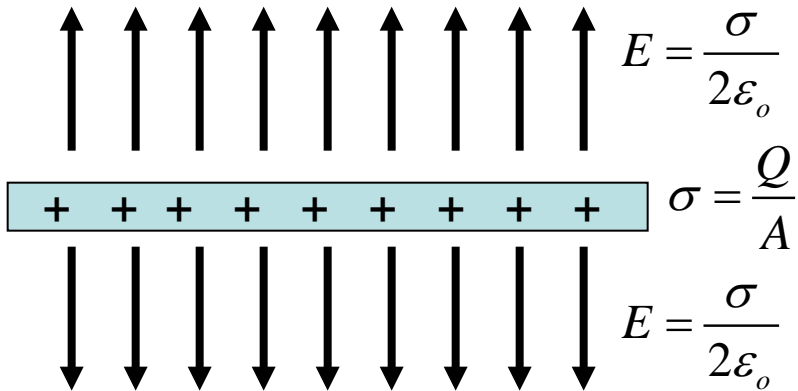


ME597/PHYS57000  
Fall Semester 2009  
Lecture 25

Using the AFM to Measure  
Electrostatic Forces

# Electrostatics of Charged Plates

Charge +Q added to plate of area A



$$E = 0$$

$$\left. \vphantom{\begin{matrix} E = 0 \\ E = 0 \end{matrix}} \right\} E_{net} = \frac{\sigma}{\epsilon_0}$$

$$E = 0$$

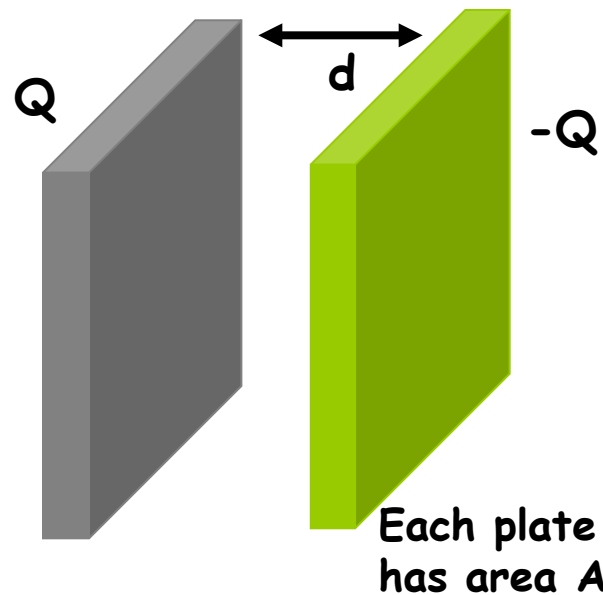
2 clicks

# Standard results - Parallel Plates

$$E_{net} = \frac{\sigma}{\epsilon_o}; \quad V = E_{net} d = \frac{Q}{\epsilon_o A} d$$

$$Q = \epsilon_o A \frac{V}{d}; \quad C \equiv \frac{Q}{V} = \frac{\epsilon_o A}{d};$$

$$\Rightarrow C = \frac{\epsilon_o A}{z}; \quad \left[ \frac{dC}{dz} \right]_{z=d} = \left[ -\frac{\epsilon_o A}{z^2} \right]_{z=d} = -\frac{\epsilon_o A}{d^2}$$



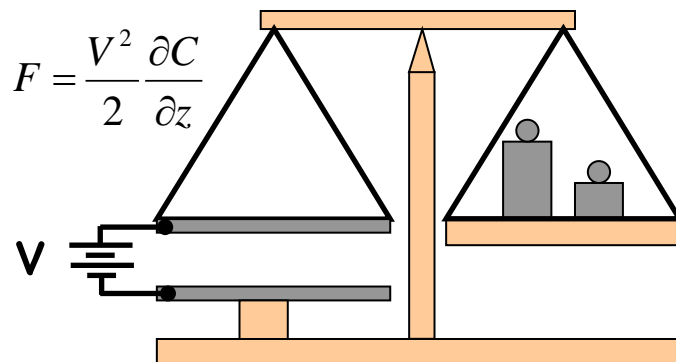
$$|F| = |q_{right\ plate} E_{left\ plate}| = Q \frac{Q}{2\epsilon_o A} = \frac{1}{2} \left( \frac{\epsilon_o A}{d^2} \right) V^2 \quad (\text{attractive})$$

(for finite size, circular plates with separation  $d$  and diameter  $D$ ):

$$F \approx \frac{\epsilon_o A V^2}{2d^2} \left( 1 + \frac{2d}{D} + \dots \right)$$

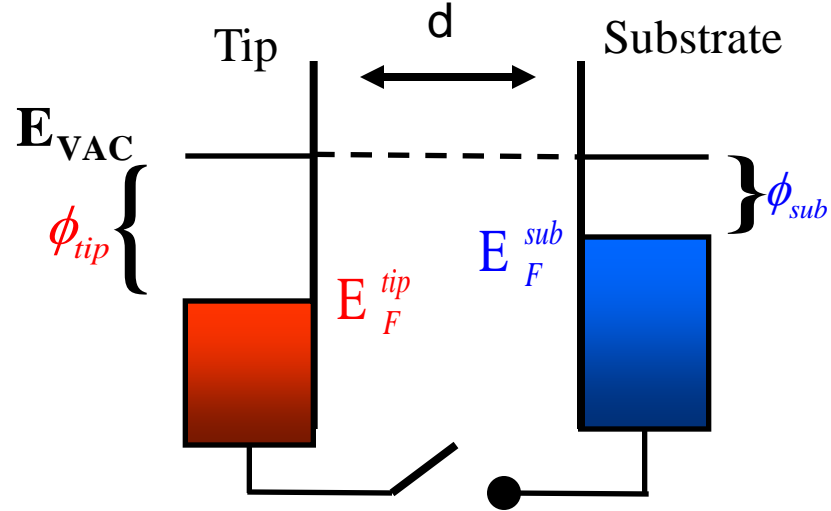
in general

$$F = -\frac{1}{2} \frac{dC}{dz} V^2$$

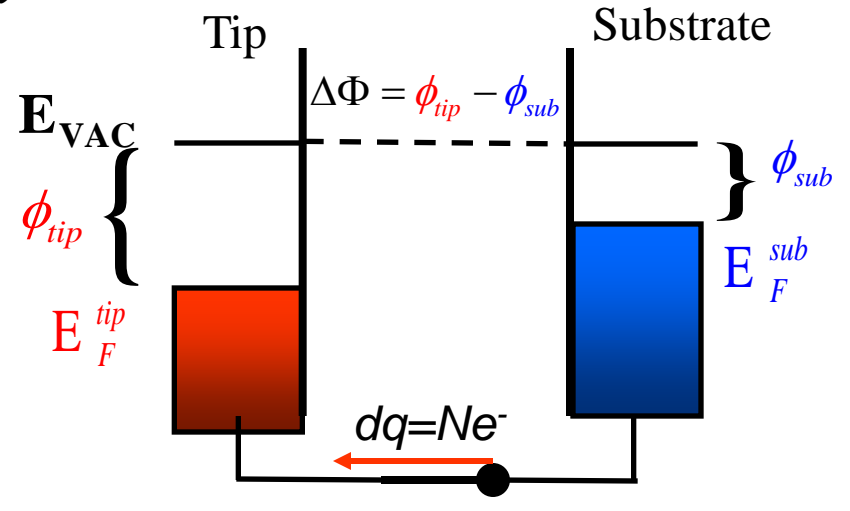


# Contact Potential Difference (CPD)

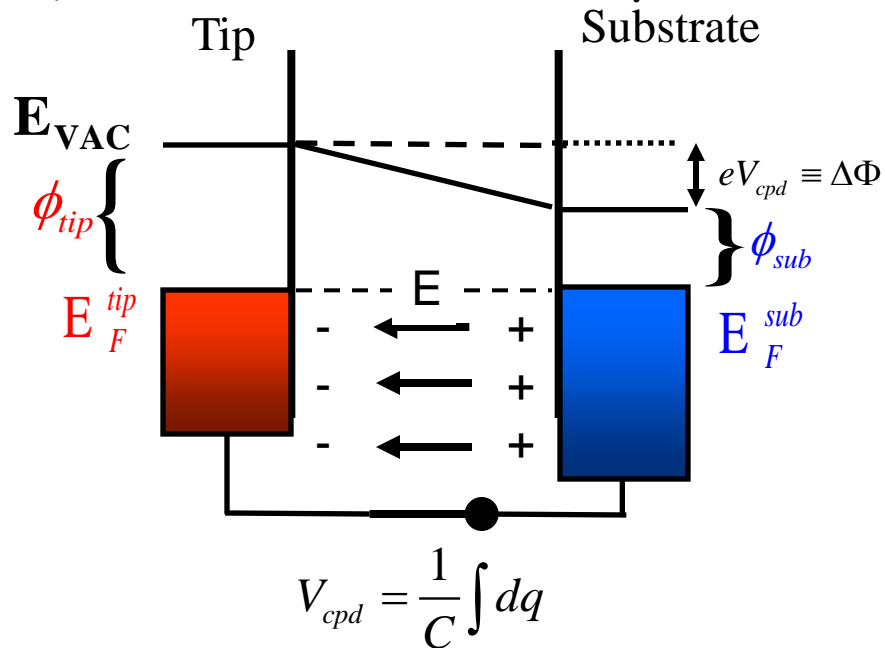
a) Work function difference



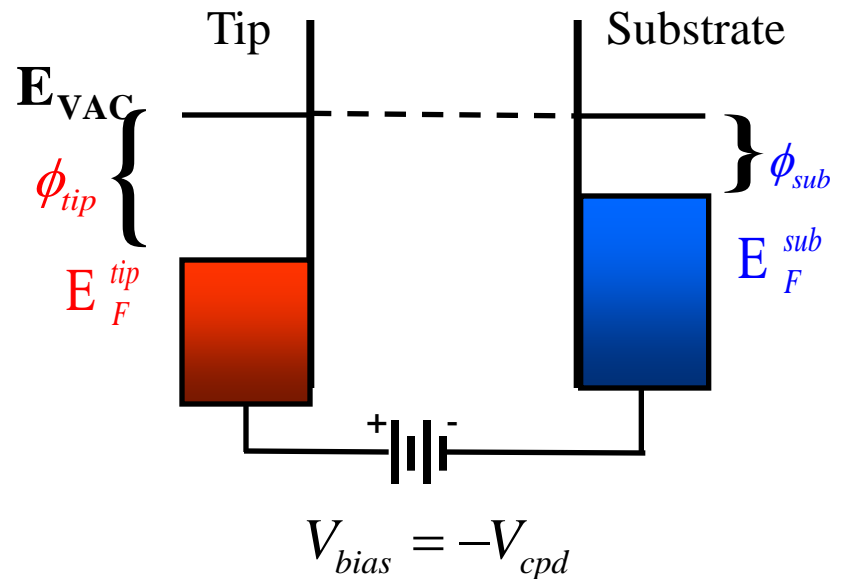
b) Establish electrical connection



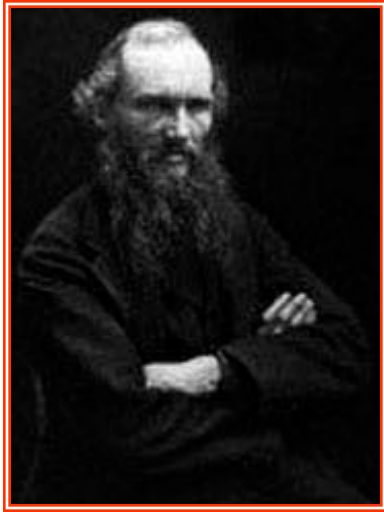
c) Electrostatic field develops



d) Electrostatic field nullified

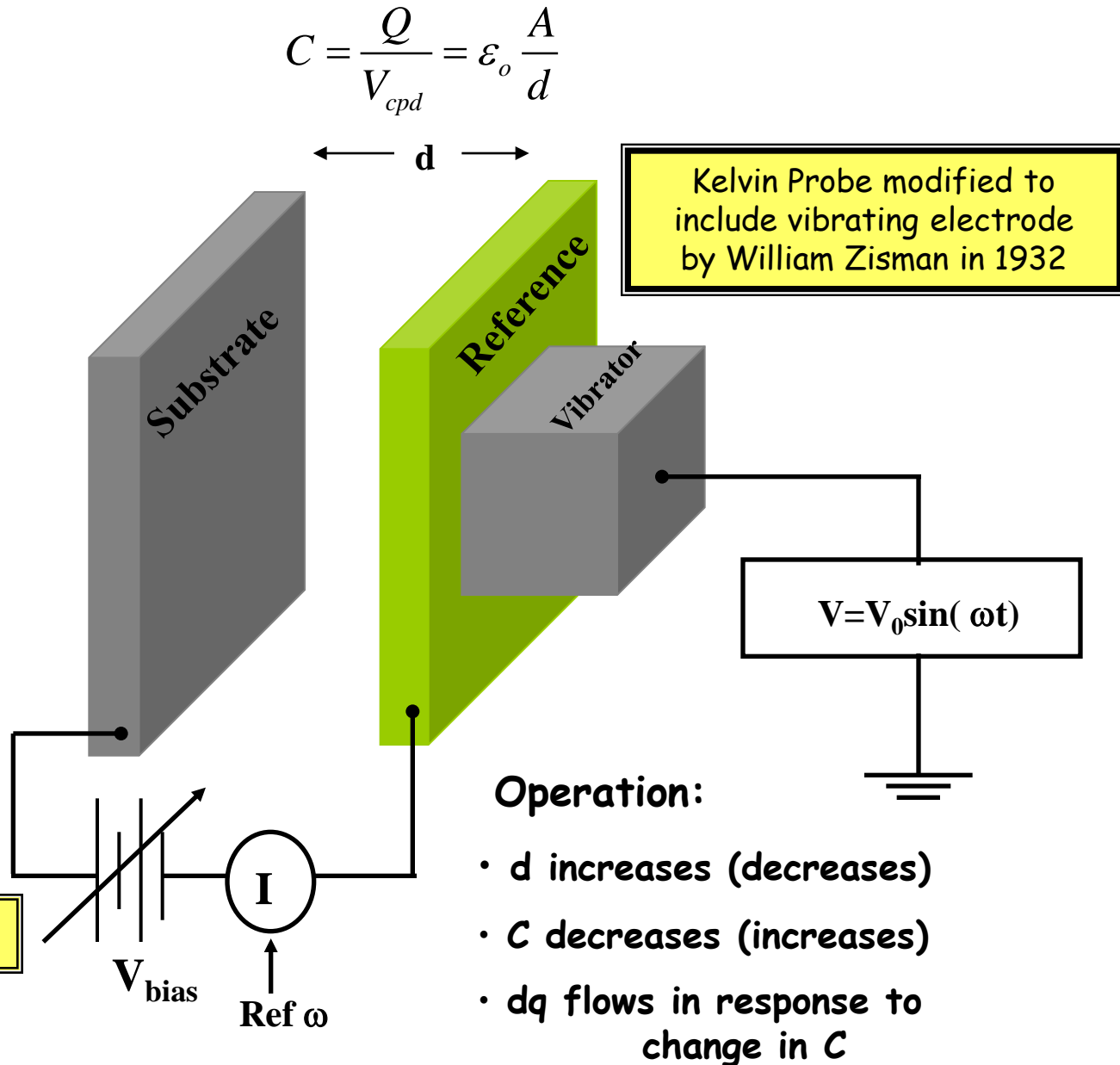


# Macroscopic Kelvin Probe Measures CPD

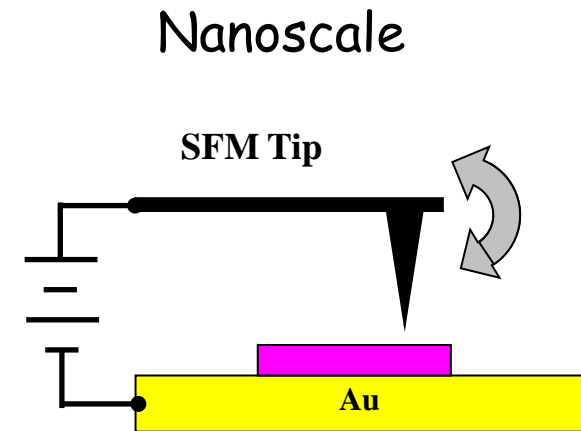
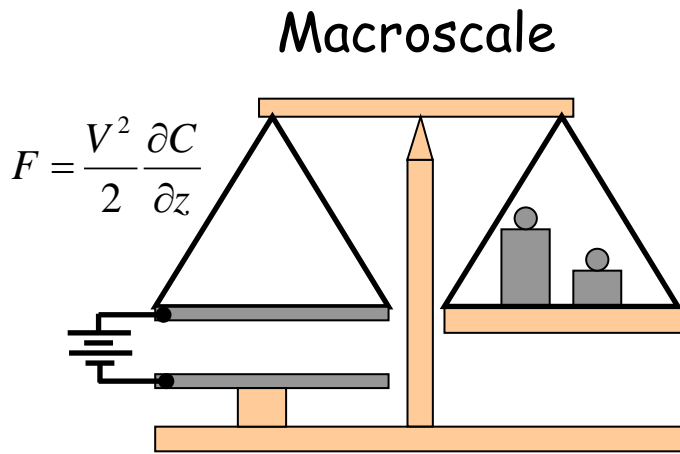


- Sir Wm. Thomson, Lord Kelvin, 1861
- Non-contact
- Non-destructive
- ~1 mV resolution (best case)

Adjust  $V_{\text{bias}}$  so  $I = 0$

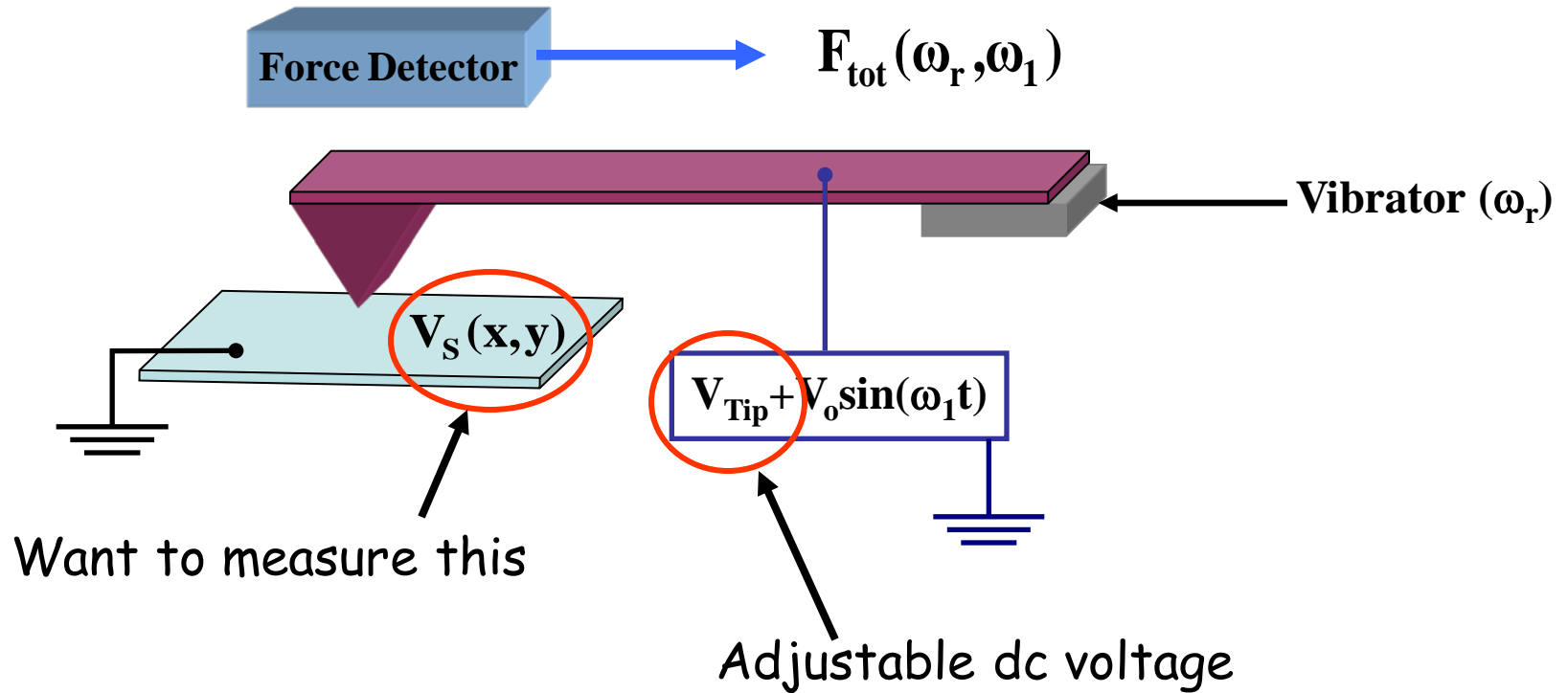


# Electrostatic Force Microscopy - The Basic Idea



M. Nonnenmacher, M. P. O'Boyle, and  
H. K. Wickramasinghe, *Appl. Phys.  
Lett.* **58**, 2921 (1991).

# The Basic Idea



The tip-sample potential difference is:

$$\Delta V = V_S(x, y) - [V_{\text{Tip}} + V_o \sin(\omega_1 t)]$$

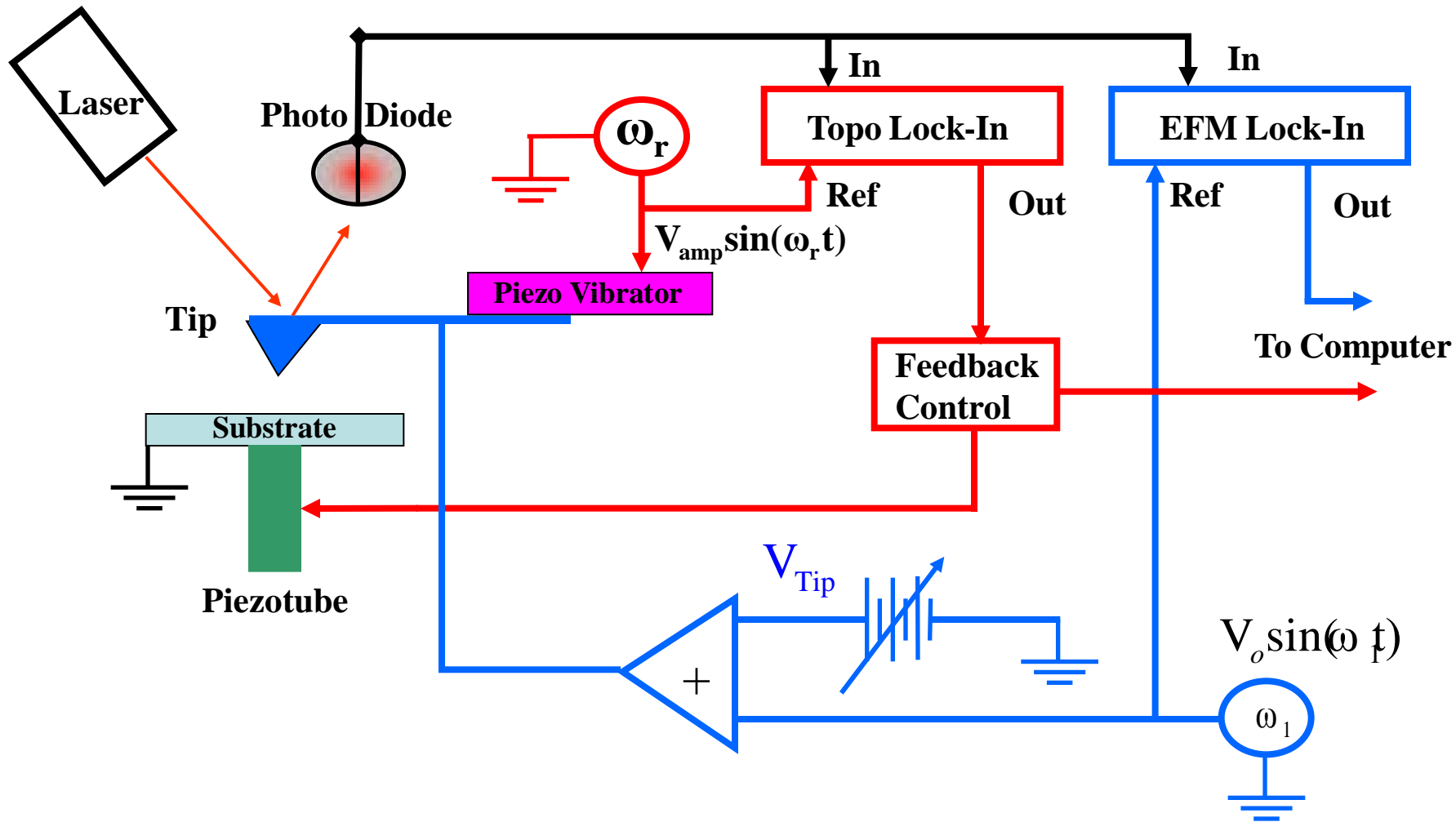
The force acting on the cantilever due to the tip-sample capacitance gradient is:

$$\begin{aligned}
 F_{electrostatic} &= -\frac{1}{2} \frac{dC}{dz} (\Delta V)^2 \\
 &= -\frac{1}{2} \frac{dC}{dz} \left[ V_s(x, y) - (V_{tip} + V_o \sin(\omega_1 t)) \right]^2 \\
 &= -\frac{1}{2} \frac{dC}{dz} \left[ \underbrace{(V_s(x, y) - V_{tip})^2}_{\text{dc term}} + \underbrace{V_o^2 \sin^2(\omega_1 t)}_{2\omega_1 \text{ term}} - \underbrace{2(V_s(x, y) - V_{tip}) V_o \sin(\omega_1 t)}_{\omega_1 \text{ term}} \right]
 \end{aligned}$$

$$F|_{\omega_1} = \frac{dC}{dz} (V_s(x, y) - V_{tip}) V_o \sin(\omega_1 t)$$



# Experimental Set-Up



The **EFM lock-in** measures the amplitude of the  $\omega_1$  component: 
$$\text{Amp} |_{\omega_1} = \frac{dC}{dz} V_o [V_s(x, y) - V_{tip}]$$

# Operational Modes

## 1) EFM Imaging Mode:

No Controlled modification of  $V_{\text{Tip}}$  ( $V_{\text{Tip}}$  is held constant).

Passively record the output of the **EFM lock-in** (the detected electrostatic force) as a function of position.

## 2) KFM Imaging Mode:

A feedback circuit modifies  $V_{\text{Tip}}$  to minimize the output of the **EFM lock-in** (eliminating the  $\omega_1$  component of the electrostatic force).

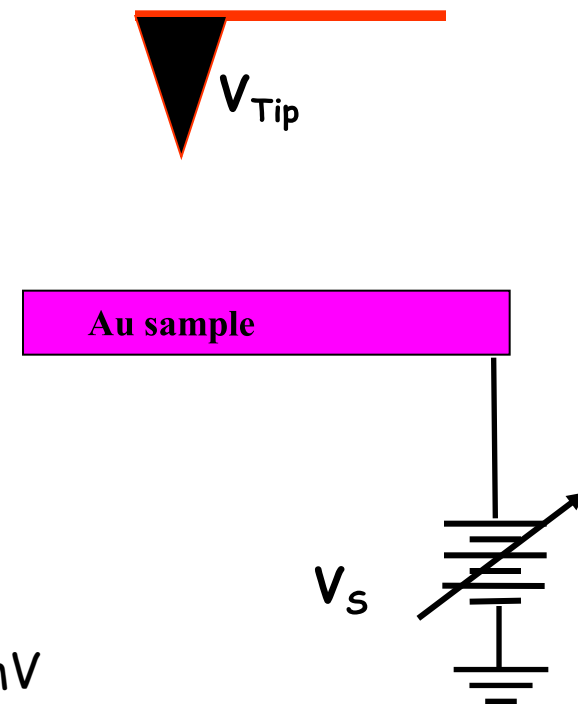
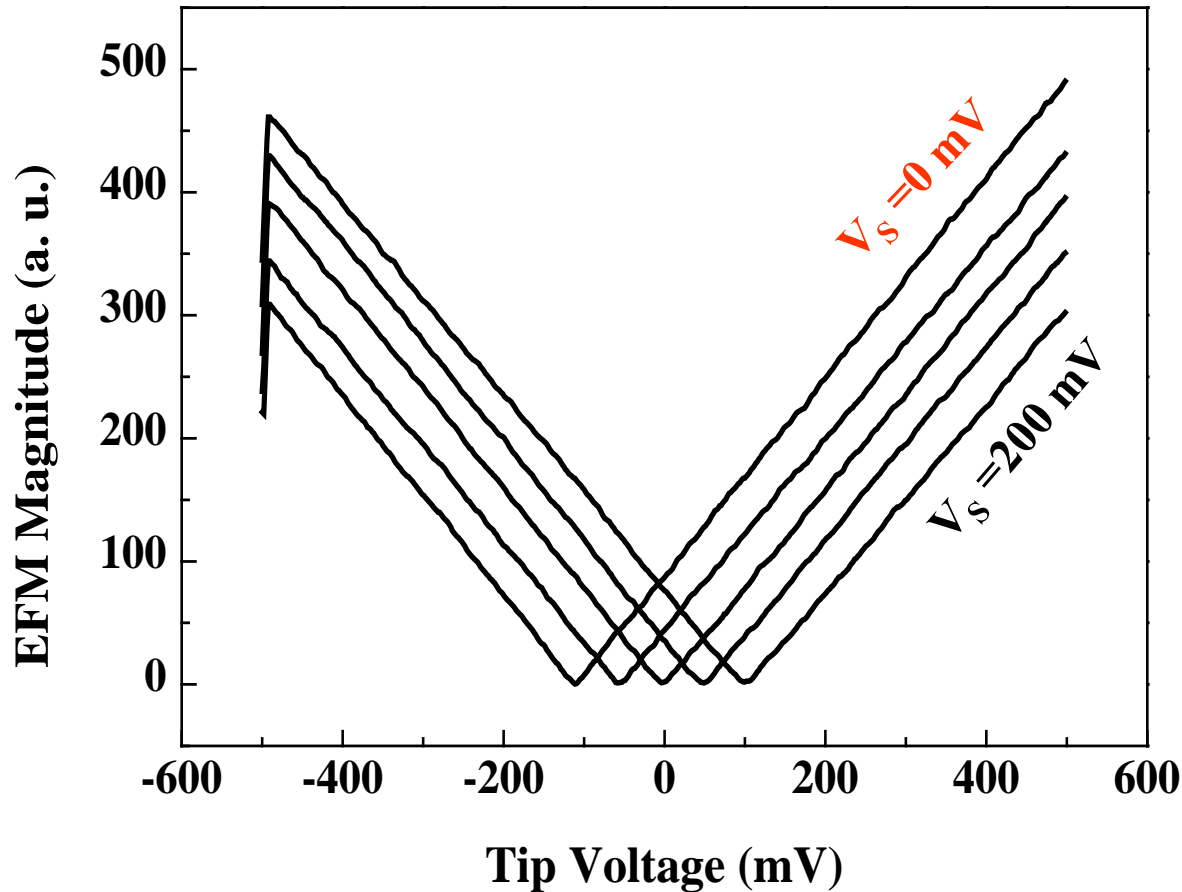
Recording  $V_{\text{Tip}}$  as a function of position produces a map of the electrostatic surface potential.

## 3) Electrostatic Force Curve:

Tip is positioned over a region of the sample.

The output of the **EFM lock-in** (the magnitude of the  $\omega_1$  component) is measured as a function of  $V_{\text{Tip}}$ .

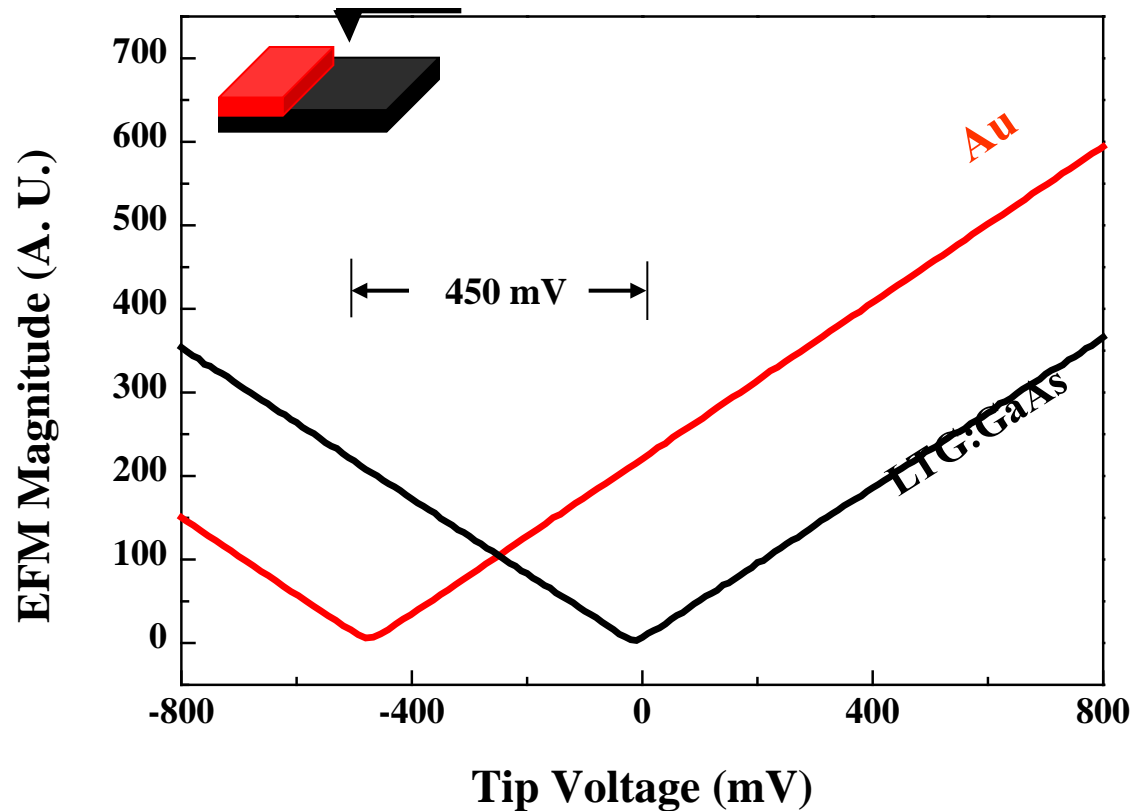
# Testing the Experimental Technique (no scan)



- Sample potential was intentionally increased by +50 mV increments by connecting it to a voltage source  $V_s$ .

# Comparison Between Different Regions (no scan)

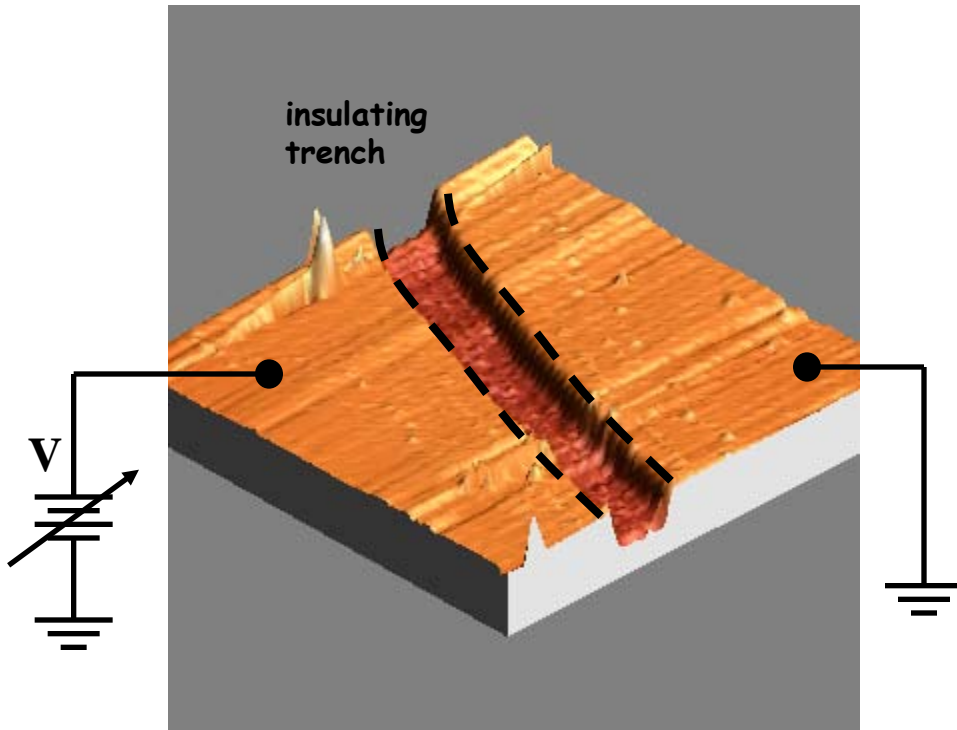
Contact Potential Difference Test



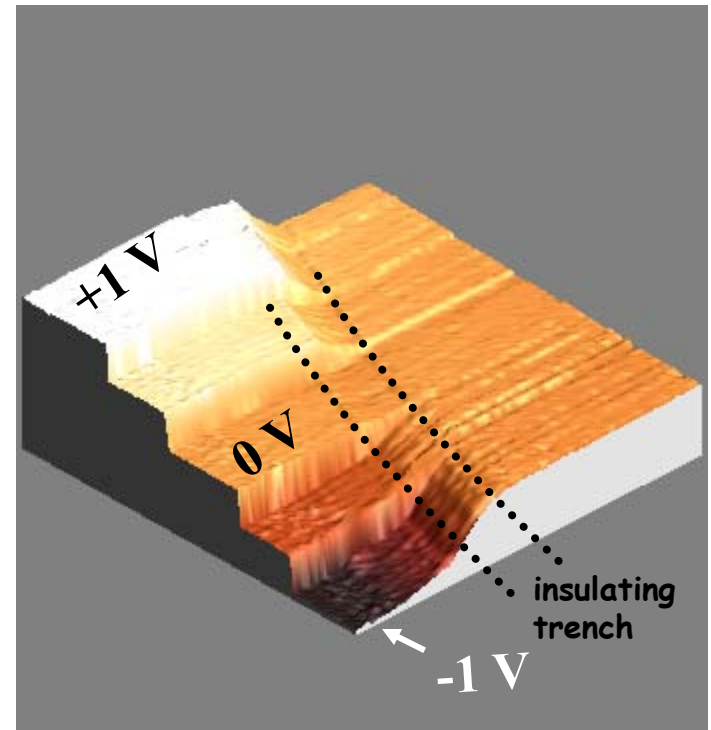
Work function difference between different regions on same sample can be determined using the same tip.

# Scanning Test Between Two Au Electrodes (voltage offset adjusted by hand)

## Topography



## KFM Image



Scan Size:  $30\ \mu\text{m} \times 30\ \mu\text{m}$

# Topographic and Electrostatic Force Scans of a Single 20 nm Au Cluster on LTG:GaAs

Topography

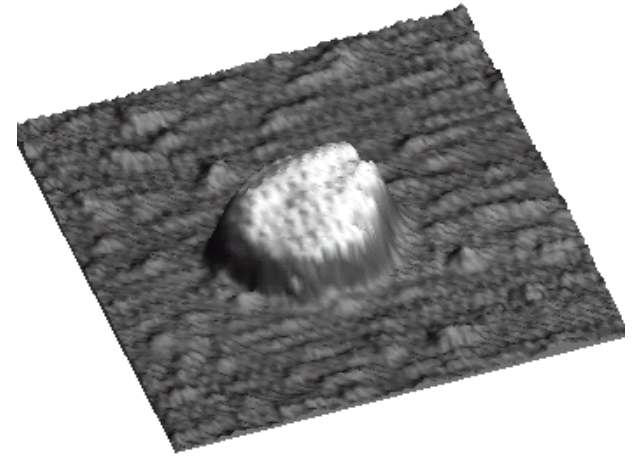


Scan Size: 100 nm × 100 nm

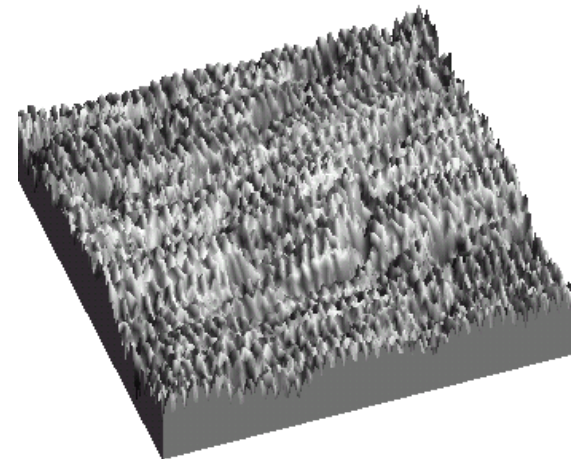
When  $V_{\text{Tip}} = -230$  mV, the electrostatic force over the substrate is the same as over the cluster .

The estimated charge on the cluster is  $\sim 1.1 \times 10^{-18}$  C. This corresponds to 7 electron charges.

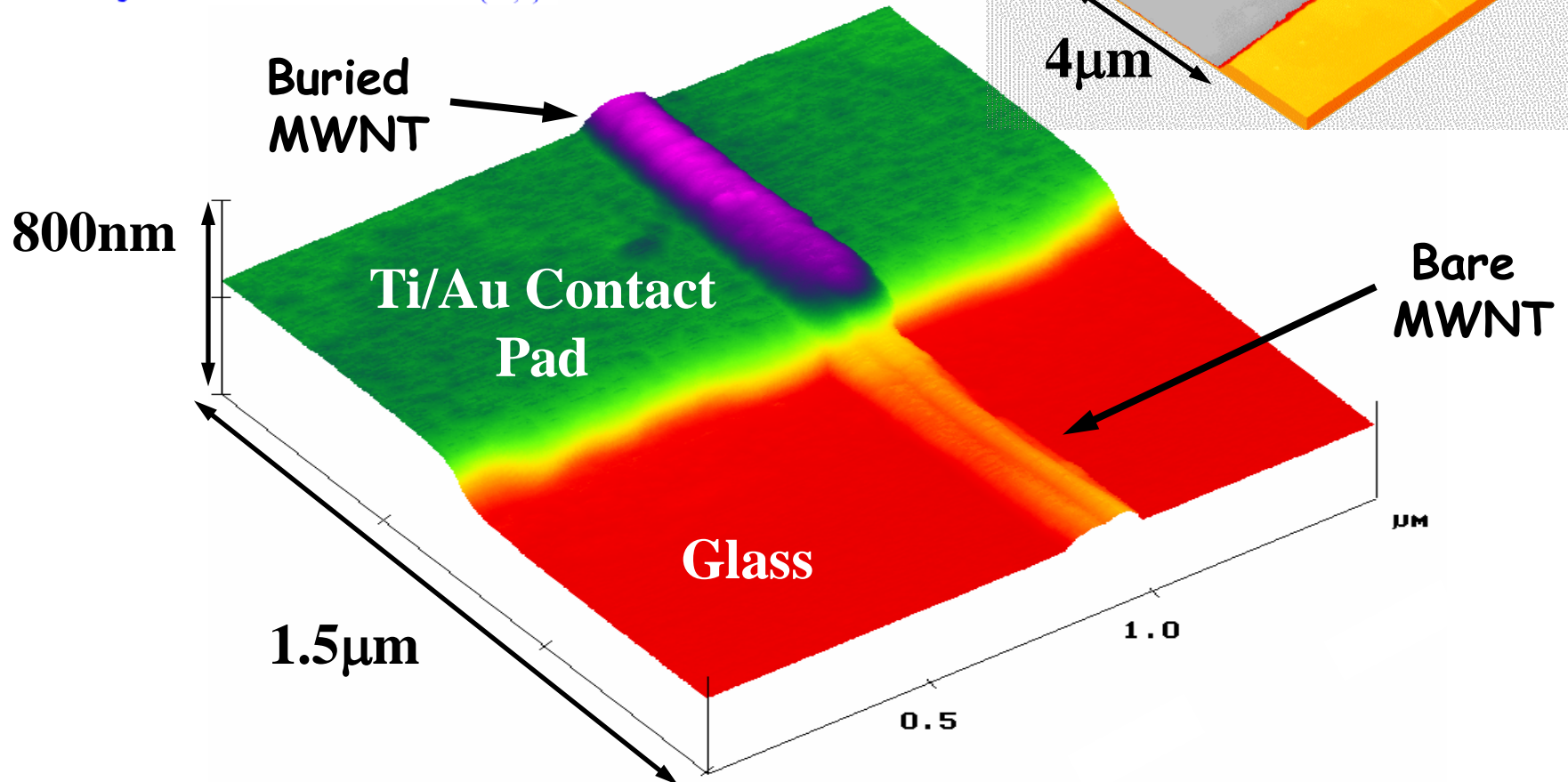
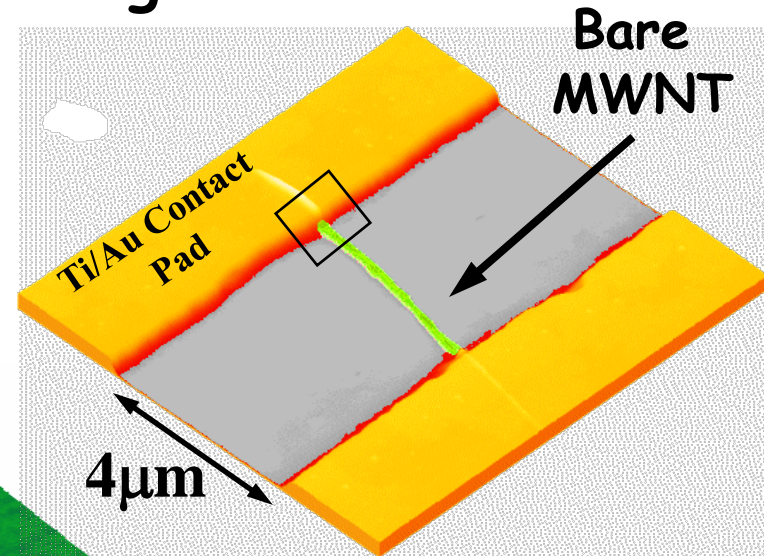
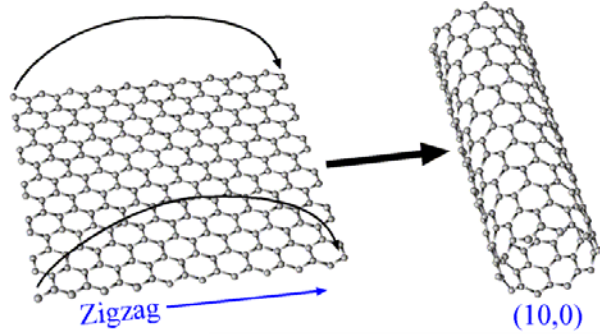
EFM Image:  $V_{\text{Tip}} = +670$  mV



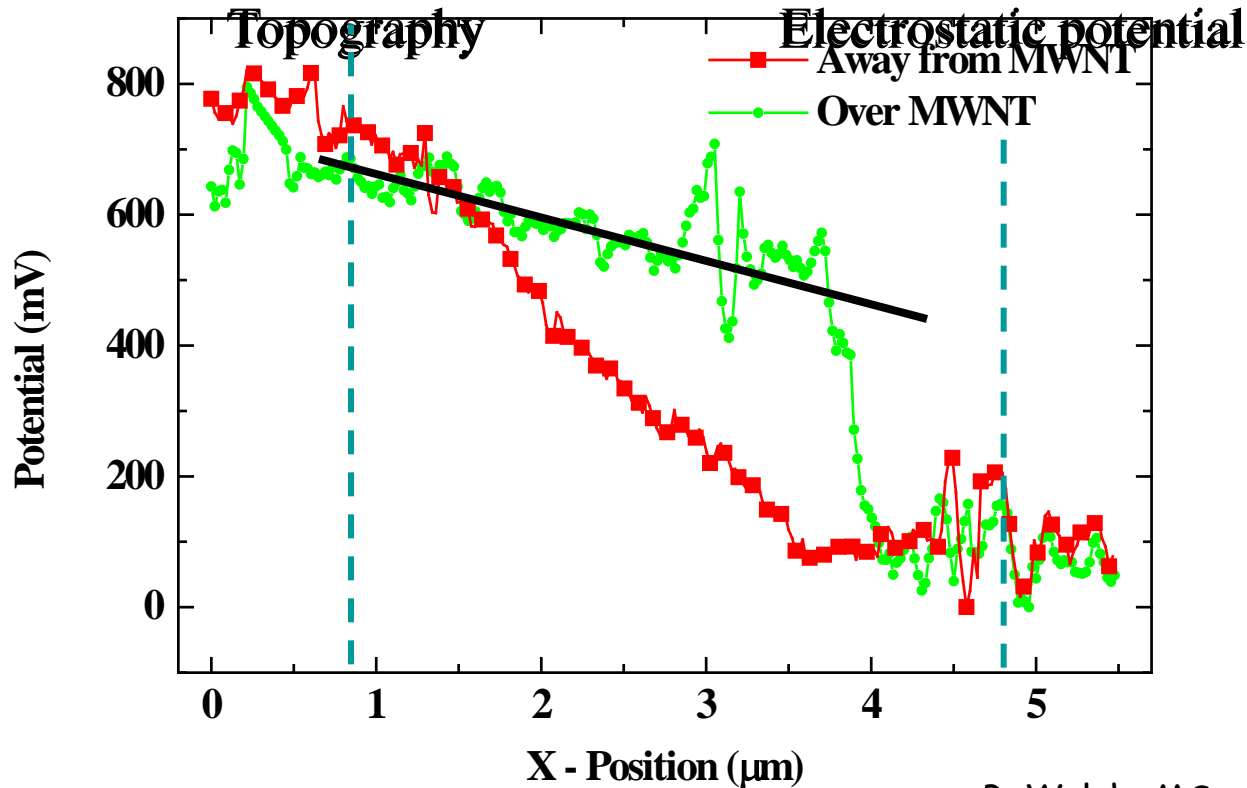
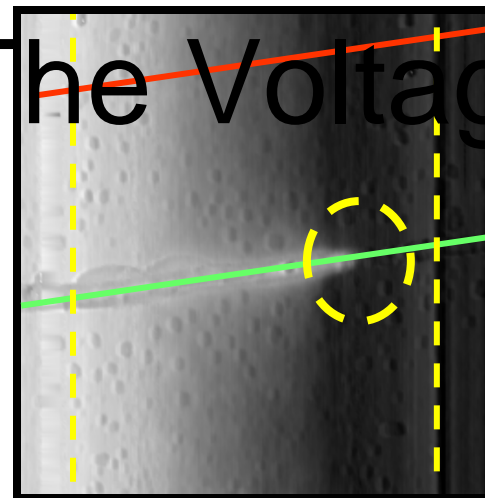
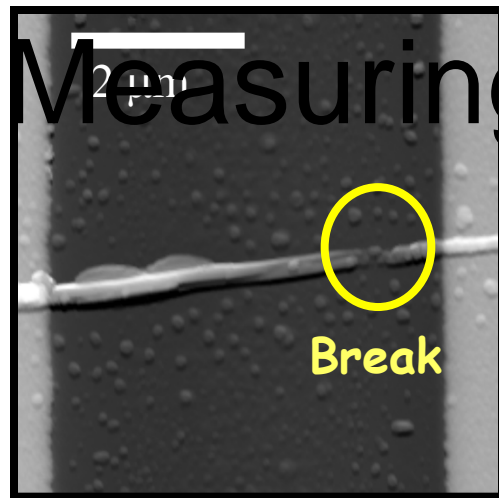
EFM Image:  $V_{\text{Tip}} = -230$  mV



# Electrically Contacting Nanotubes

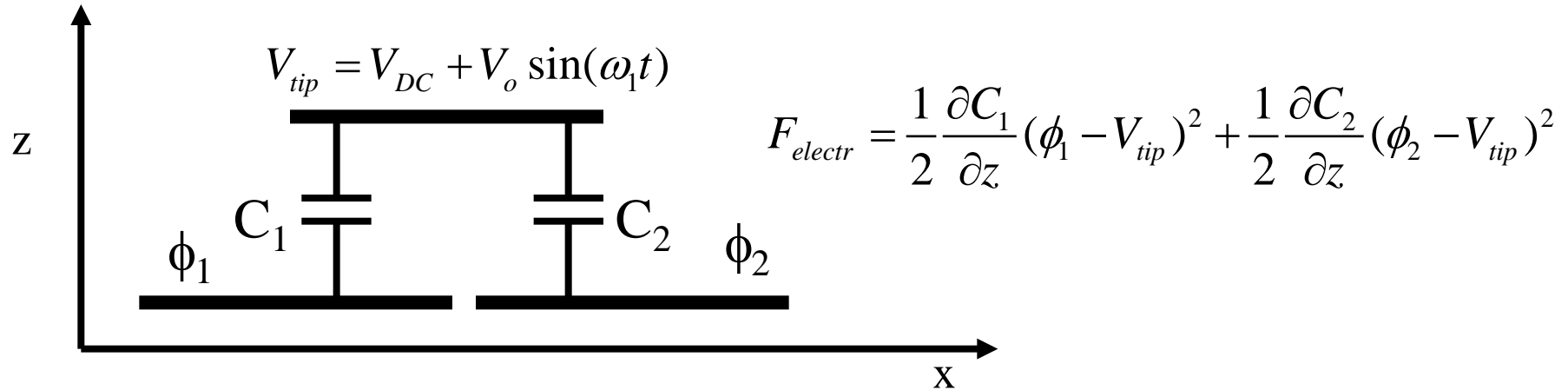


# Measuring the voltage drop along a MWCNT





# Convolution Effects



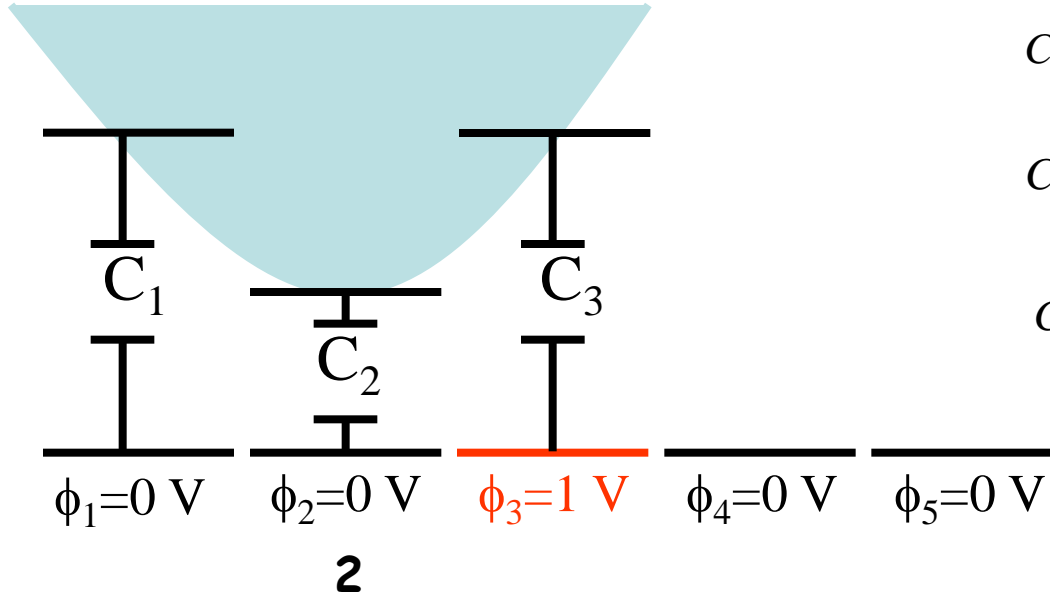
$$|F_{\omega_1, electr}| = \frac{1}{2} \frac{\partial C_1}{\partial z} (\phi_1 - V_{DC}) V_o + \frac{1}{2} \frac{\partial C_2}{\partial z} (\phi_2 - V_{DC}) V_o = 0$$

$F_{electr} = 0$  when

$$V_{DC} = \frac{\frac{\partial C_1}{\partial z} \phi_1 + \frac{\partial C_2}{\partial z} \phi_2}{\frac{\partial C_1}{\partial z} + \frac{\partial C_2}{\partial z}}$$

Measurements of the electrostatic potential will be distorted due to the non-uniform capacitive coupling between the tip and various parts of the surface.

# Effects of Convolution - Simple Model -



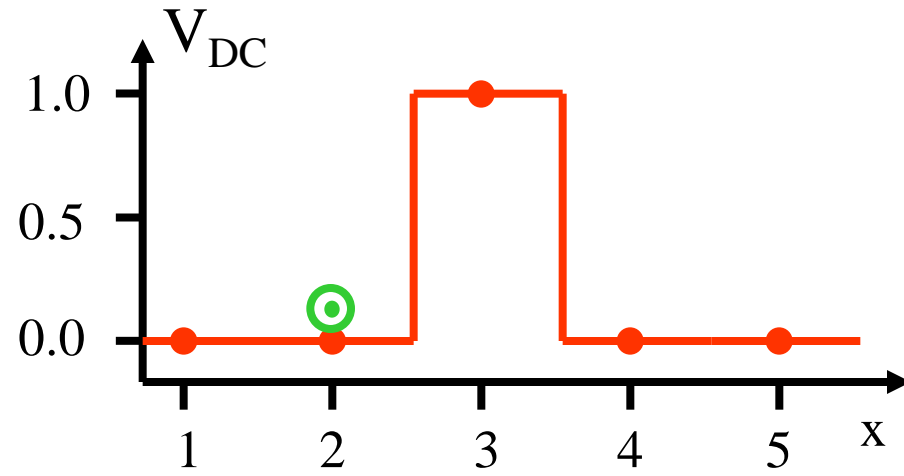
$$C_1 = \frac{\epsilon_0 A}{z_1}; \quad z_1 = 2 \mu\text{m}$$

$$C_2 = \frac{\epsilon_0 A}{z_2}; \quad z_2 = 1 \mu\text{m}$$

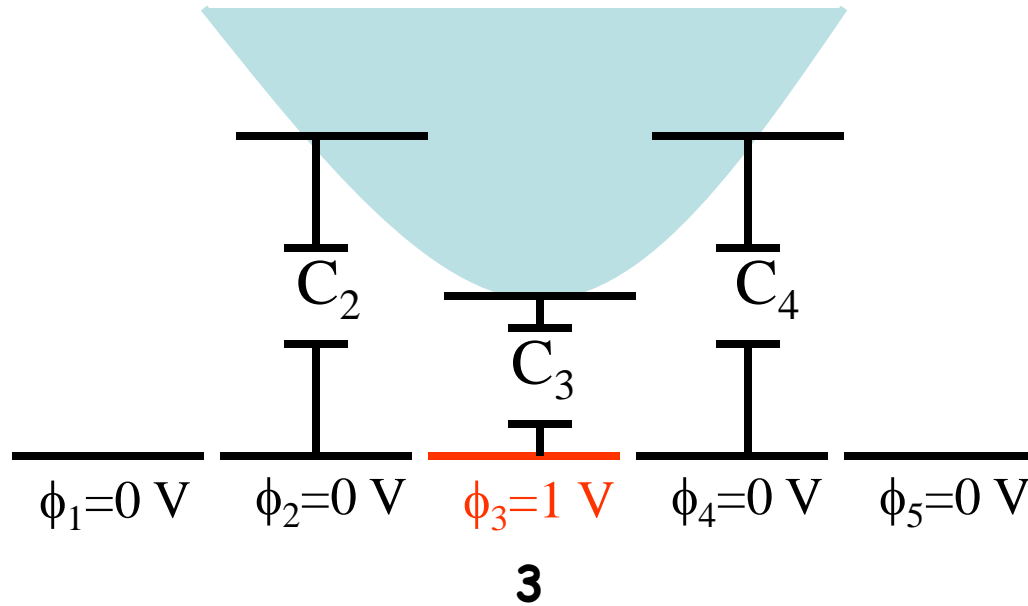
$$C_3 = \frac{\epsilon_0 A}{z_3}; \quad z_3 = 2 \mu\text{m}$$

$$V_{DC} = \frac{\frac{1}{2} \frac{\partial C_1}{\partial z} \phi_1 + \frac{1}{2} \frac{\partial C_2}{\partial z} \phi_2 + \frac{1}{2} \frac{\partial C_3}{\partial z} \phi_3}{\frac{1}{2} \frac{\partial C_1}{\partial z} + \frac{1}{2} \frac{\partial C_2}{\partial z} + \frac{1}{2} \frac{\partial C_3}{\partial z}} = \frac{\frac{1}{4} \cdot 0 + \frac{1}{1} \cdot 0 + \frac{1}{4} \cdot 1}{\frac{1}{4} + \frac{1}{1} + \frac{1}{4}} = \frac{0.25}{1.5}$$

$V_{DC} = 0.17 \text{ V}$



# Effects of Convolution - Simple Model -



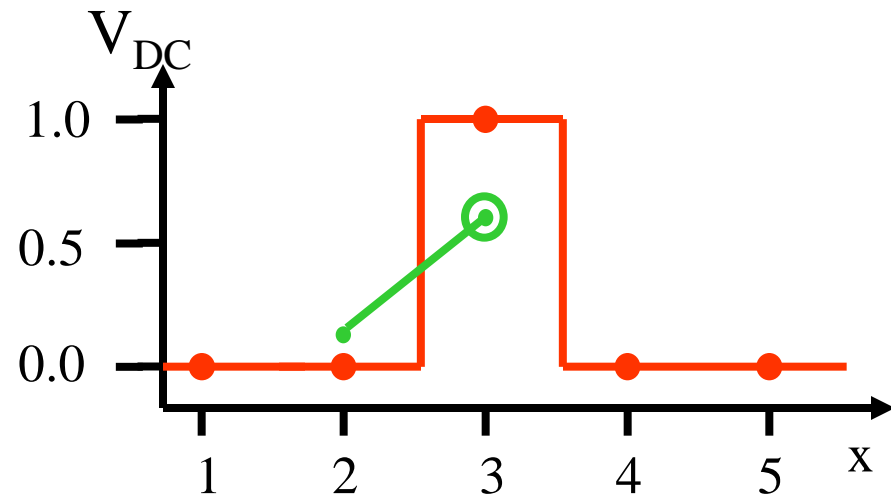
$$C_2 = \frac{\epsilon_0 A}{z_2} ; z_2 = 2\ \mu\text{m}$$

$$C_3 = \frac{\epsilon_0 A}{z_3} ; z_3 = 1\ \mu\text{m}$$

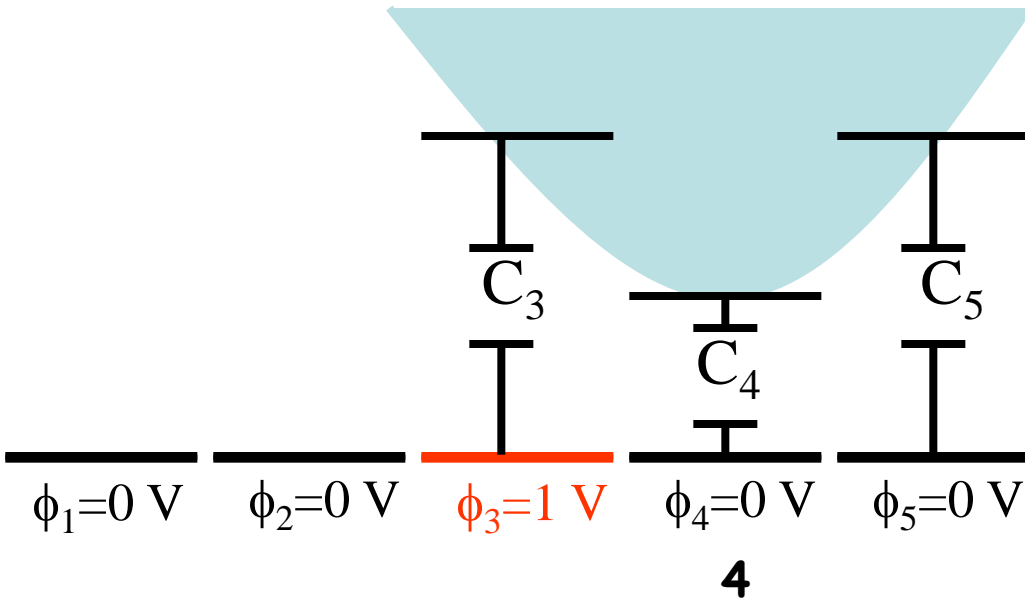
$$C_4 = \frac{\epsilon_0 A}{z_4} ; z_4 = 2\ \mu\text{m}$$

$$V_{DC} = \frac{\frac{1}{2} \frac{\partial C_2}{\partial z} \phi_2 + \frac{1}{2} \frac{\partial C_3}{\partial z} \phi_3 + \frac{1}{2} \frac{\partial C_4}{\partial z} \phi_4}{\frac{1}{2} \frac{\partial C_2}{\partial z} + \frac{1}{2} \frac{\partial C_3}{\partial z} + \frac{1}{2} \frac{\partial C_4}{\partial z}} = \frac{\frac{1}{4} \cdot 0 + \frac{1}{1} \cdot 1 + \frac{1}{4} \cdot 0}{\frac{1}{4} + \frac{1}{1} + \frac{1}{4}} = \frac{1}{1.5}$$

$$V_{DC} = 0.67\text{ V}$$



# Effects of Convolution - Simple Model -



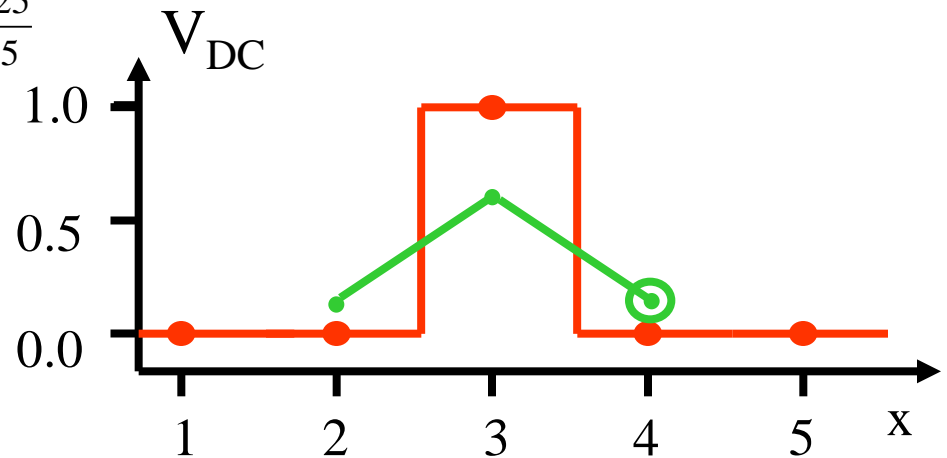
$$C_3 = \frac{\epsilon_0 A}{z_3} ; z_3 = 2 \mu\text{m}$$

$$C_4 = \frac{\epsilon_0 A}{z_4} ; z_4 = 1 \mu\text{m}$$

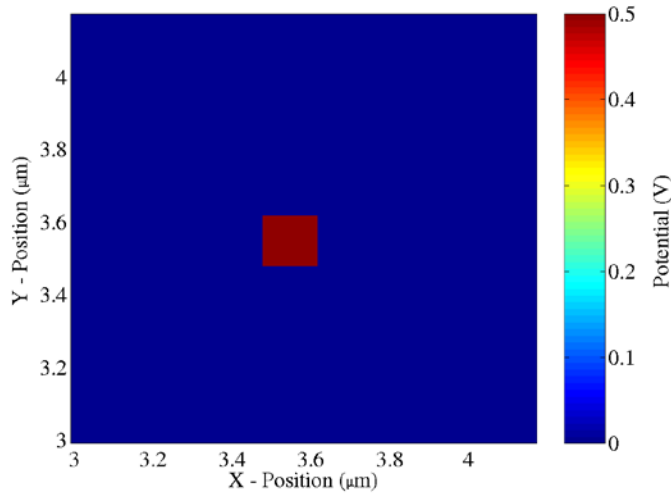
$$C_5 = \frac{\epsilon_0 A}{z_5} ; z_5 = 2 \mu\text{m}$$

$$V_{DC} = \frac{\frac{1}{2} \frac{\partial C_3}{\partial z} \phi_{23} + \frac{1}{2} \frac{\partial C_4}{\partial z} \phi_4 + \frac{1}{2} \frac{\partial C_5}{\partial z} \phi_5}{\frac{1}{2} \frac{\partial C_3}{\partial z} + \frac{1}{2} \frac{\partial C_4}{\partial z} + \frac{1}{2} \frac{\partial C_5}{\partial z}} = \frac{\frac{1}{4} \cdot 1 + \frac{1}{1} \cdot 0 + \frac{1}{4} \cdot 0}{\frac{1}{4} + \frac{1}{1} + \frac{1}{4}} = \frac{0.25}{1.5}$$

**$V_{DC} = 0.17 \text{ V}$**

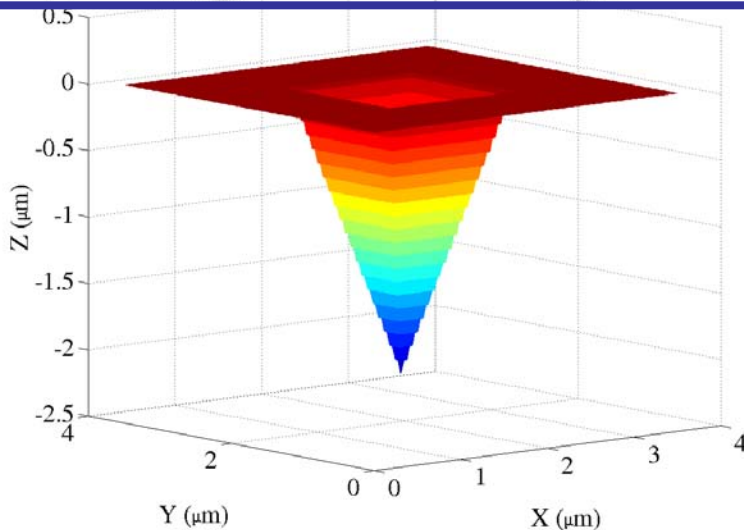


# More Realistic Model

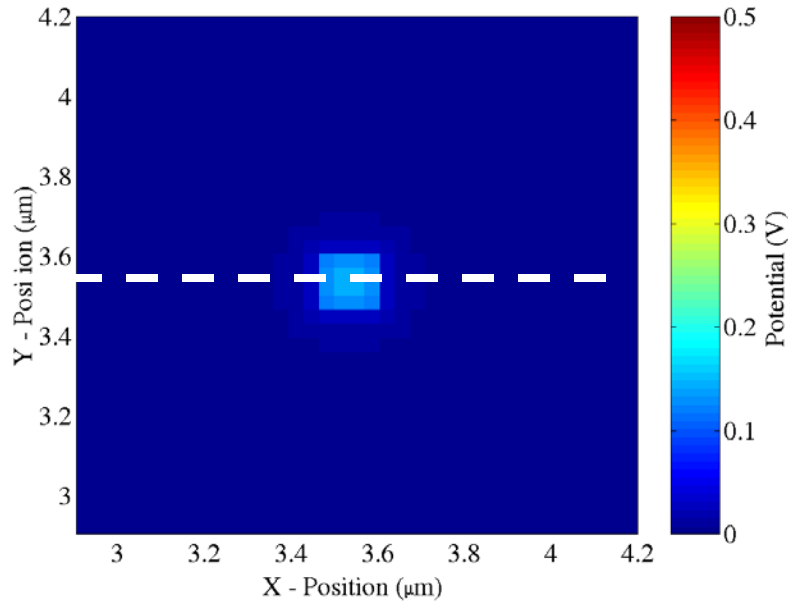


The electrostatic potential is specified in two dimensions by  $V(x,y)$

The tip is moved relative to the specified potential, and the weighted average is calculated at each point.

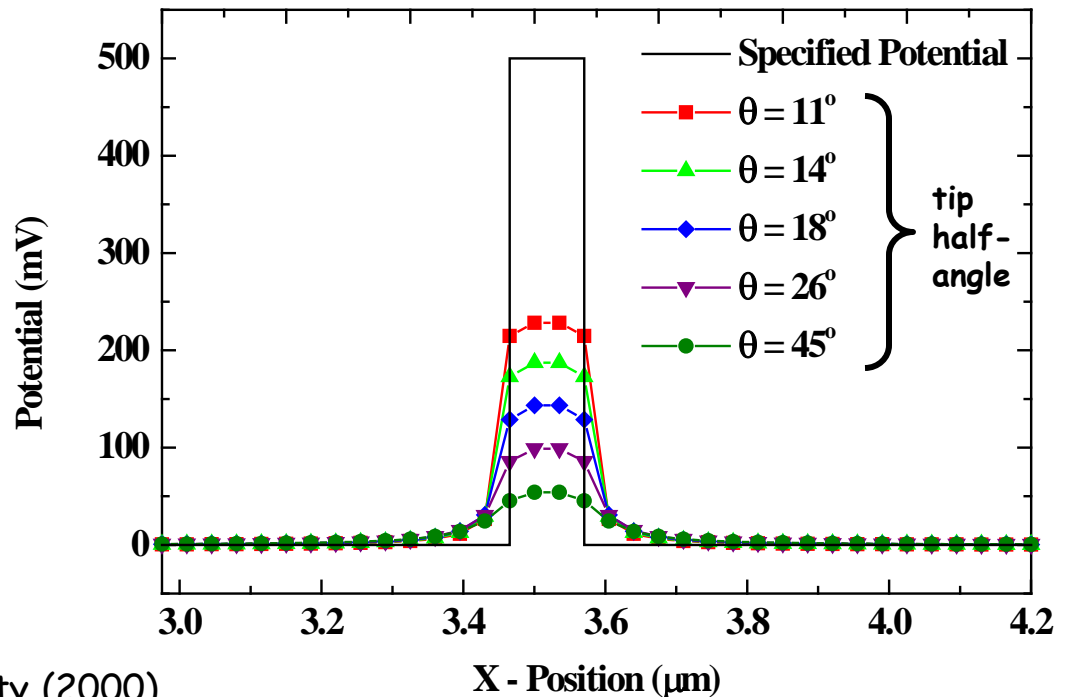


$$V_{DC} = \frac{\sum_{surface}^i \left( \frac{\partial C_i}{\partial z} \cdot \phi_{surface} \right)}{\sum_{surface}^i \left( \frac{\partial C_i}{\partial z} \right)}$$

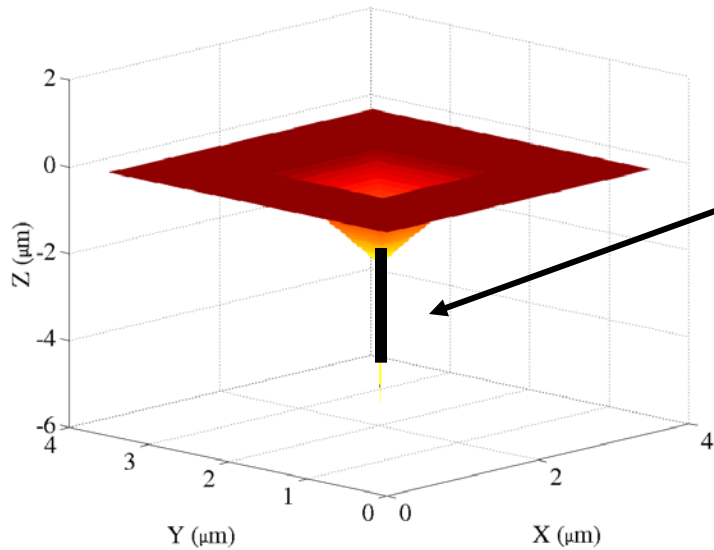


The output of the convolution program is a 2-D electrostatic potential map.

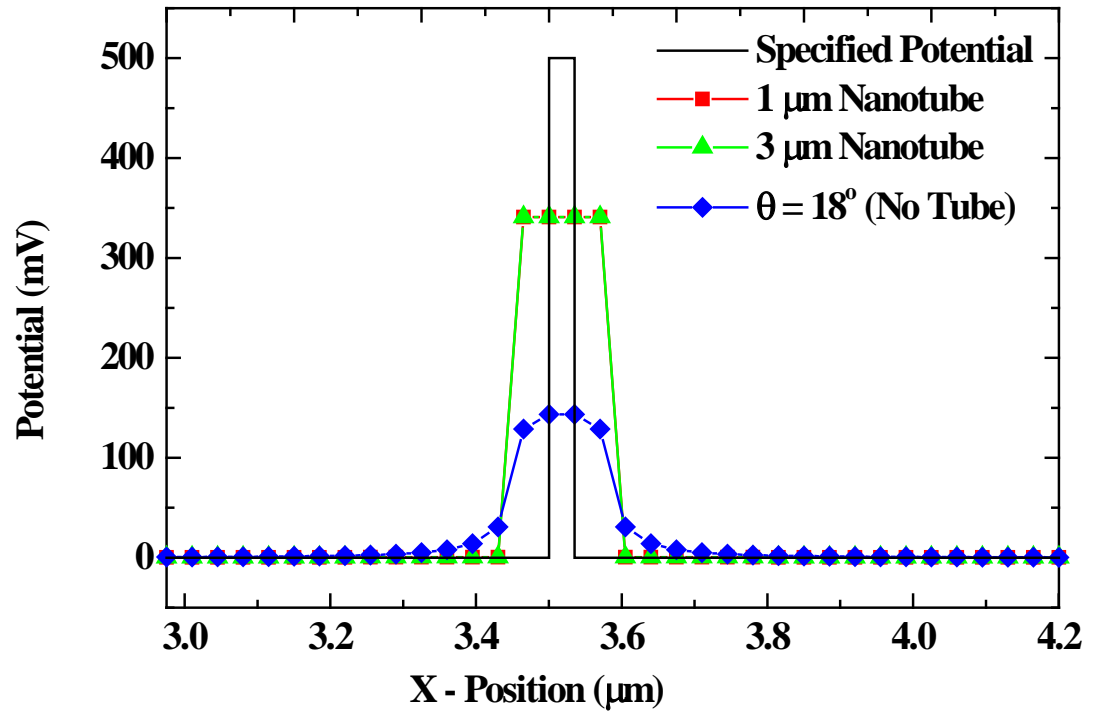
Plot of profiles of the electrostatic potential along dotted white line.



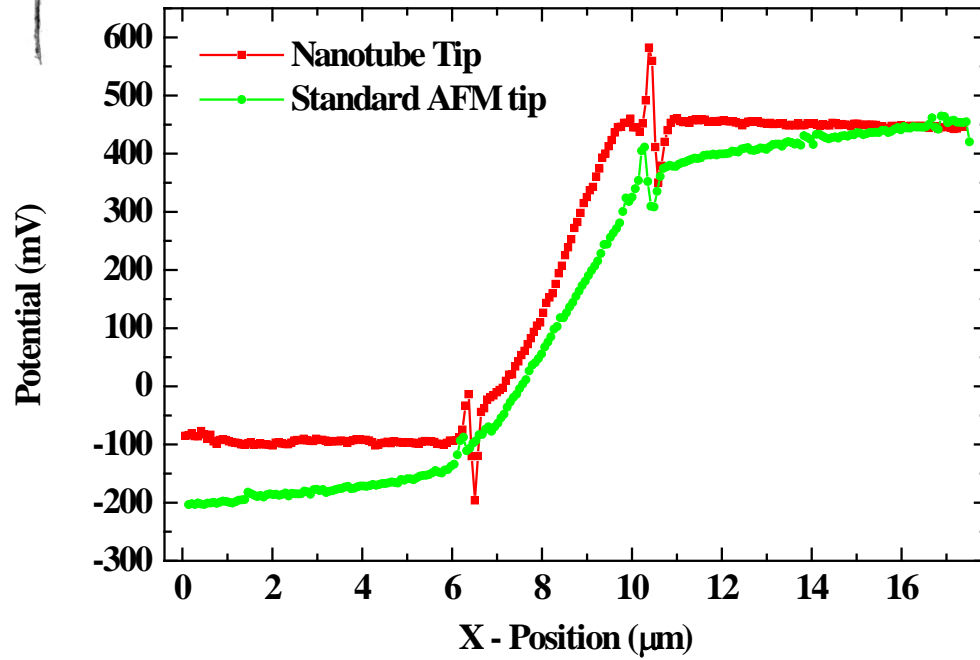
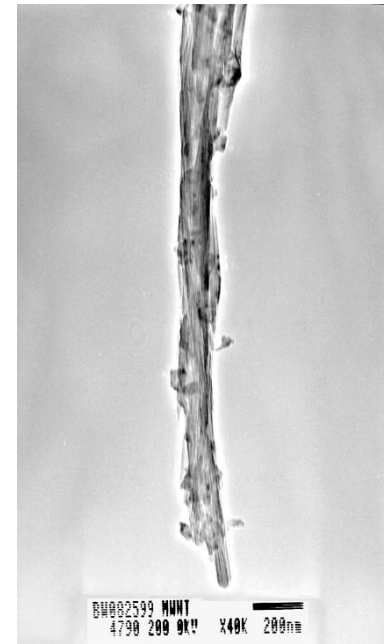
# Improving the Situation



1 μm and 3 μm long extensions to the tip were modeled.



# Experiment: MWCNT tip





# Recent Development: sharp tips with high aspect ratio

