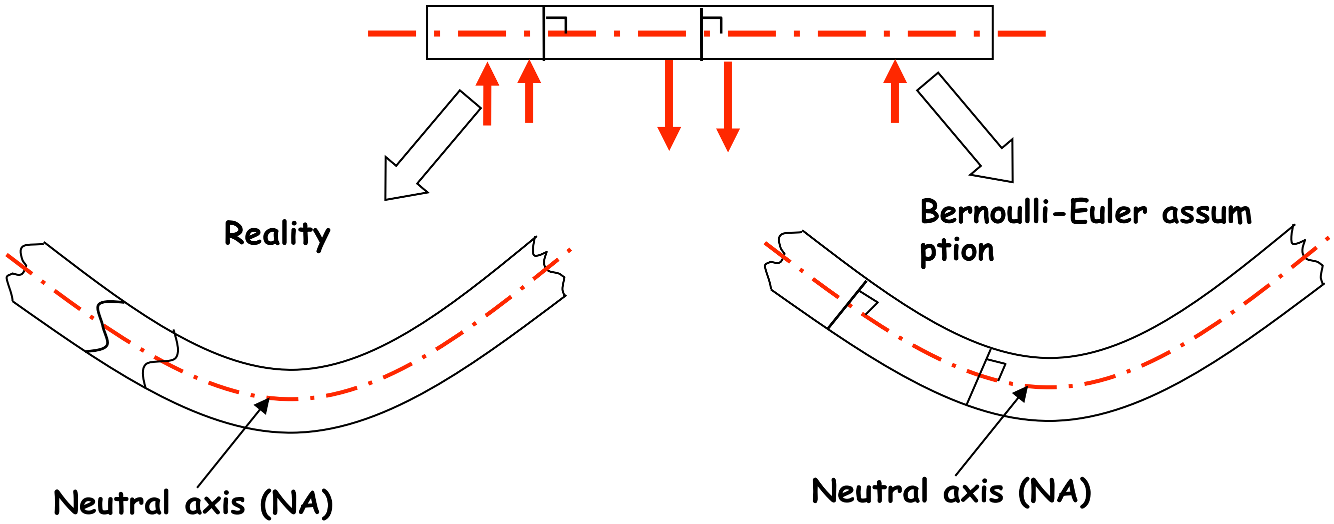
V(x) :Internal shear force (N)
F(x): Internal axial force (N)
M(x): Internal bending moment (N.m)

Classical beam theory

- Relationship between V(x) and M(x)

Key Equation	Applied Load	Derivation
$\frac{dV}{dx} = p(x)$	p(x) N/m 	 $V(x) + p(x)\Delta x - V(x+\Delta x) = 0,$ $as\ \Delta x \to 0\ we\ get\ \frac{dV}{dx} = p(x)$
$\frac{dM}{dx} = V(x)$		$M(x+\Delta x) - M(x) - V(x)\Delta x - p(x)\Delta x\left(\frac{\Delta x}{2}\right) = 0,$ $as\ \Delta x \to 0\ we\ get\ \frac{dM}{dx} = V(x)$

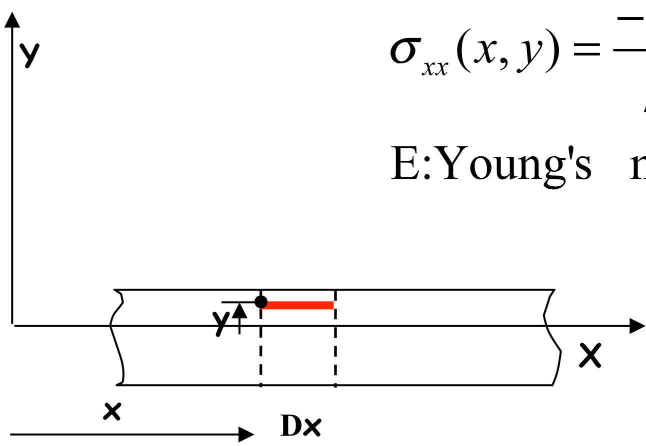
Classical beam theory- stress/strain



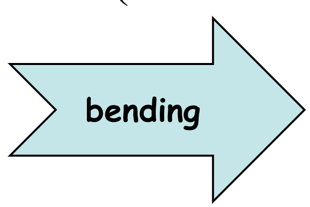
$$\varepsilon_{xx}(x,y) =_{\lim \Delta x \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta} = \frac{-y}{\rho}$$

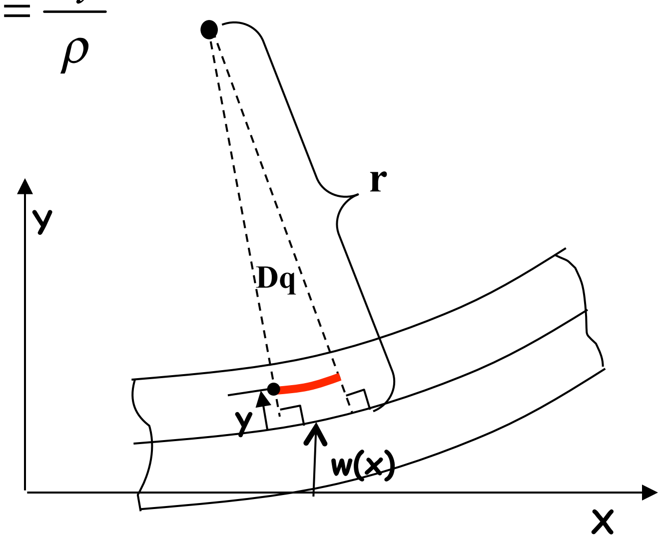
$$\sigma_{xx}(x,y) = \frac{-Ey}{\rho} \sim -Ey\frac{d^2 w}{dx^2}$$

$$\text{E:Young's modulus (N/m}^2\text{or Pa)}$$



Undeformed beam segment





Deformed beam segme
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- Key: s_{xx} varies linearly across section

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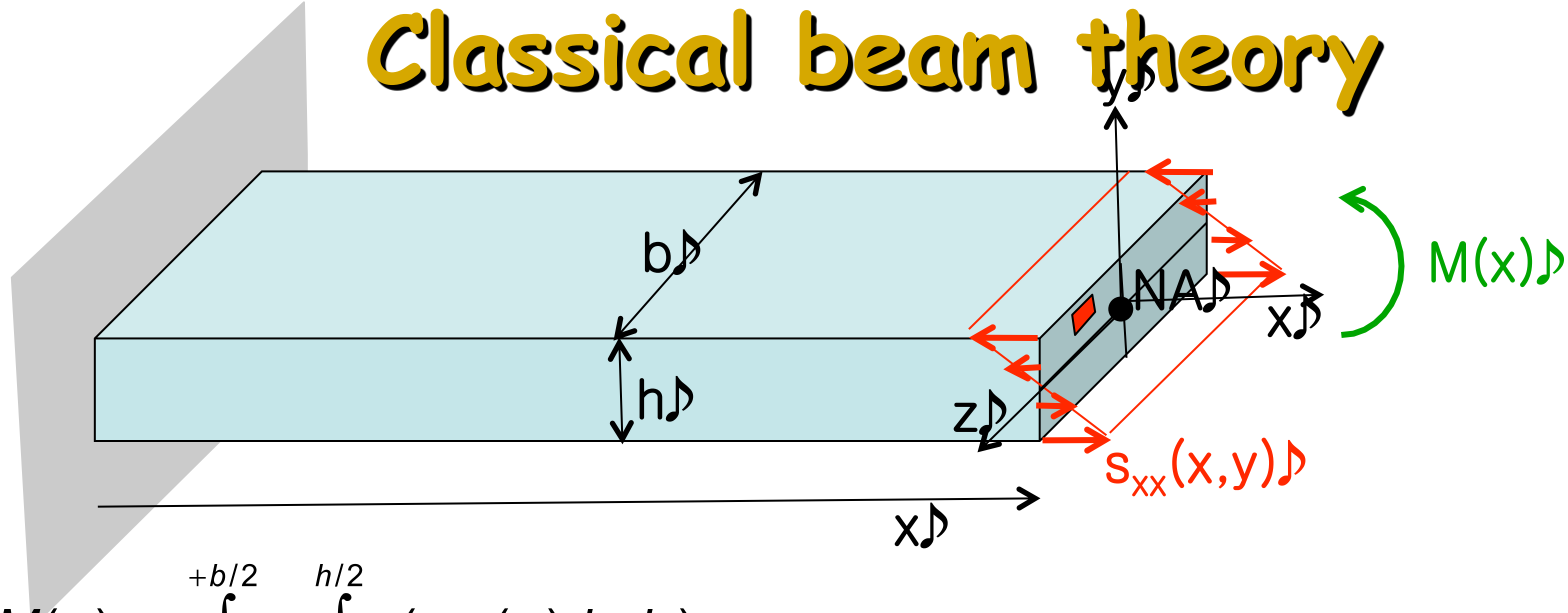
Classical beam theory



$$M(x) = \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{h/2} (\sigma_{xx}(x)dydz)y$$

$$or \quad M(x) = \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{h/2} Ey^2 \frac{d^2w(x)}{dx^2} dydz = E\frac{d^2w(x)}{dx^2} \int_{z=-b/2}^{+b/2} \int_{y=-h/2}^{h/2} y^2 dydz$$

$$or \quad M(x) = EI_{zz}\frac{d^2w(x)}{dx^2} \quad where \ I_{zz} = \frac{bh^3}{12} \quad (area\ moment\ m^4)$$

$$Likewise \quad V(x) = EI_{zz}\frac{d^3w(x)}{dx^3}$$

$$Finally \quad EI\frac{d^4w(x)}{dx^4} = p(x) \ to\ be\ solved\ with\ boundary\ conditions\ at\ x = 0, L$$

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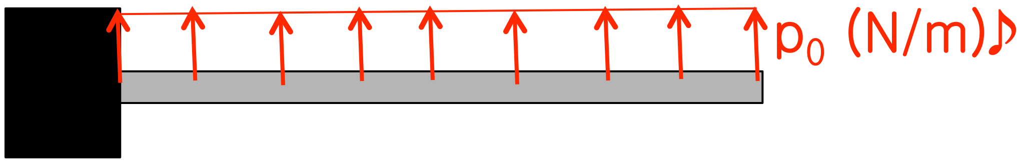
Classical beam theory

- Example1



$$EI\frac{d^4w(x)}{dx^4} = p_0 \Rightarrow \frac{d^3w(x)}{dx^3} = \frac{p_0}{EI}x + c_1$$

$$\Rightarrow \frac{d^2w(x)}{dx^2} = \frac{1}{2}\frac{p_0}{EI}x^2 + c_1x + c_2 \Rightarrow \frac{dw(x)}{dx} = \theta(x) = \frac{1}{6}\frac{p_0}{EI}x^3 + \frac{1}{2}c_1x^2 + c_2x + c_3$$

$$w(x) = \frac{1}{24}\frac{p_0}{EI}x^4 + \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

Boundary conditions

$$w(0) = \theta(0) = 0 \quad EI\frac{d^2w(L)}{dx^2} = EI\frac{d^3w(L)}{dx^3} = 0$$

(*no point moments or force applied at* $x = L$)

$$\Rightarrow c_3 = c_4 = 0,\ c_1 = -\frac{p_0L}{EI}, c_2 = \frac{1}{2}\frac{p_0L^2}{EI}$$

$$w(L) = \delta = \frac{1}{8}\frac{p_0L^4}{EI}, \quad \theta(L) = \theta = \frac{1}{6}\frac{p_0L^3}{EI} \Rightarrow \frac{\theta}{\delta} = \frac{4}{3}L$$

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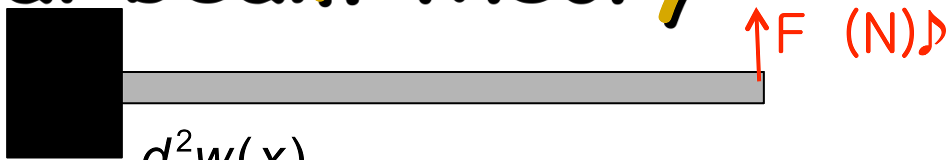
Classical beam theory

- Example2



$$EI\frac{d^4w(x)}{dx^4} = 0 \Rightarrow \frac{d^3w(x)}{dx^3} = c_1 \Rightarrow \frac{d^2w(x)}{dx^2} = c_1x + c_2$$

$$\Rightarrow \frac{dw(x)}{dx} = \theta(x) = \frac{1}{2}c_1x^2 + c_2x + c_3$$

$$w(x) = \frac{1}{6}c_1x^3 + \frac{1}{2}c_2x^2 + c_3x + c_4$$

Boundary conditions

$$w(0) = \theta(0) = 0 \quad EI\frac{d^2w(L)}{dx^2} = 0 \; EI\frac{d^3w(L)}{dx^3} = -F$$

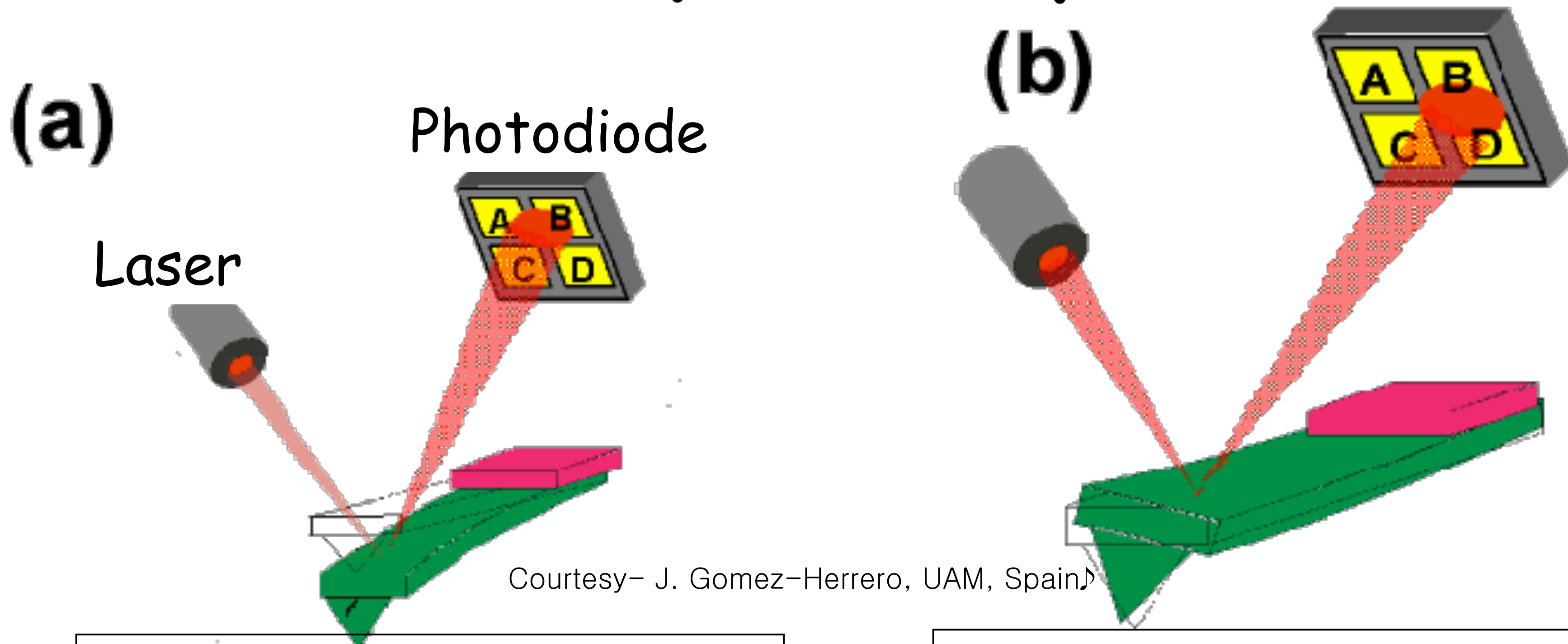
(*no point moment applied at* $x = L$)

$$\Rightarrow c_3 = c_4 = 0,\; c_1 = -\frac{F}{EI}, c_2 = \frac{FL}{EI}$$

$$w(L) = \delta = \frac{1}{3}\frac{FL^3}{EI}, \quad \theta(L) = \theta = \frac{1}{2}\frac{FL^2}{EI} \Rightarrow \frac{\theta}{\delta} = \frac{2}{3}L$$

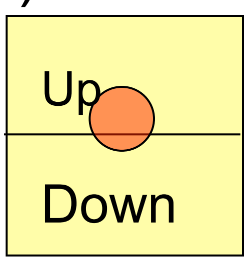
$F = k\delta$, *where* $k = \dfrac{3EI}{L^3}$ *is the static bending stiffness of thecantilever*

The four-quadrant photodiode



Courtesy– J. Gomez–Herrero, UAM, Spain

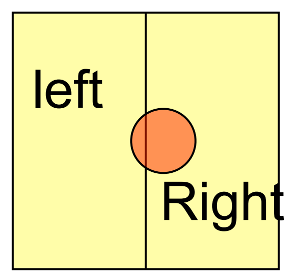
a) Vertical bending



A+B= UP

C+D=DOWN

b)Lateral/torsion motion

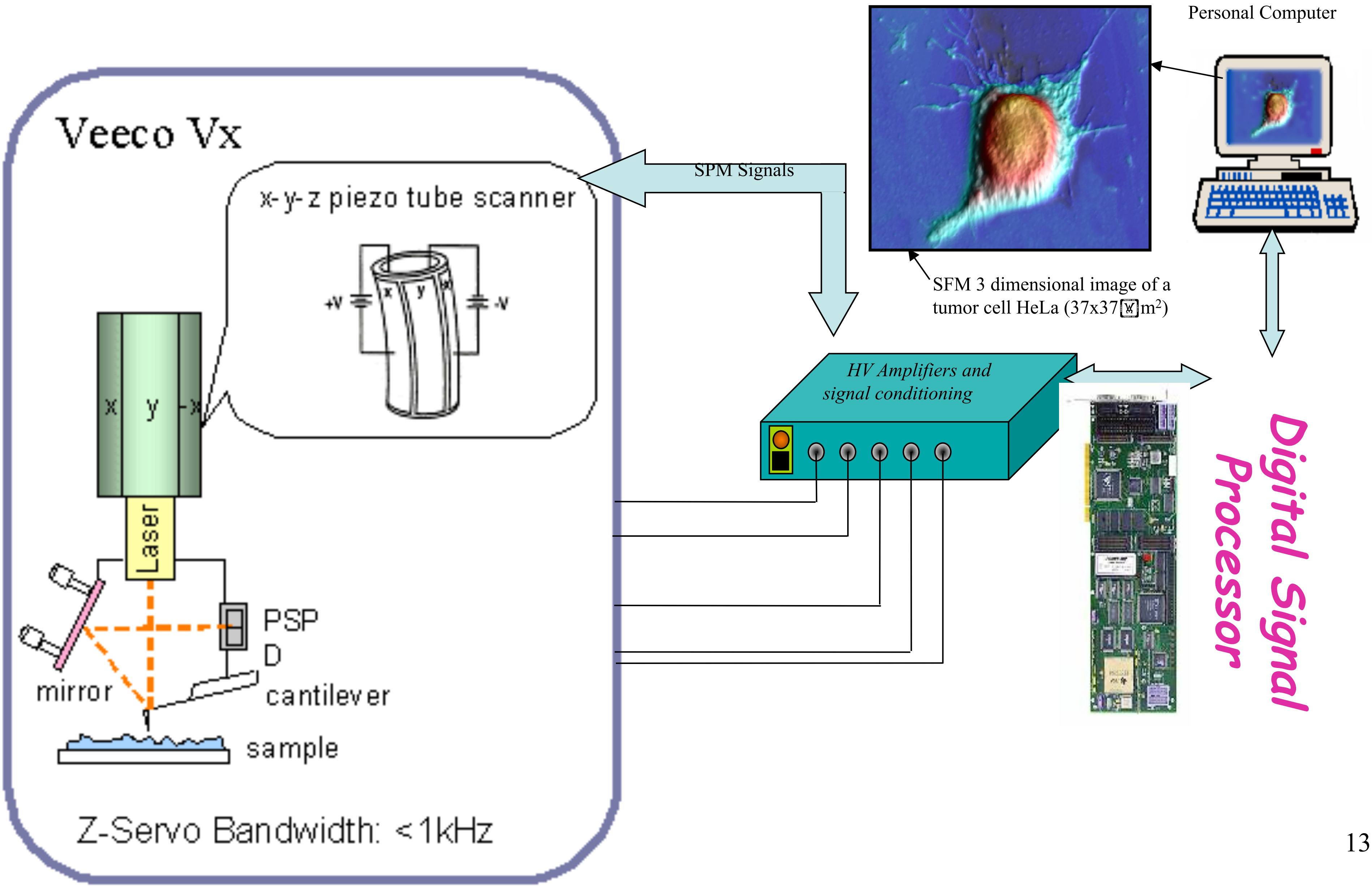


A+C= LEFT

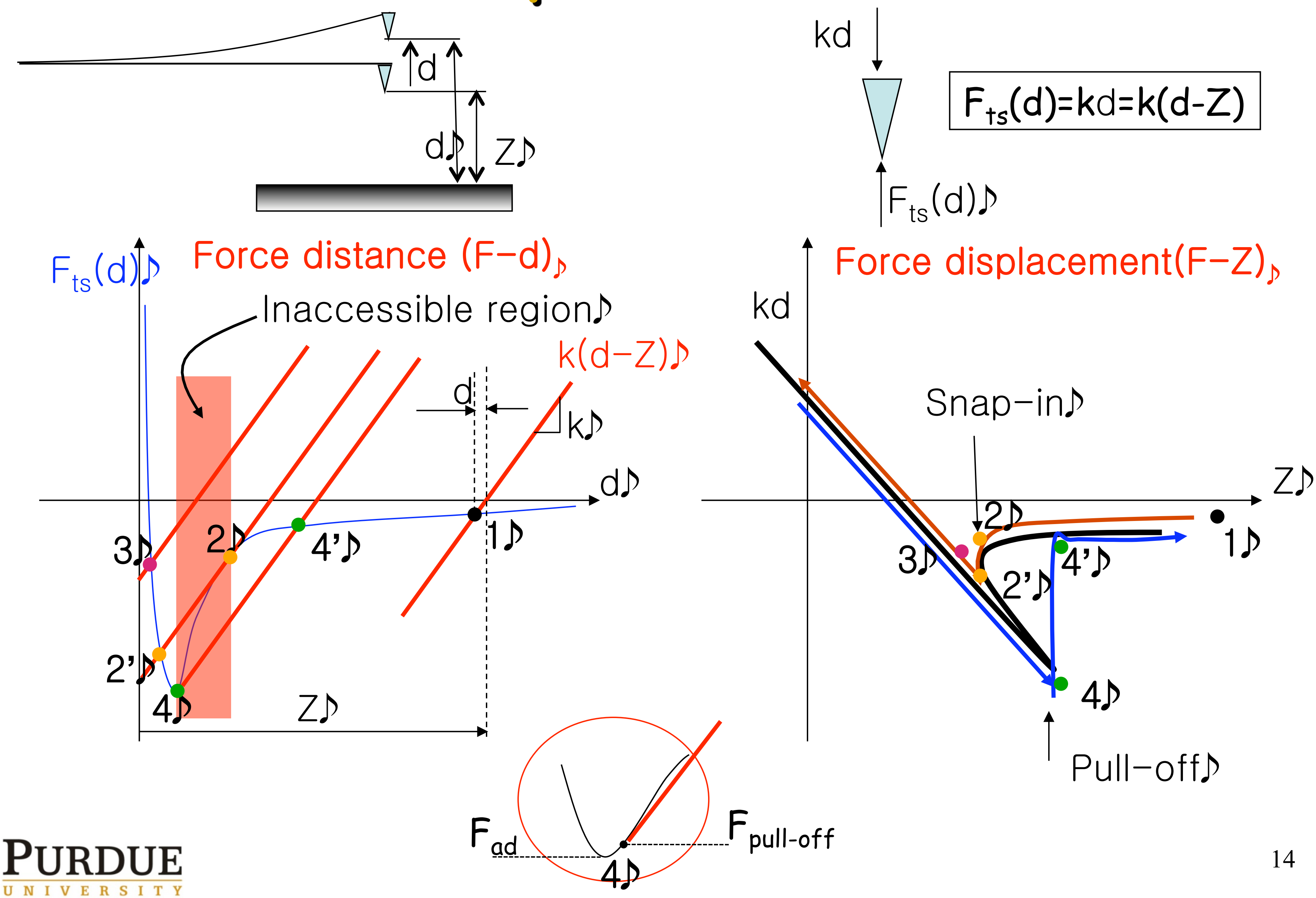
B+D=Right

Commercial AFMs measure the rotation angle!!! (bending or torsion)

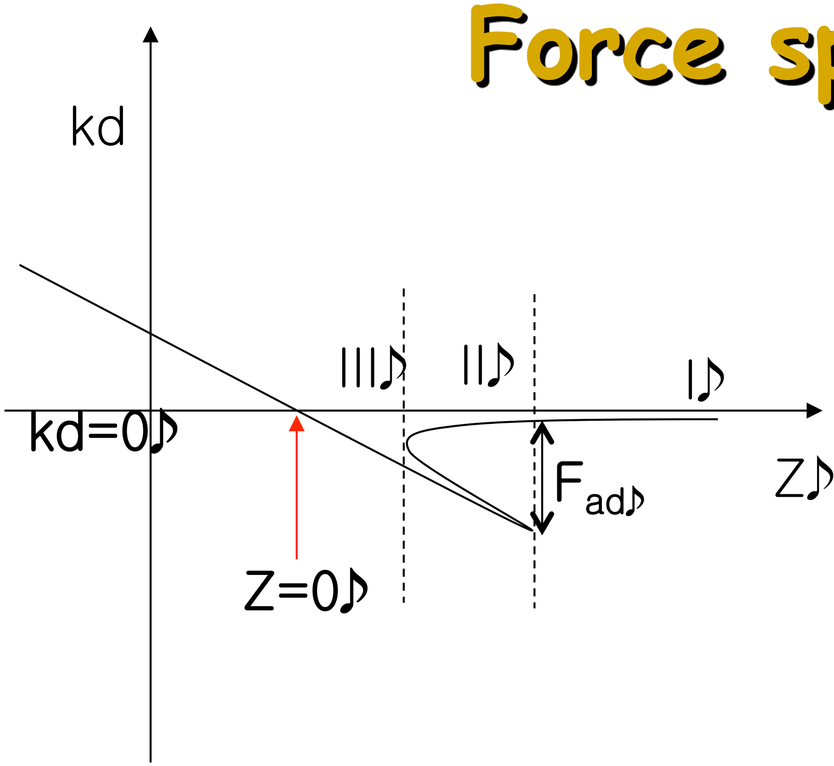
AFM Block Diagram

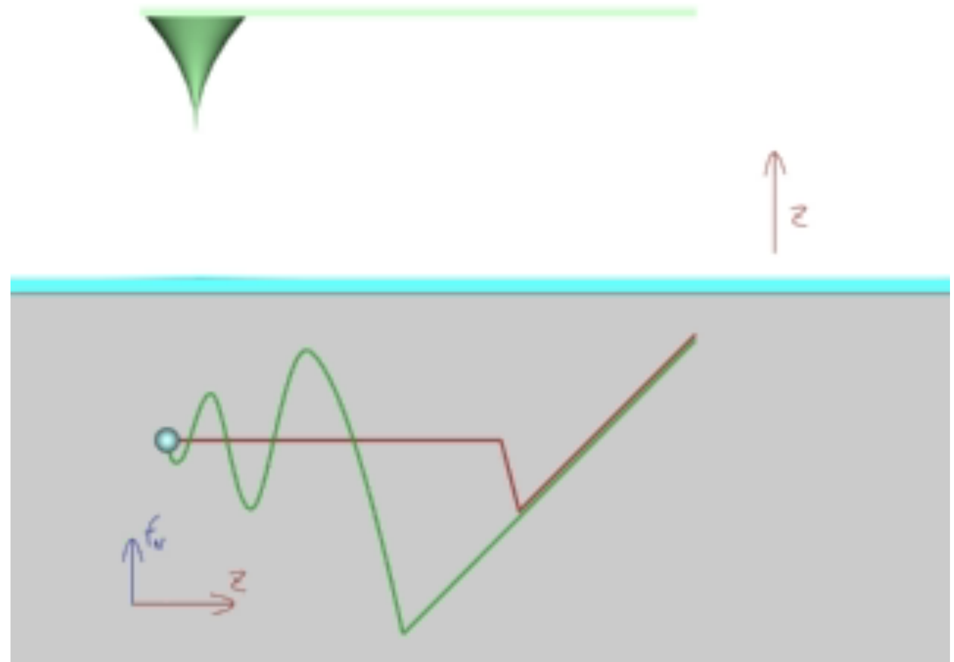


Force-displacement curves



Force spectroscopy





- Three distinct regions
- If k is known then from the static-force distance curve, F(d) can be calculated for all d except for inaccesible range near snap -in
- Pull-off force ~ F_{Ad} which can be converted to W_{132} (work of adhesion between two infinitely wide planes)
- Slope in III is good measure of repulsive forces (local elasticity)

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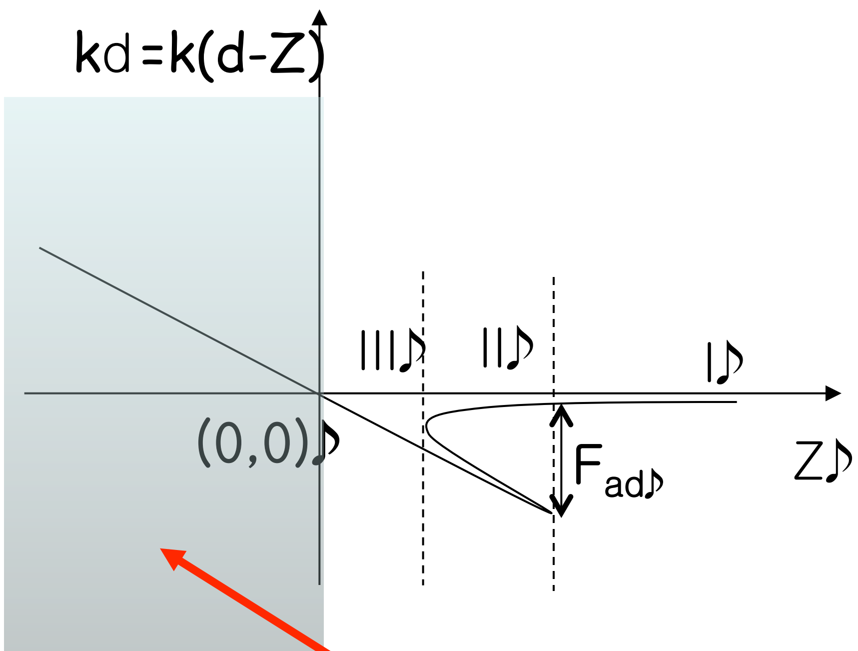
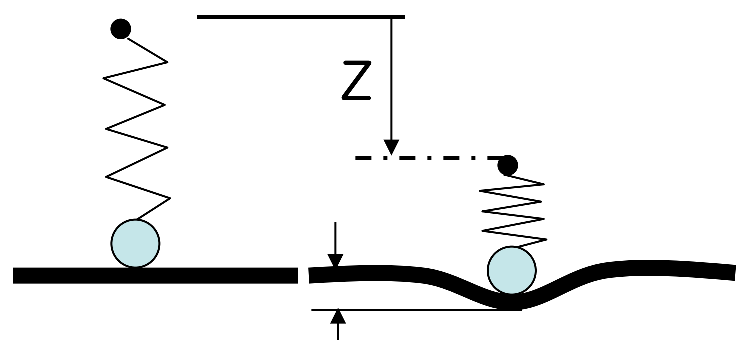
Animation courtesy J. Gomez-Herrero, UAM, Spain

How to extract force-indentation curves from force-distance curves

- Often AFM is used to measure the nanoscale mechanical properties of soft materials such as polymer films, biological tissue etc
- For such materials we would like to know the so called f orce-indentation curves
- With the calibrated force-distance curve, first recenter the Z=0 w hen cantilever deflection is zero (not necessary, but often done)



Data regime where local elasticity is calculated



Measured data: k(d-Z) vs. Z
Convert to: k(d-Z) vs. d

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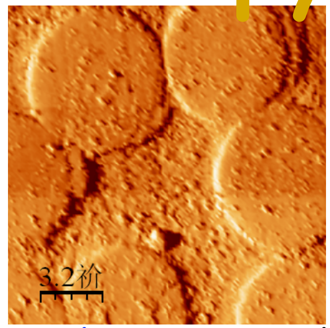
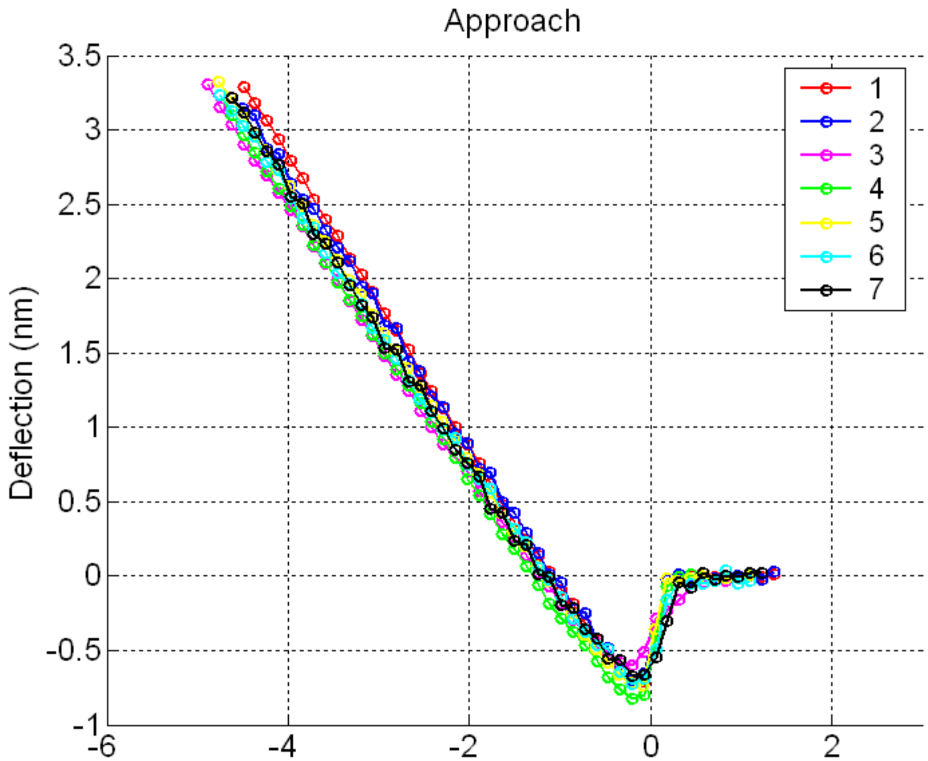
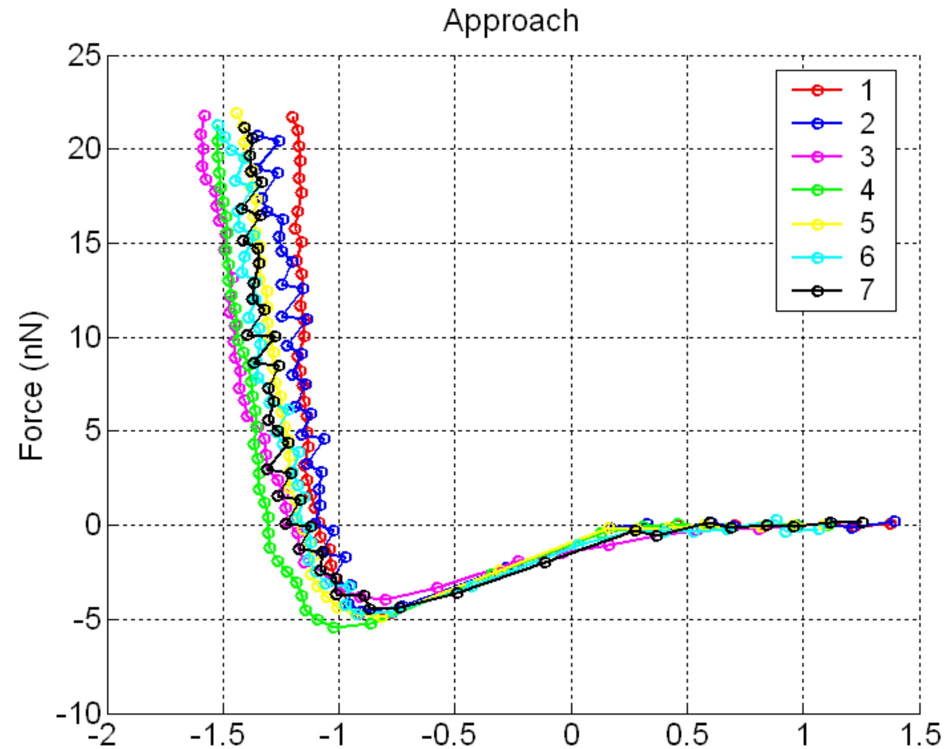
Force spectroscopy - an example



Convert deflection vs. displacement curves to force vs. distance (gap) curves



Piezo displacement Z (nm)



Tip-sample distance d (nm)

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Usually d axis is recentered to zero where force is a minimum

Next class

- Practical aspects of force distance curves