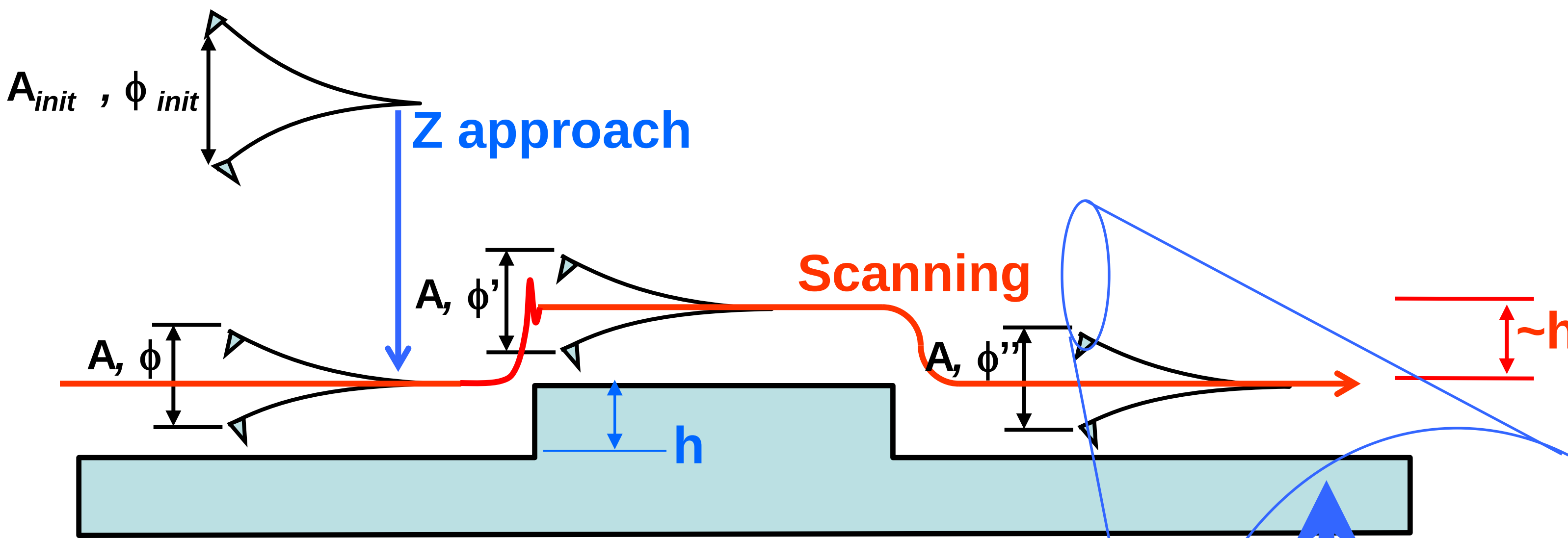
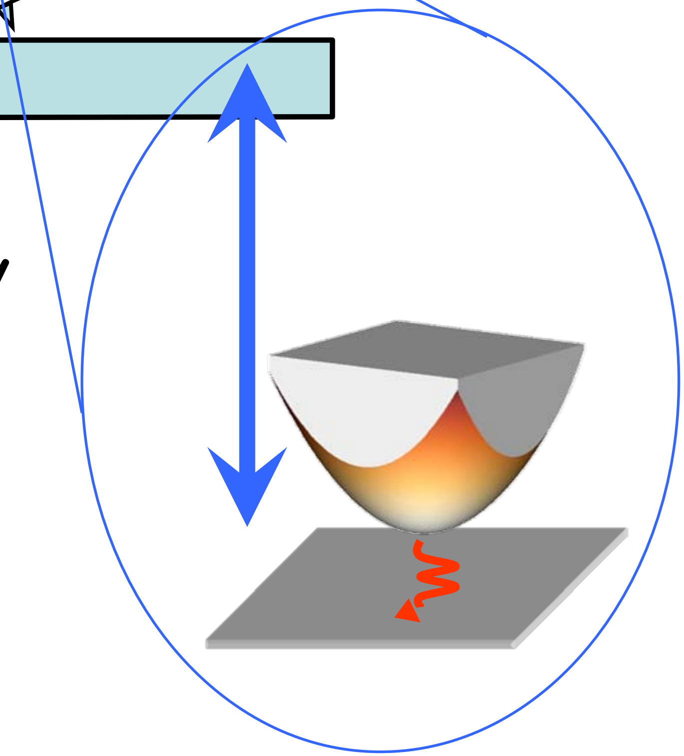
Lecture 13
Point mass models, modeling AM-AFM

Arvind Raman
Mechanical Engineering
Birck Nanotechnology Center

AM-AFM (aka Tapping Mode, IC mode)





Key points

- Drive frequency is always fixed ω, usually near ω_0
- During approach A, ϕ change due to tip-sample interaction forces
- During scan ϕ is a free variable and changes naturally while scanning

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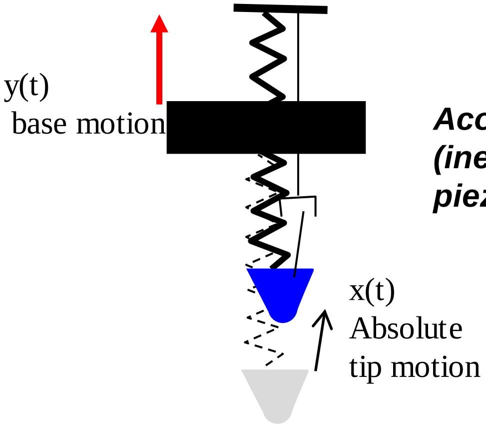
Response of acoustically excited levers



$$m\ddot{x} = -k(x-y) - c\dot{x}$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = y(t);\ with\quad \omega_0 = \sqrt{\frac{k}{m}},\ Q = \frac{m\omega_0}{c}$$

$$Measured\ motion\ z(t) = x(t) - y(t)$$

$$\frac{\ddot{z}}{\omega_0^2} + z + \frac{1}{\omega_0 Q}\dot{z} = -\frac{\ddot{y}}{\omega_0^2} - \frac{1}{\omega_0 Q}\dot{y}$$

$$y(t) = Y_0\ \sin(\omega t)$$

$$z^p(t) = A\sin(\omega t - \phi_{inertial})$$

$$|H_{inertial}(\omega)| = \frac{A}{Y_0} = \left(\frac{r^4 + (r/Q)^2}{(1-r^2)^2 + (r/Q)^2}\right)^{1/2}$$

$$\phi_{inertial}(\omega) = \tan^{-1}\left(\frac{Q}{r(1+Q^2r^2 - Q^2)}\right)$$

$$where\ r = \frac{\omega}{\omega_0}$$

- ω_0 is the natural freq, ω is the drive freq
- Maximum amplitude occurs when $\omega > \omega_0$!
- Base motion amplitude at r=1 is A/Q!

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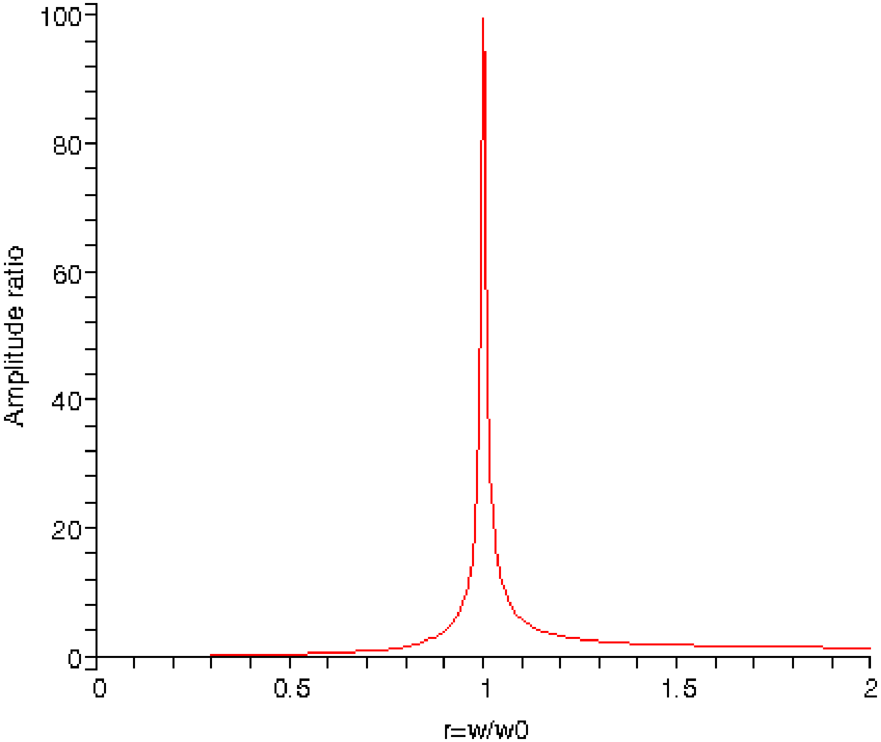
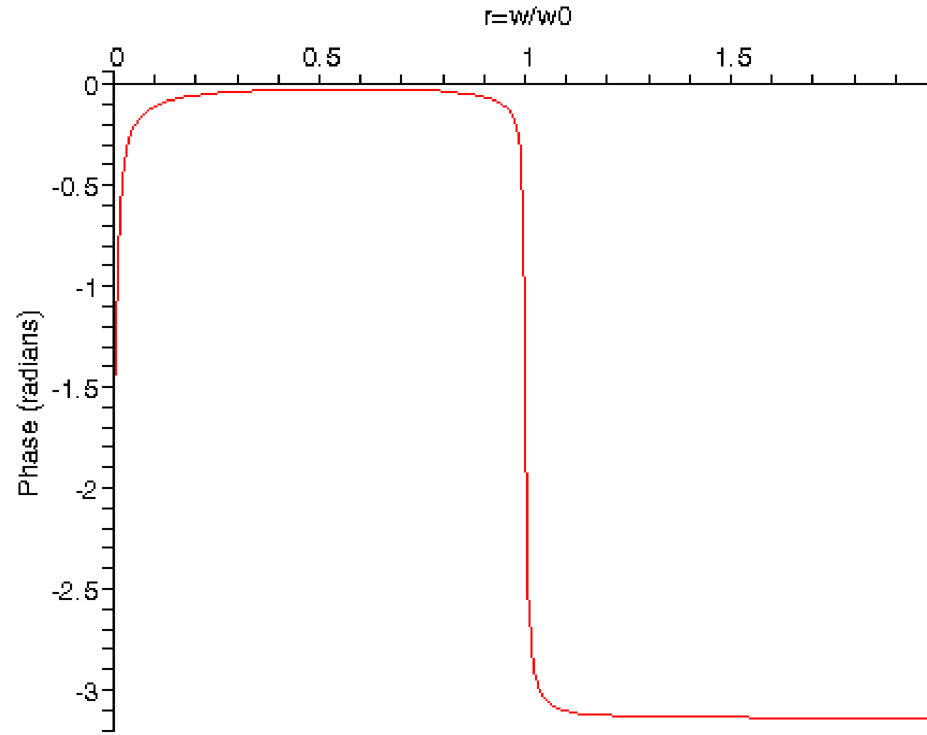
Response of acoustically excited levers





- For Q=100, see response above
- Asymmetric peak, amplitude greater when $\omega > \omega_0$

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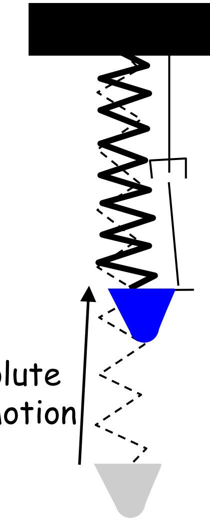
Response of directly excited AFM levers



$$m\ddot{x} = -kx - c\dot{x} + F_{mag}(t)$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = \frac{1}{k}F_{mag}(t);\ with \quad \omega_0 = \sqrt{\frac{k}{m}},\ Q = \frac{m\omega_0}{c}$$

$$Measured\ motion = x(t)$$

$$F_{mag}(t) = F_0 \sin(\omega t)$$

$$x^p(t) = A\sin(\omega t - \phi_{magnetic})$$

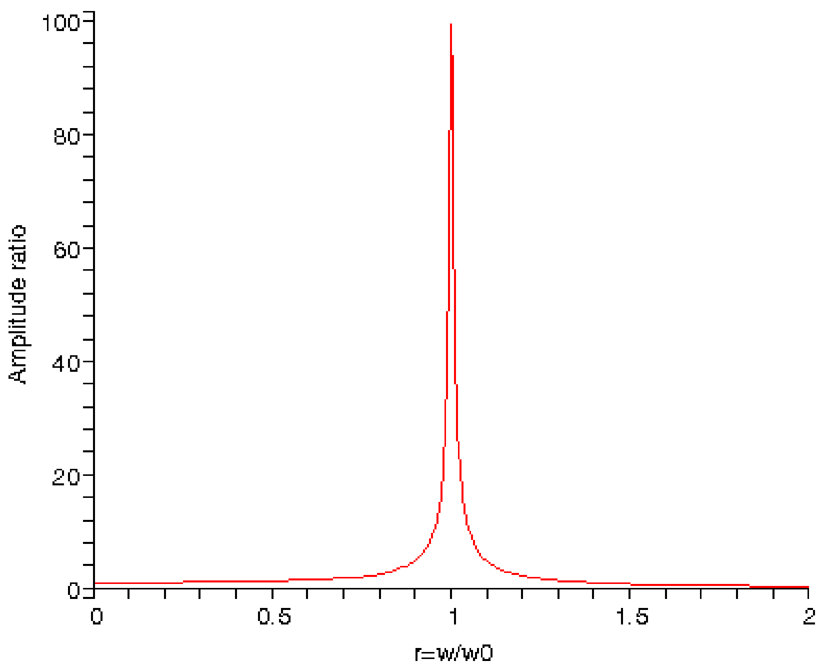
$$|H_{mag}(\omega)| = \frac{A}{F_0 / k} = \left(\frac{1}{(1-r^2)^2 + (r/Q)^2}\right)^{1/2}$$

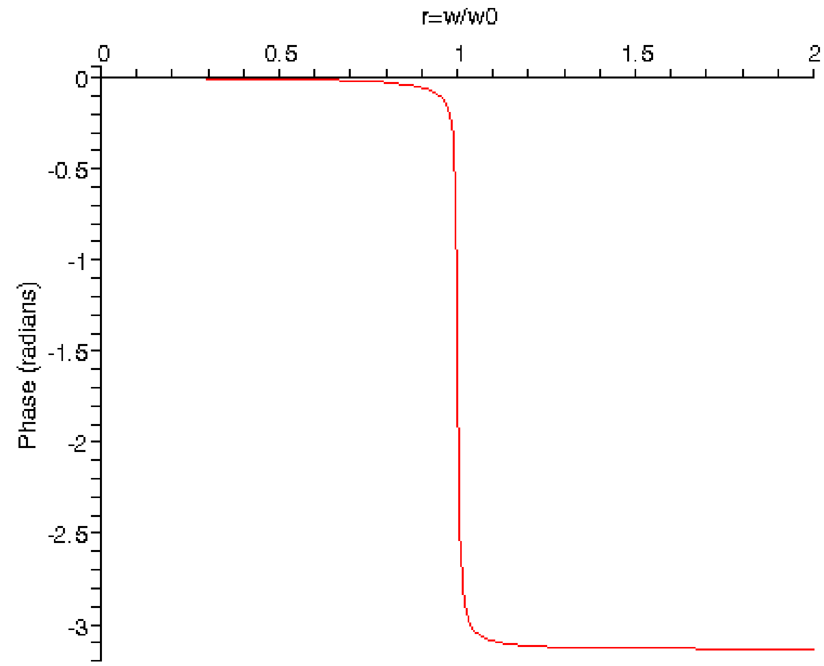
$$\phi_{mag}(\omega) = \tan^{-1}\left(\frac{r}{Q(r^2-1)}\right)$$

$$where\ r = \frac{\omega}{\omega_0}$$

- ω_0 is the natural freq, ω is the drive freq
- Maximum amplitude occurs when $\omega<\omega_0$!
- For $\omega<<\omega_0$ $A=F_{mag}/k$!

Response of directly excited AFM levers





- Asymmetric response with greater amplitude when $\omega<\omega_0$!
- Classical phase response

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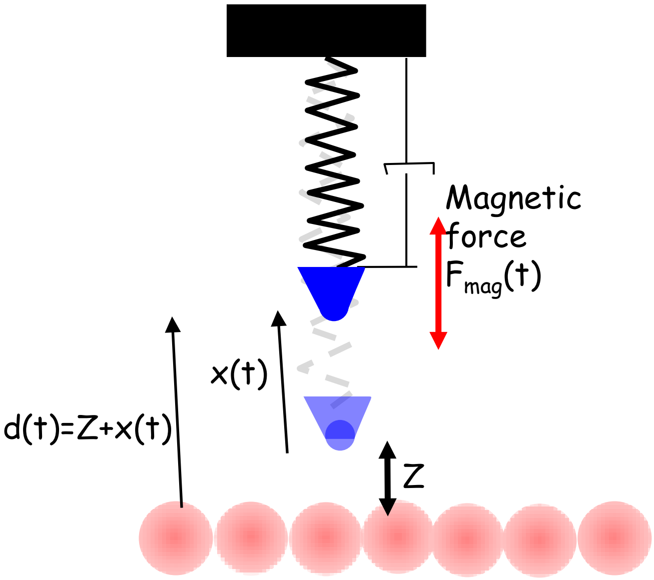
Driven point mass model with tip-sample interaction



Magnetic

$$m\ddot{x} = -kx - c\dot{x} + F_{mag}(t) + F_{ts}(Z + x(t))$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = \frac{1}{k}\left(F_{mag}(t) + F_{ts}(Z + x(t))\right);$$

$$\text{with} \quad \omega_0 = \sqrt{\frac{k}{m}},\ Q = \frac{m\omega_0}{c}$$

$$\text{Measured motion} = x(t)$$

$$F_{mag}(t) = F_0 \sin(\omega t)$$

Acoustic excitation

$$\frac{\ddot{z}}{\omega_0^2} + z + \frac{1}{\omega_0 Q}\dot{z} = -\frac{\ddot{y}}{\omega_0^2} - \frac{1}{\omega_0 Q}\dot{y} + \frac{F_{ts}(Z + y(t) + x(t))}{\omega_0^2}$$

- Highly nonlinear ordinary differential equation

Linearized analysis

Magnetic excitation

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = \frac{1}{k}\left(F_0 \sin(\omega t) + F_{ts}(Z + x(t))\right); \qquad d(t) = Z + x(t) \qquad (1)$$

At a given Z the equilibrium deflection is

$$x^* = \frac{1}{k}F_{ts}(Z + x^*) \quad where \quad d^* = Z + x^* \qquad (2)$$

$$Let \quad x(t) = x^* + \bar{x}(t) \qquad (3)$$

Include time – dependent terms

$$\frac{\left(\ddot{\bar{x}} + \ddot{x}^*\right)}{\omega_0^2} + \left(\bar{x} + x^*\right) + \frac{1}{\omega_0 Q}(\dot{\bar{x}} + \dot{x}^*) = \frac{1}{k}\left(F_0 \sin(\omega t) + F_{ts}(Z + x^* + \bar{x})\right) \qquad (4)$$

$$\frac{\ddot{\bar{x}}}{\omega_0^2} + \left(\bar{x} + x^*\right) + \frac{1}{\omega_0 Q}\dot{\bar{x}} = \frac{1}{k}\left(F_0 \sin(\omega t) + F_{ts}(Z + x^* + \bar{x})\right) \qquad (5)$$

If $\bar{x} << Z + x^*$ *or when* $\bar{x} << d^*$ *then*

$$\frac{\ddot{\bar{x}}}{\omega_0^2} + \left(\bar{x} + x^*\right) + \frac{1}{\omega_0 Q}\dot{\bar{x}} = \frac{1}{k}\left(F_0 \sin(\omega t) + F_{ts}(Z + x^*) + \left.\frac{\partial F_{ts}(d)}{\partial d}\right|_{d=d^*}\bar{x}\right) \qquad (6)$$

$\Rightarrow$

$$\frac{\ddot{\bar{x}}}{\omega_0^2} + \left(1 - \frac{1}{k}\left.\frac{\partial F_{ts}(d)}{\partial d}\right|_{d=x^*}\right)\bar{x} + \frac{1}{\omega_0 Q}\dot{\bar{x}} = \frac{1}{k}\left(F_0 \sin(\omega t)\right) \qquad (7)$$

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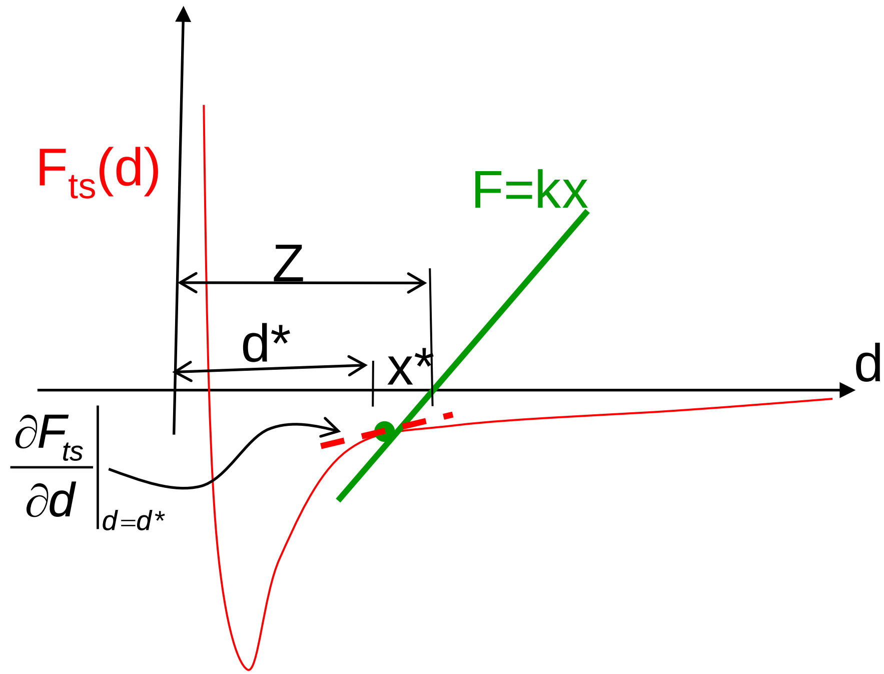
Linearized analysis

$$\frac{\ddot{\bar{x}}}{\omega_0^2}+\left(1-\frac{1}{k}\frac{\partial F_{ts}(d)}{\partial d}\bigg|_{d=d^*}\right)\bar{x}+\frac{1}{\omega_0 Q}\dot{\bar{x}}=\frac{1}{k}\left(F_0\ \sin(\omega t)\right)$$

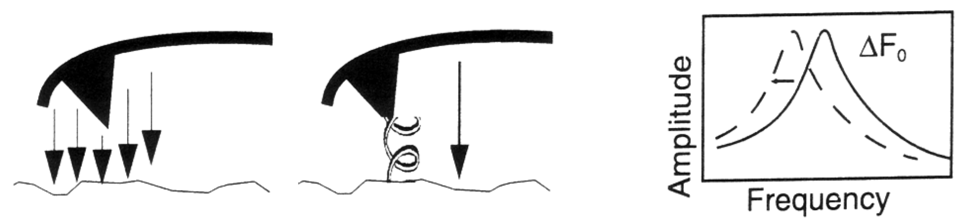
Or

$$\ddot{\bar{x}}+\omega_0^2\left(1-\frac{1}{k}\frac{\partial F_{ts}(d)}{\partial d}\bigg|_{d=d^*}\right)\bar{x}+\frac{\omega_0}{Q}\dot{\bar{x}}=\frac{\omega_0^2}{k}\left(F_0\ \sin(\omega t)\right)$$

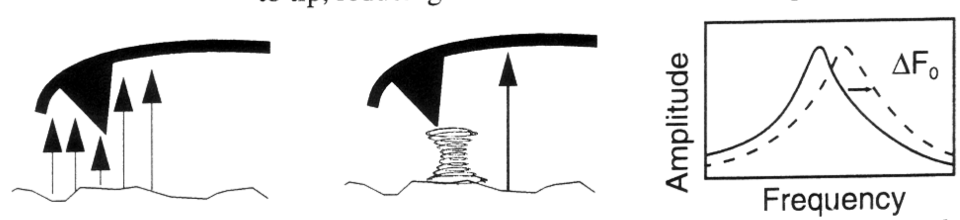
$$\hat{\omega}_0^2=\omega_0^2\left(1-\frac{1}{k}\frac{\partial F_{ts}(d)}{\partial d}\bigg|_{d=d^*}\right)$$



- When $\frac{\partial F_{ts}}{\partial d}\Big|_{d=d^*}>0$ attractive force and natural frequency decreases
- When $\frac{\partial F_{ts}}{\partial d}\Big|_{d=d^*}<0$ rep. regime and natural frequency increases



Attractive gradient equivalent to additional spring in tension attached to tip, reducing the cantilever resonance frequency.



Repulsive gradient equivalent to additional spring in compression attached to tip, increasing the cantilever resonance frequency.

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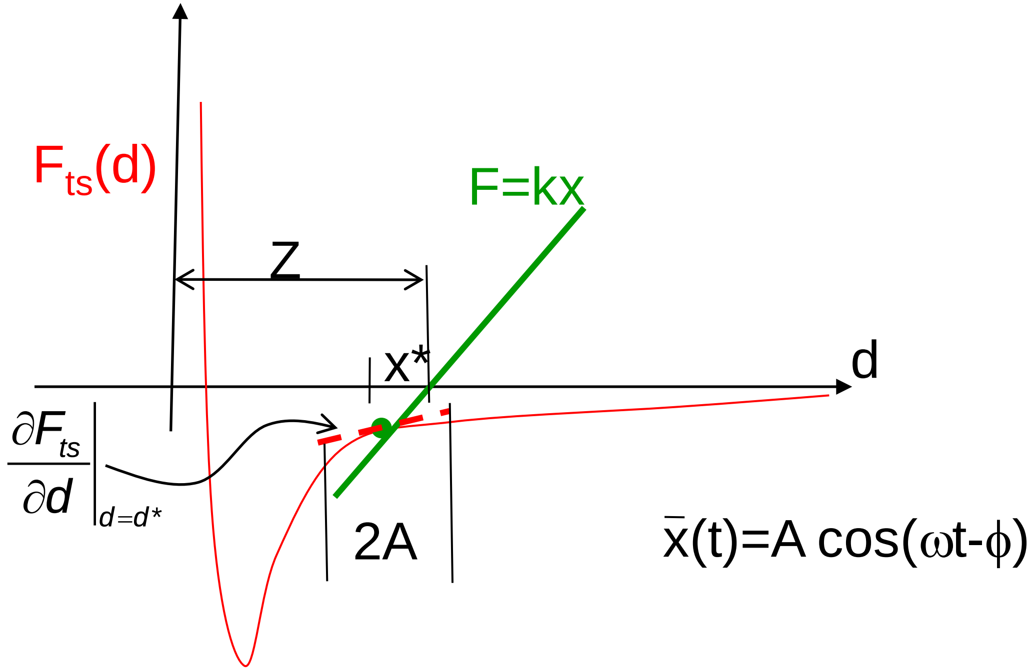
Limitations of the linearized analysis



- For what driven oscillation amplitude is this approximation valid?
-

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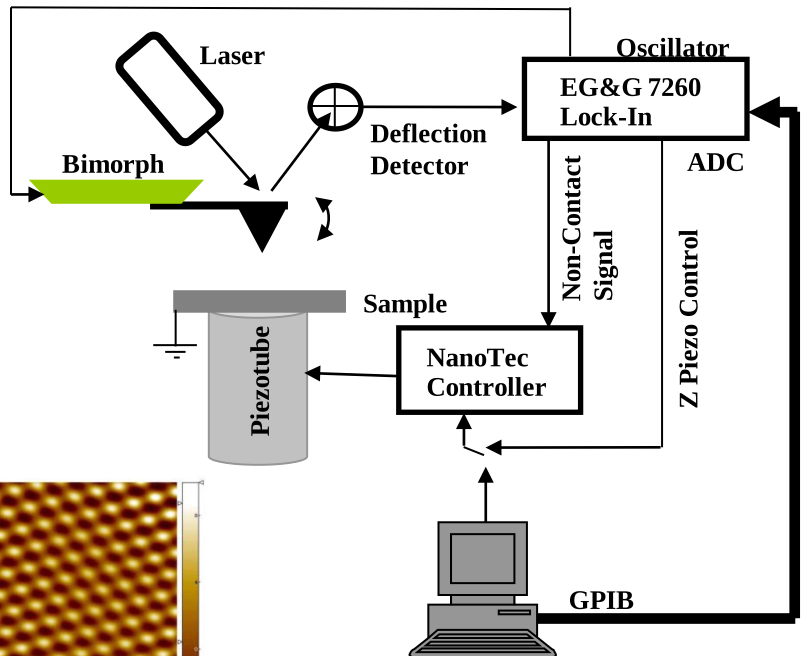
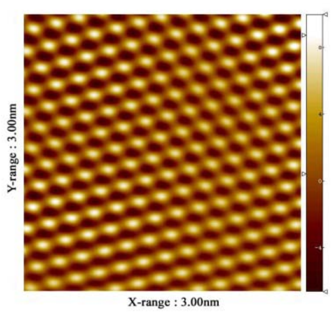
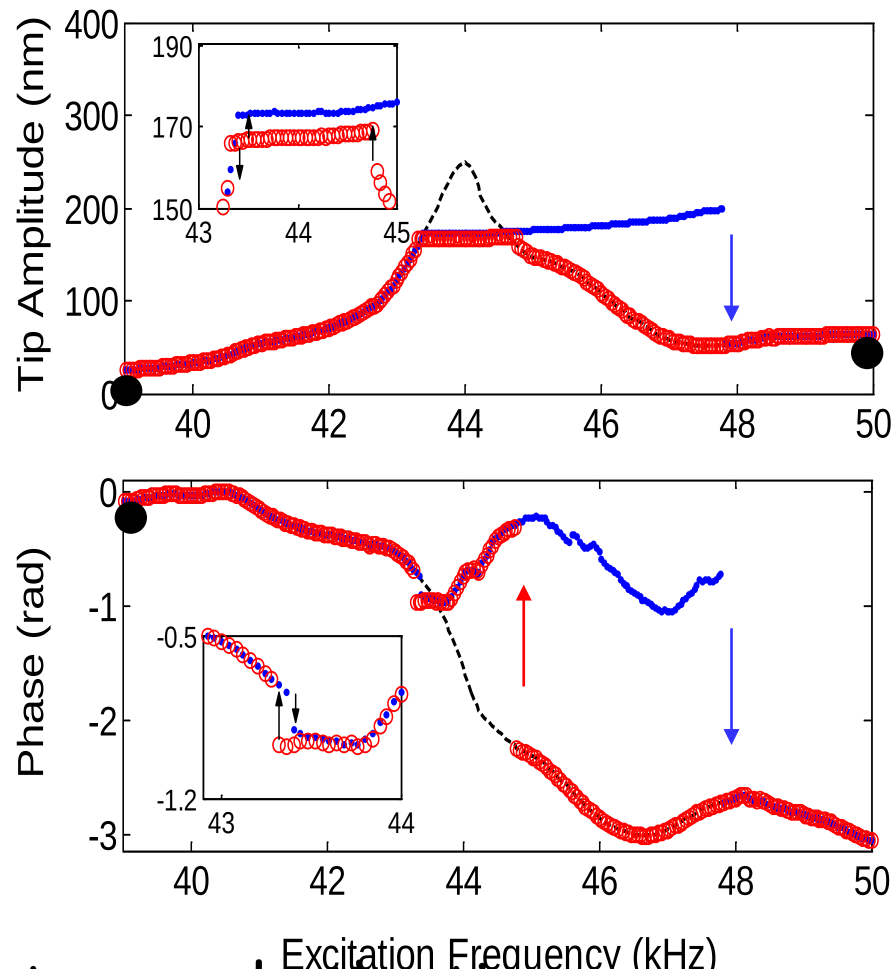
Experiments with conventional tips

- **Si tip / HOPG sample z=90 nm, frequency sweep**







- When brought closer to sample the tip sometimes sticks to the sample

Lee et al, Phys Rev B (2002)

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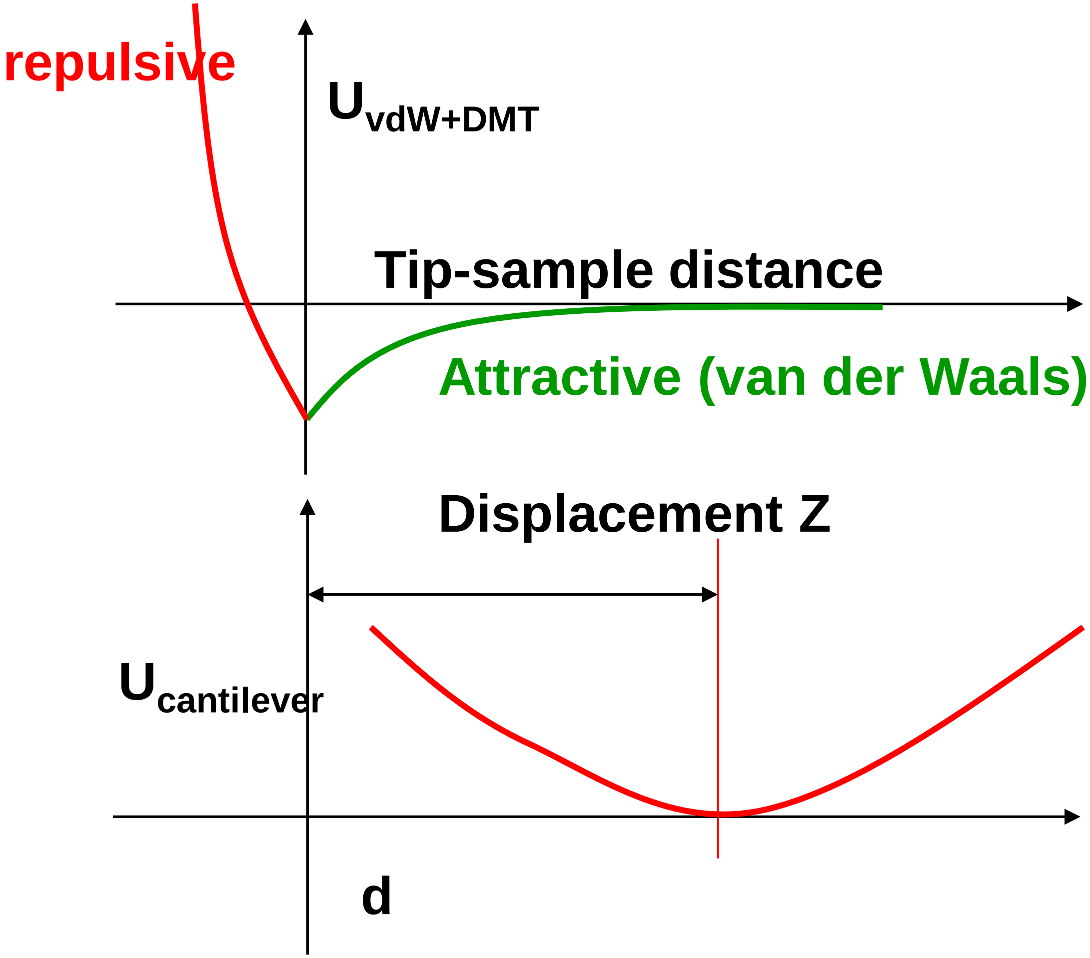
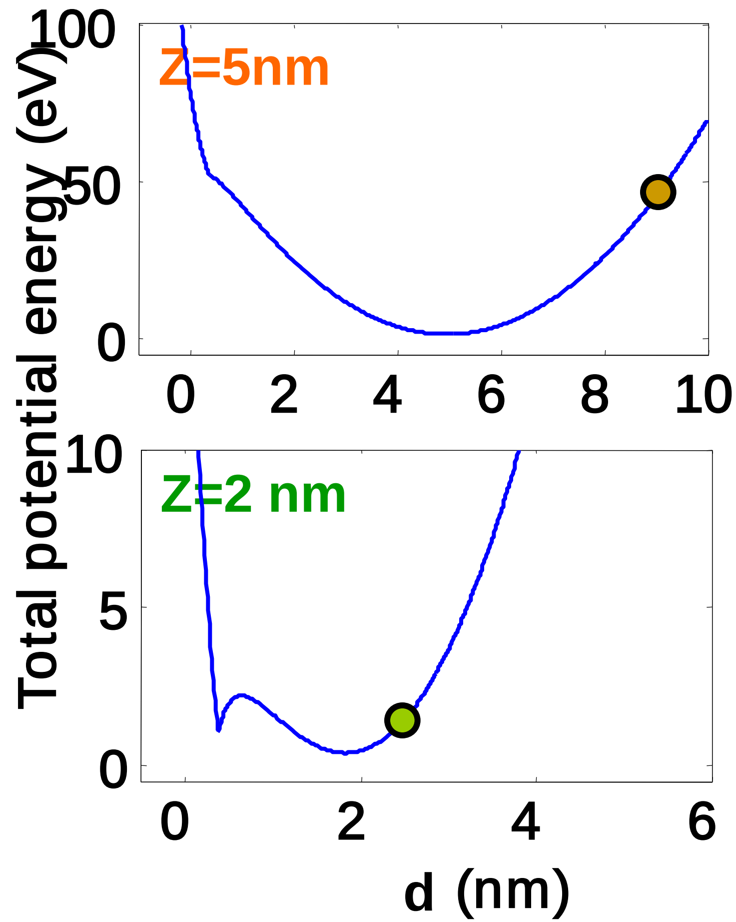
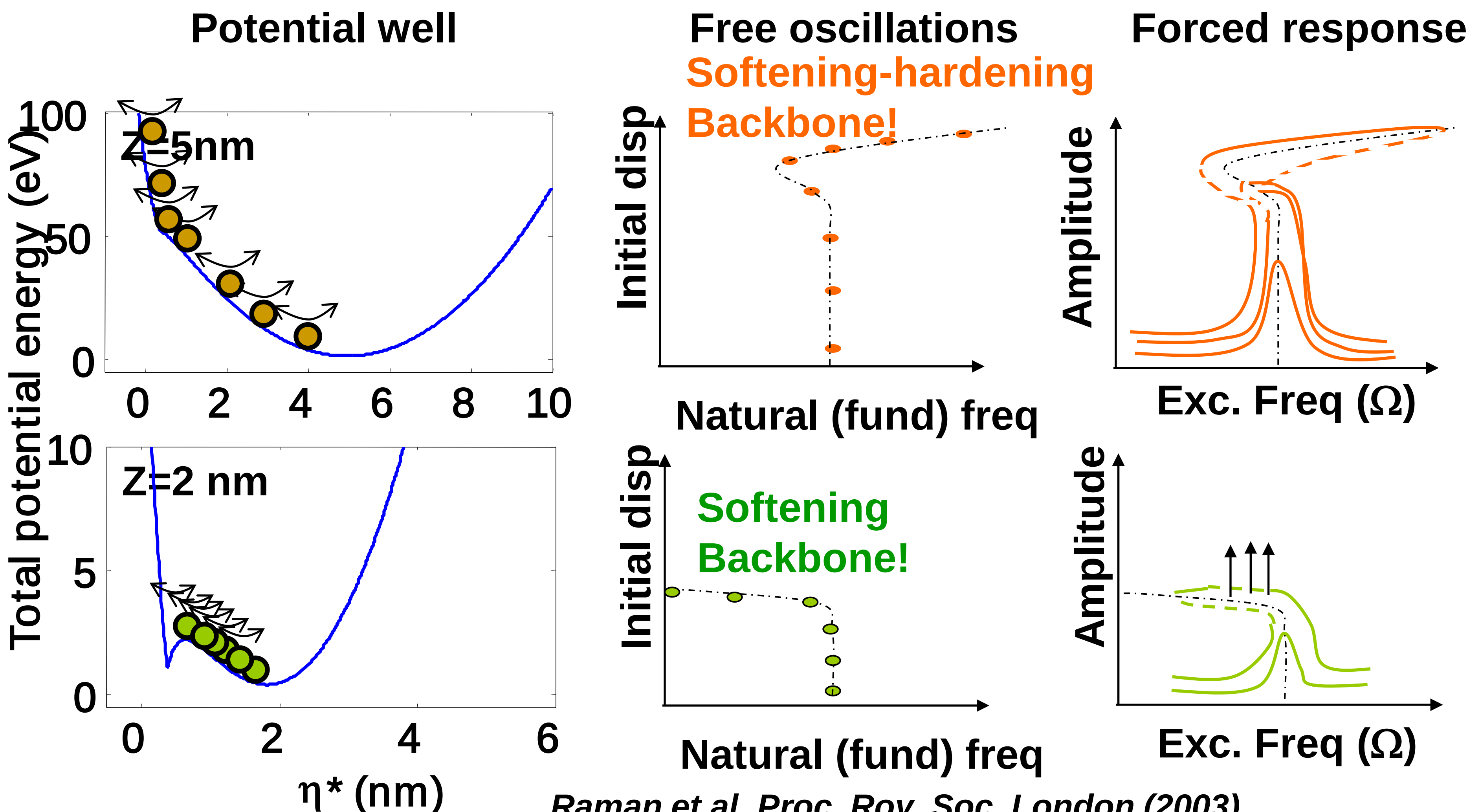
Physical mechanisms





- Total potential energy from interaction + beam elasticity
- Number of equilibria changes with Z

Physical mechanisms



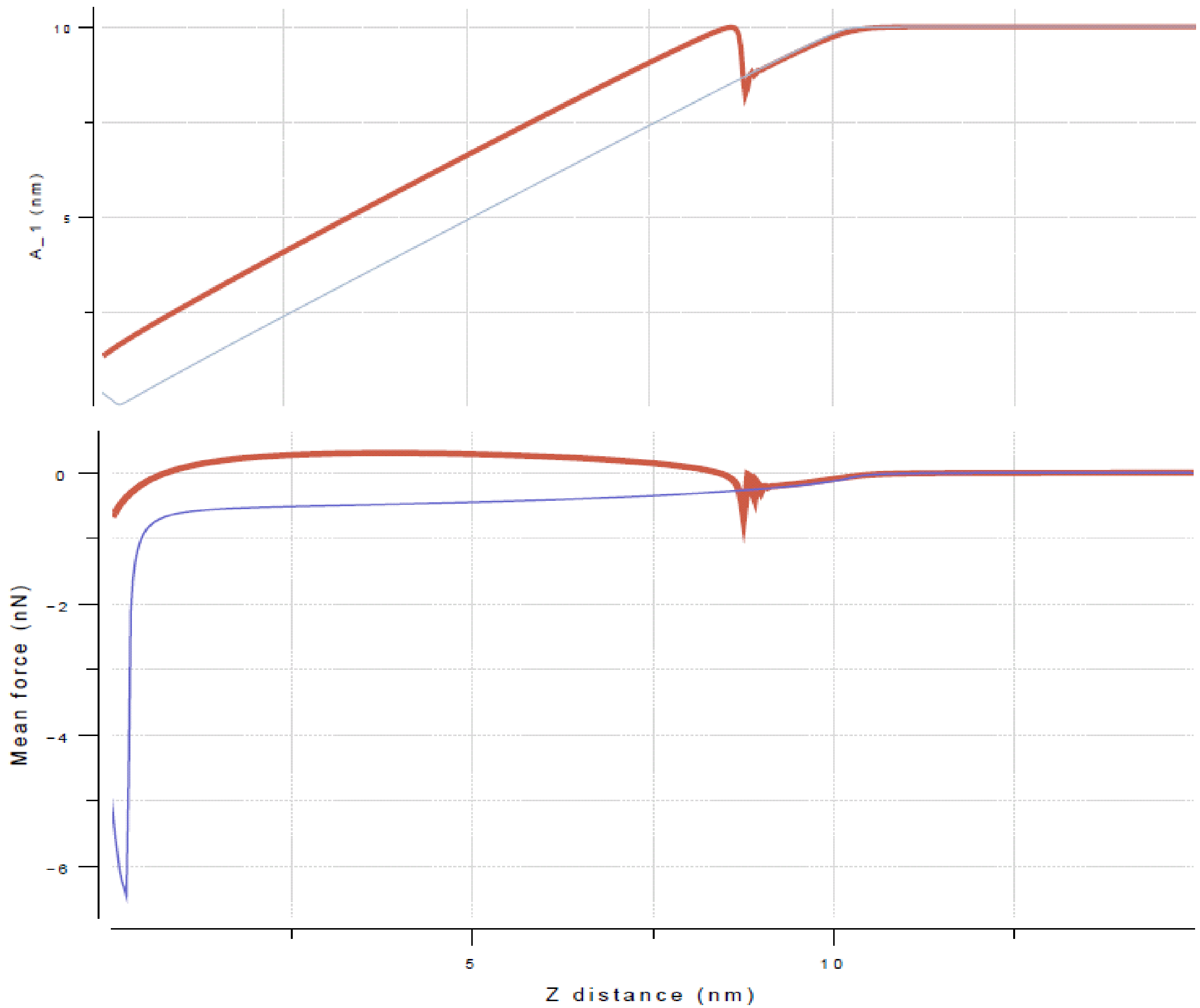
Raman et al, Proc. Roy. Soc. London (2003)

- Softening nonlinearity ⬄van der Waals forces (attractive)
- Hardening nonlinearity ⬄ sample elasticity (repulsive)

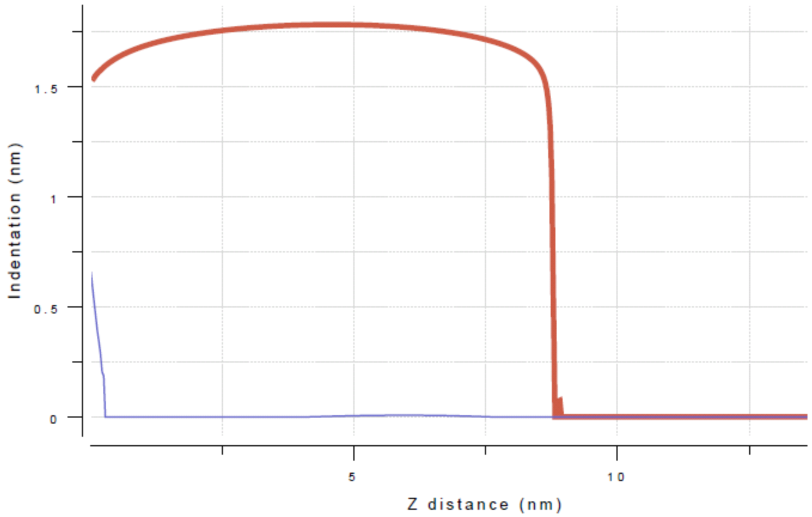
Implications for AM-AFM

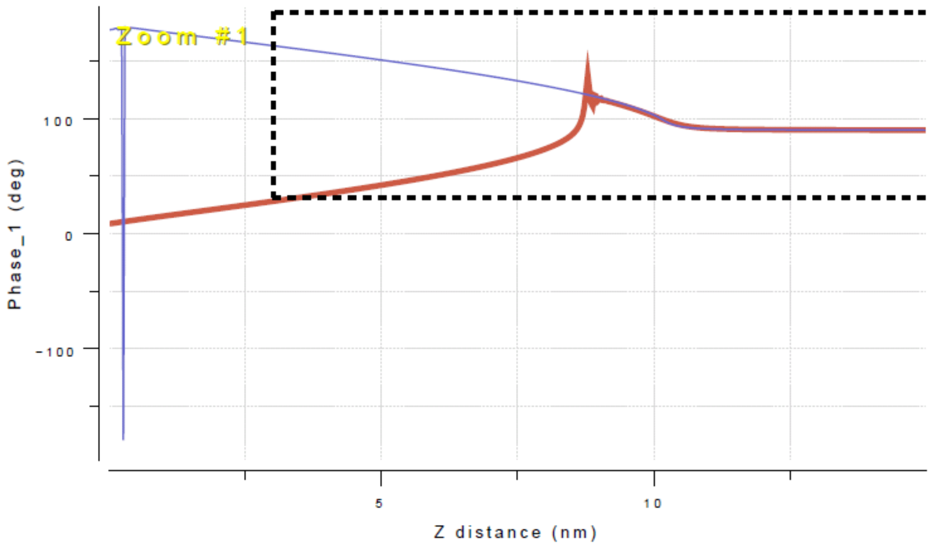
- Consider the following dynamic approach retract curves using VEDA (parameters following example in Garcia and Perez)

The dependence of the low and high oscillation solutions on the rest of the tip–surface separation for a system characterised by R, A_0, $f_0 = f$, k, Q, H, γ and E^* of 20 nm, 10 nm, 350 kHz, 40 N/m, 400, 6.4×10^{-20} J, 30 mJ/m^2 and 1.51 GPa, respectively, are plotted in Fig. 7(a). The collection of L and H



Recognizing attractive and repulsive regimes





- In attractive regime, phase lag is greater than 90 degrees while in repulsive regime it is less than 90 degrees

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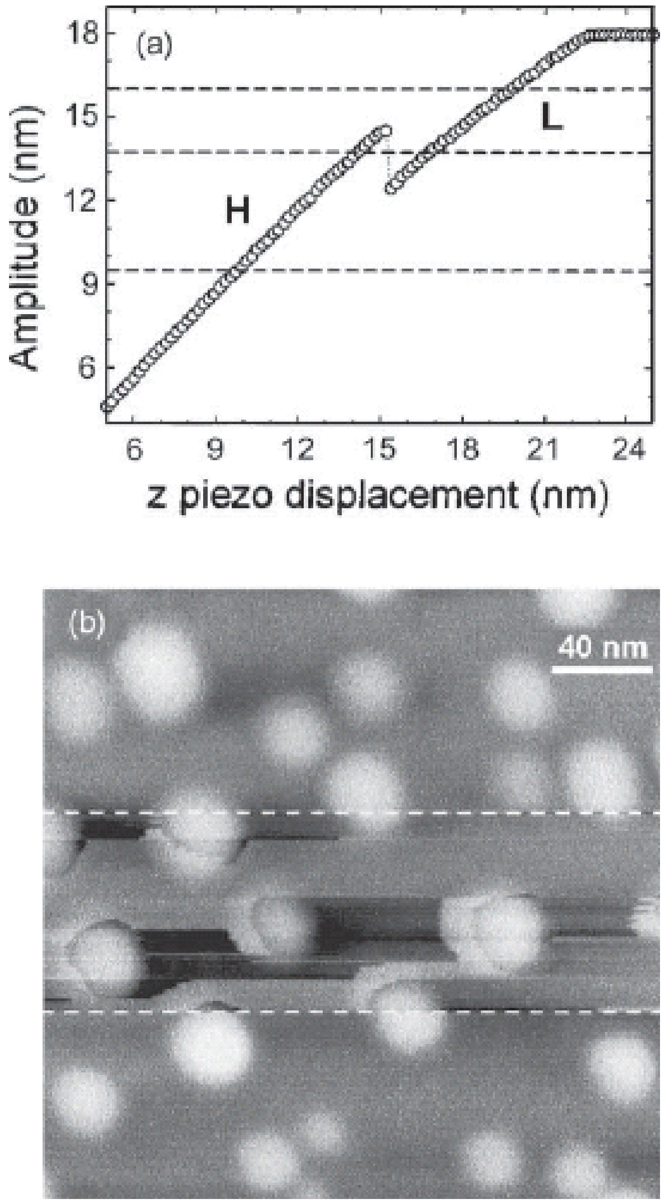
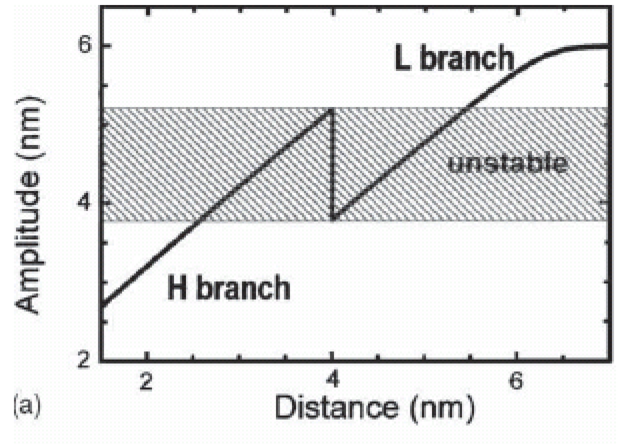
Attractive-repulsive instability





- Soft cantilevers, small amplitudes-> more attractive regime
- Stiff levers, larger amplitudes -> repulsive regime

Fig. 11. Experimental determination of the low and high amplitude branches. (a) Amplitude curve, the L and H branches are plotted by open circles. Dashed lines indicate the A_{sp} values used to image a 200 × 200 nm^2 InAs quantum dot sample. (b) The system evolves from stable imaging in the L state A_{sp} = 16 nm (top) to unstable imaging due to switching between H and L states A_{sp} = 13.8 nm (middle) and finally to stable imaging in the H state A_{sp} = 9.5 nm (bottom). Adapted from [56].