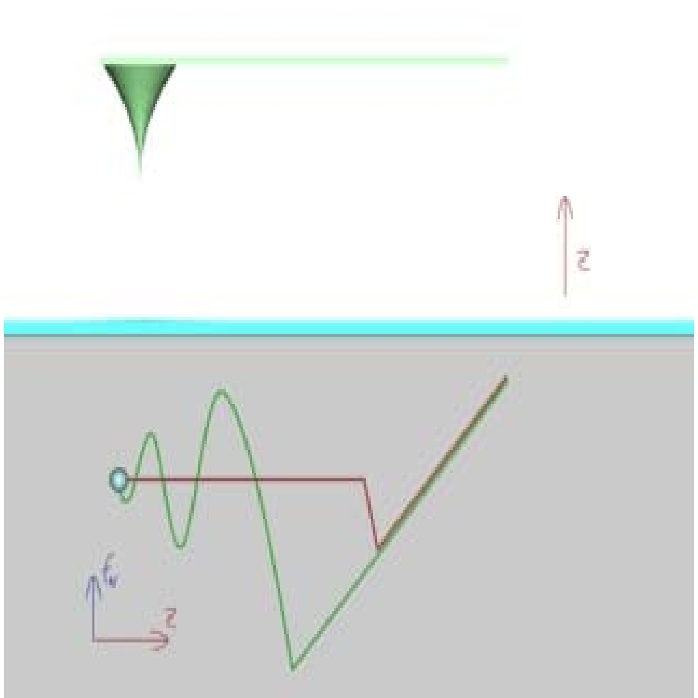
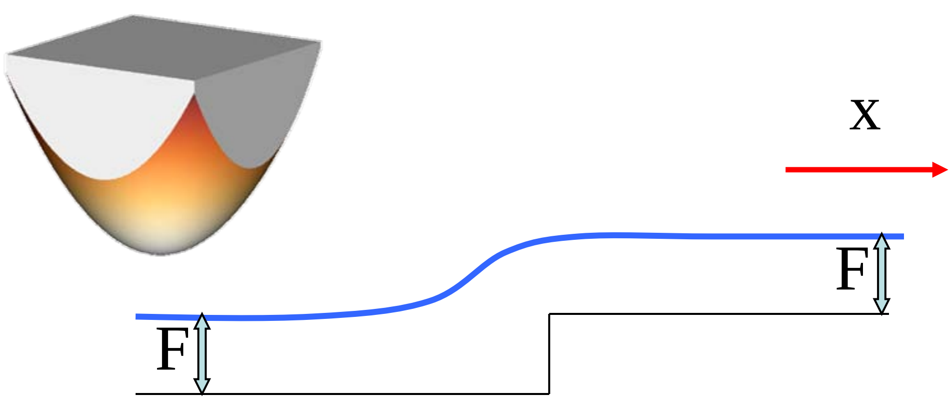
# Lecture 12
# Introduction to dynamic AFM - point mass approximation

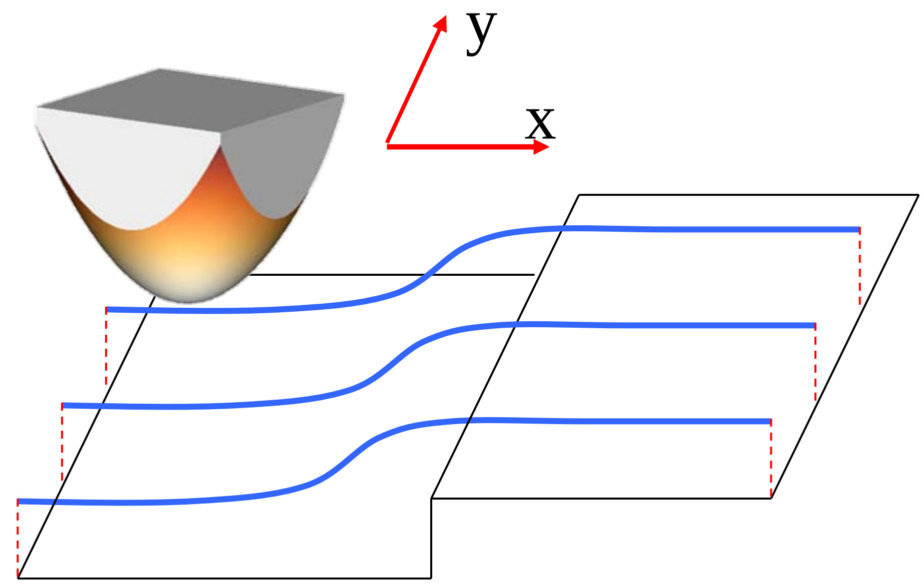
Arvind Raman
*Mechanical Engineering*
*Birck Nanotechnology Center*

PURDUE UNIVERSITY

# Contact Mode Imaging

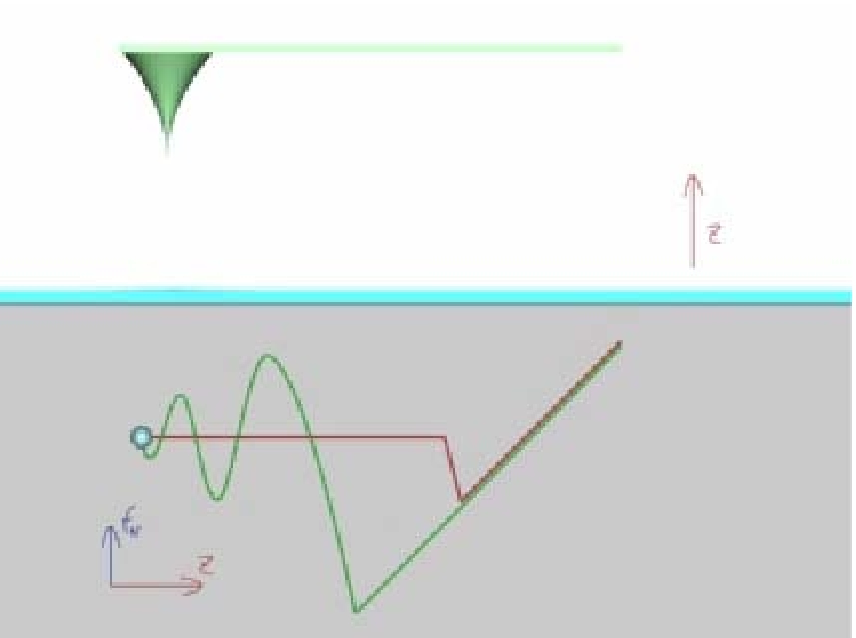


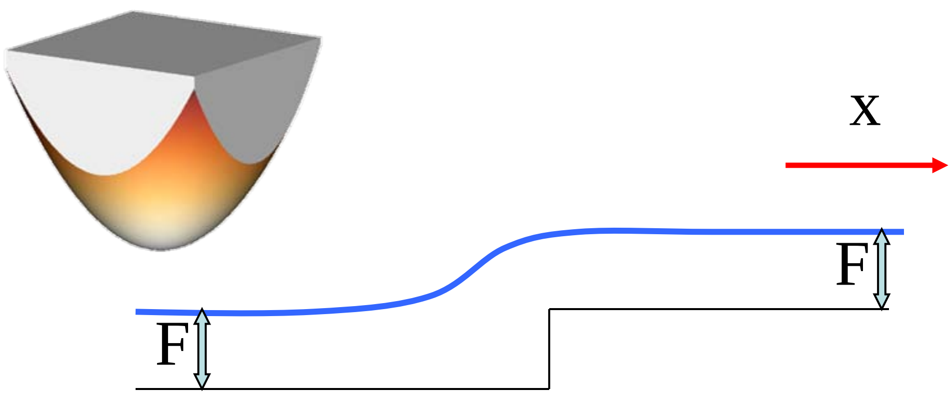


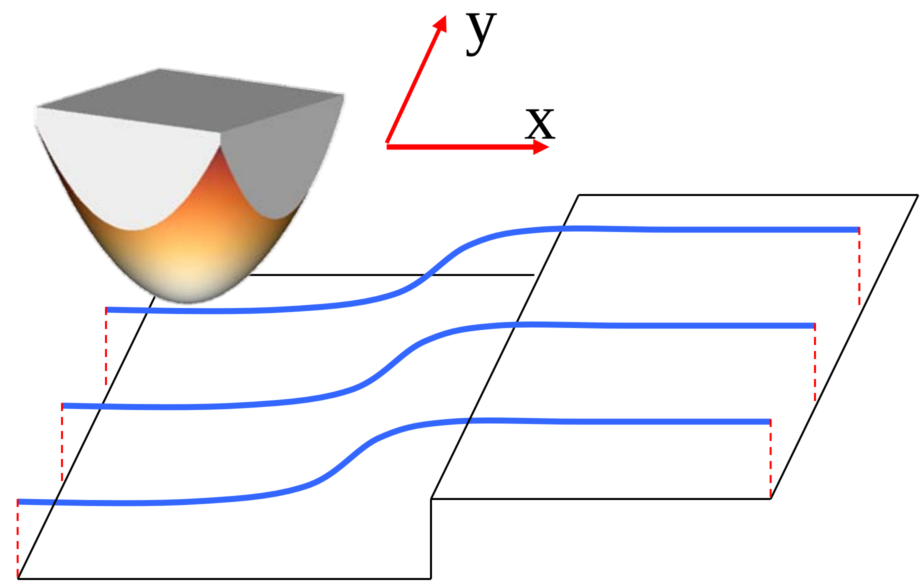




First tip contacts surface with some setpoint normal force which is kept constant during the scan

# Contact Mode Imaging





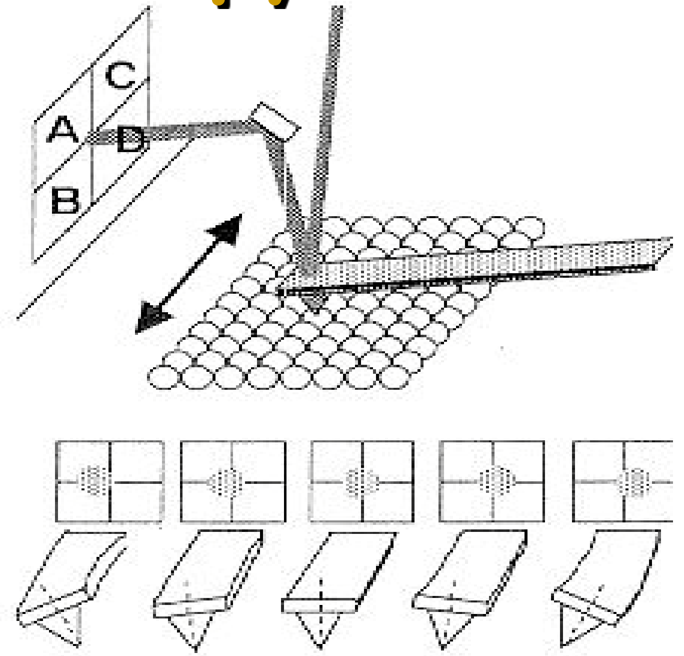



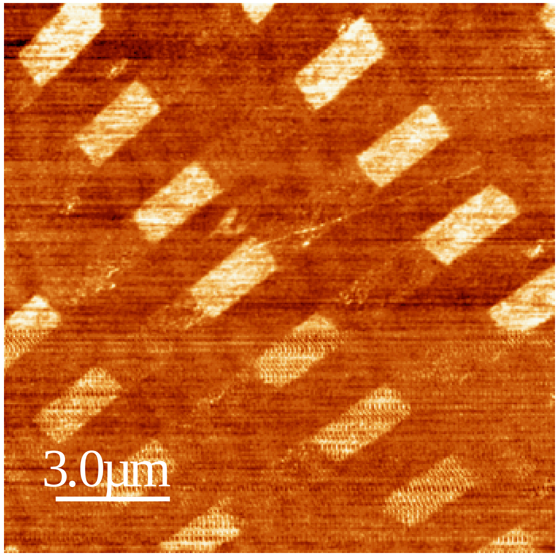

First tip contacts surface with some setpoint normal force which is kept constant during the scan

PURDUE UNIVERSITY

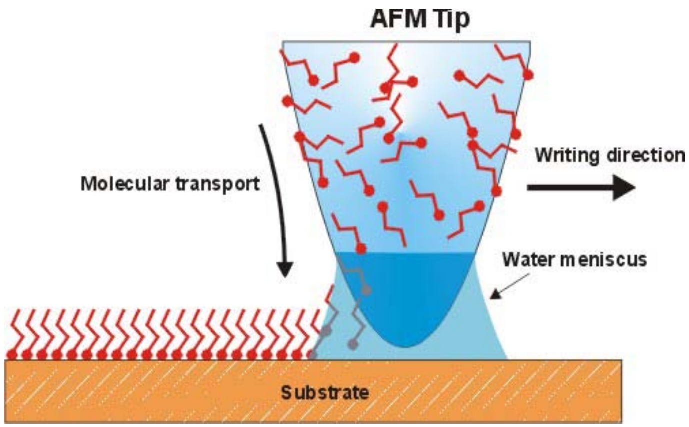
# Friction Force Microscopy

- Torsional deflections due to atomic and molecular friction
- Lateral forces are specific
- Applications to nanotribology, probe based lithography

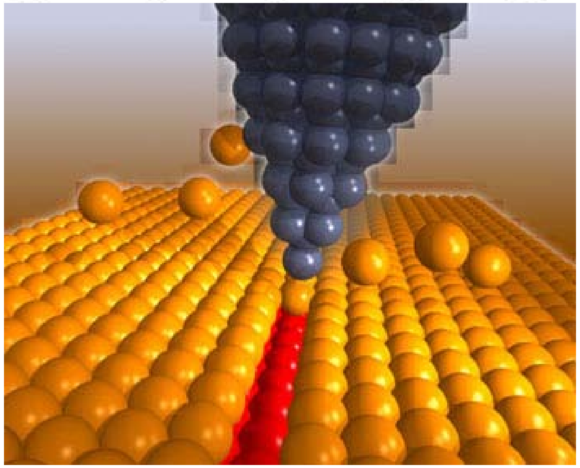



Friction force image of a self assembled monolayer (Riefenberger Group)

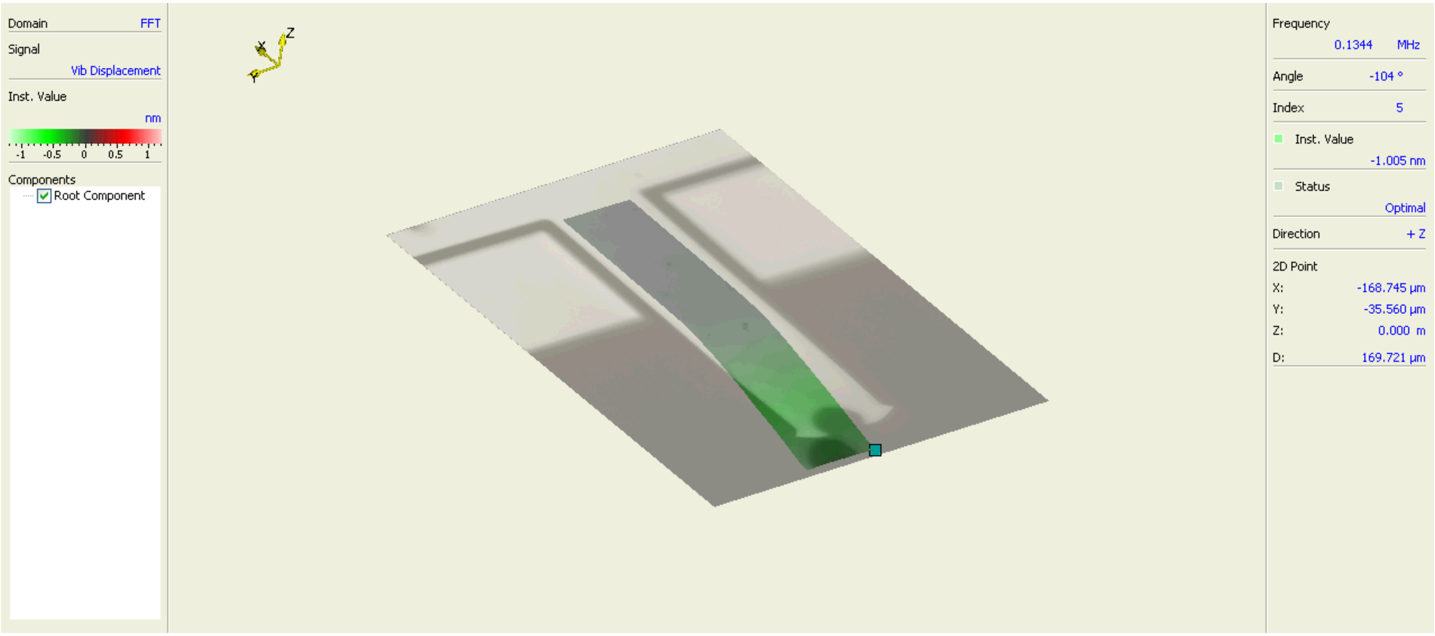



www.chem.nwu.edu/~mkngrp/
Dip-pen lithography



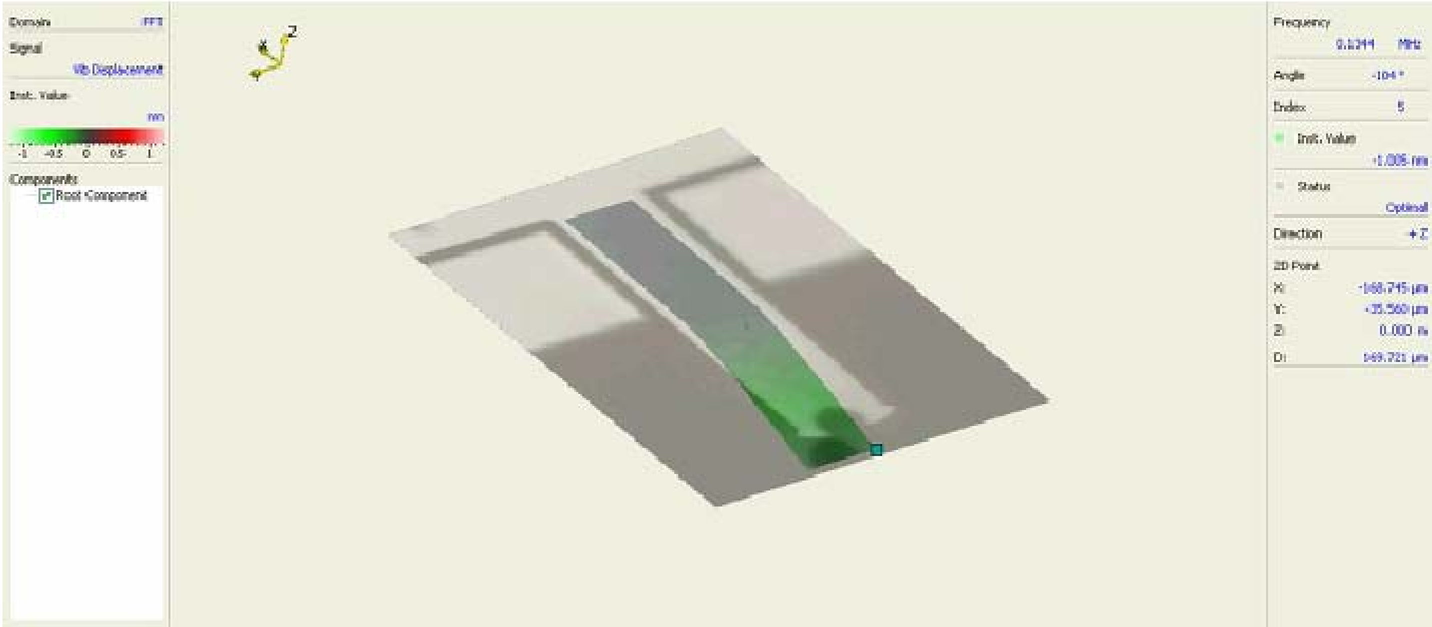
Contact mode oxidation lithography

# Dynamic AFM




- Cantilever driven near resonance
- The cantilever's resonant frequency, phase and amplitude are affected by short-scale force gradients
- In Amplitude Modulated AFM (AM-AFM) or tapping mode, driving frequency is fixed while cantilever approaches the sample
- In Frequency Modulated AFM (FM-AFM) the phase and amplitude are held constant while approaching the sample

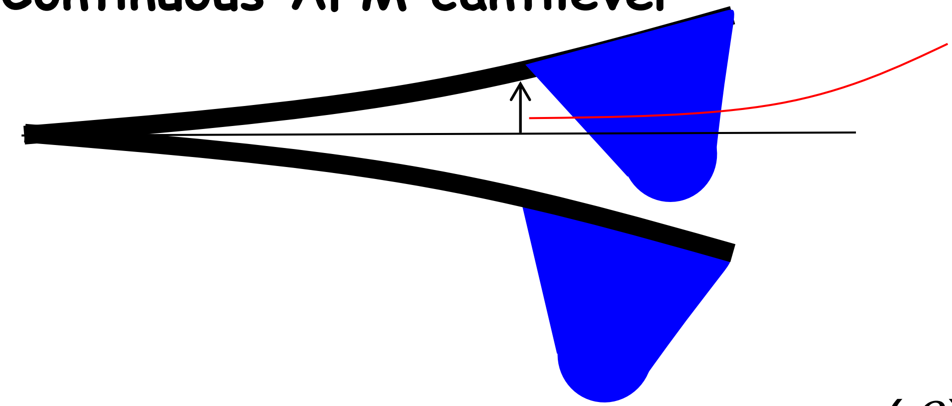
# Dynamic AFM




- Cantilever driven near resonance
- The cantilever's resonant frequency, phase and amplitude are affected by short-scale force gradients
- In Amplitude Modulated AFM (AM-AFM) or tapping mode, driving frequency is fixed while cantilever approaches the sample
- In Frequency Modulated AFM (FM-AFM) the phase and amplitude are held constant while approaching the sample

# The point mass model

**Continuous AFM cantilever**

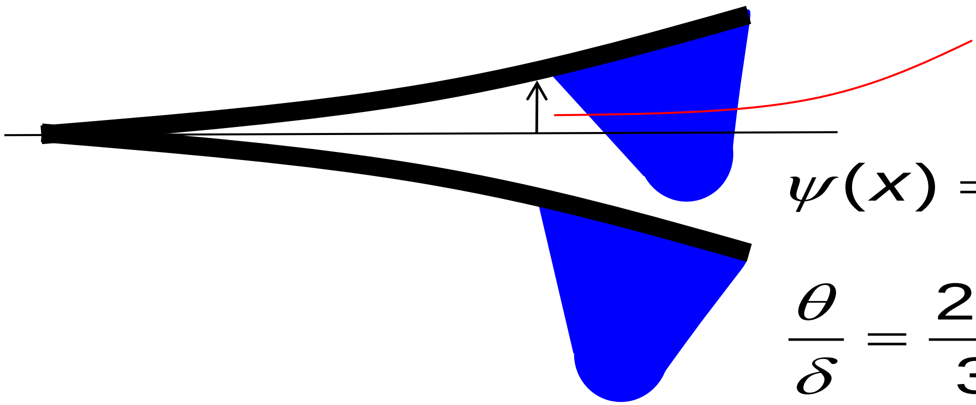


w(x,t) = A sin(ωt) ψ(x)

θ/δ=?

$$\psi(x)=\cos(\beta\frac{x}{L})-\cosh(\beta\frac{x}{L})-\frac{\cos(\beta)+\cosh(\beta)}{\sin(\beta)+\sinh(\beta)}\left[\sin(\beta\frac{x}{L})-\sinh(\beta\frac{x}{L})\right]$$

**Point mass model**



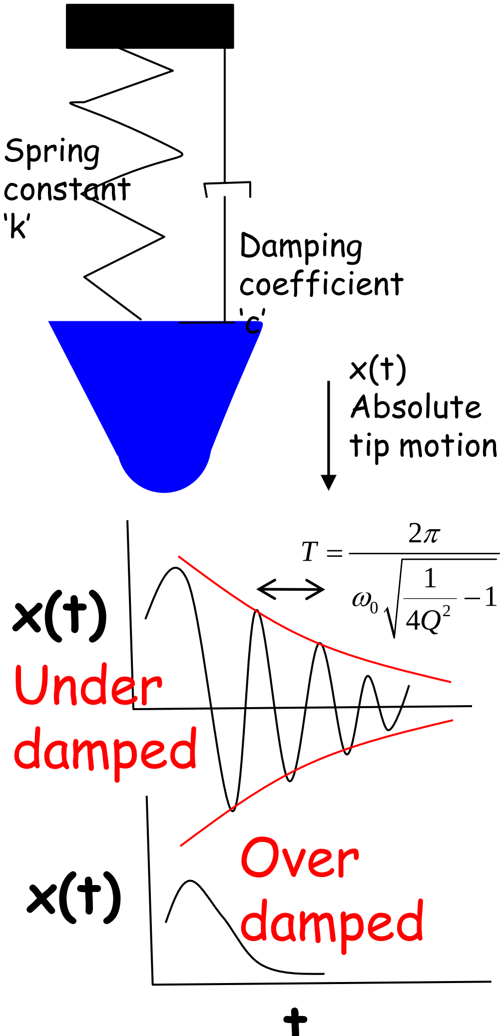
w(x,t) = A sin(ωt) ψ(x)

$$\psi(x)=-\frac{L^3}{6EI}\left(\frac{x}{L}\right)^3 x^3+\frac{1}{2}\frac{L^3}{EI}\left(\frac{x}{L}\right)^2$$

$$\frac{\theta}{\delta}=\frac{2L}{3}$$

- Tip is massive, cantilever inertia negligible
- Replace cantilever by a spring of spring constant= static bending stiffness of lever
- Cantilever oscillates such that θ/δ=2L/3

# Point mass model – free oscillations




$$m\ddot{x} = -kx - c\dot{x}$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = 0; \ with \quad \omega_0 = \sqrt{\frac{k}{m}},\ Q = \frac{m\omega_0}{c} = \frac{\sqrt{mk}}{c}$$

*General solution of type*

$$x(t) = e^{\lambda t} \Rightarrow \frac{\lambda^2}{\omega_0^2} + 1 + \frac{\lambda}{\omega_0 Q} = 0 \Rightarrow \lambda_{1,2} = -\frac{\omega_0'}{2Q} \pm \omega_0\sqrt{\frac{1}{4Q^2} - 1}$$

$x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$, **integration constants to be determined from initial conditions** $x(0), \dot{x}(0)$

$$x(t) = e^{-\frac{\omega_0}{2Q}t}\left( x(0)\cos\left(\sqrt{1 - \frac{1}{4Q^2}}\,\omega_0 t\right) + \frac{\dot{x}(0) + \omega_0\frac{x(0)}{2Q}}{\left(\sqrt{1 - \frac{1}{4Q^2}}\right)\omega_0}\cos\left(\sqrt{1 - \frac{1}{4Q^2}}\,\omega_0 t\right)\right)$$

$if \ Q > \frac{1}{2}$ **Underdamped oscillation**

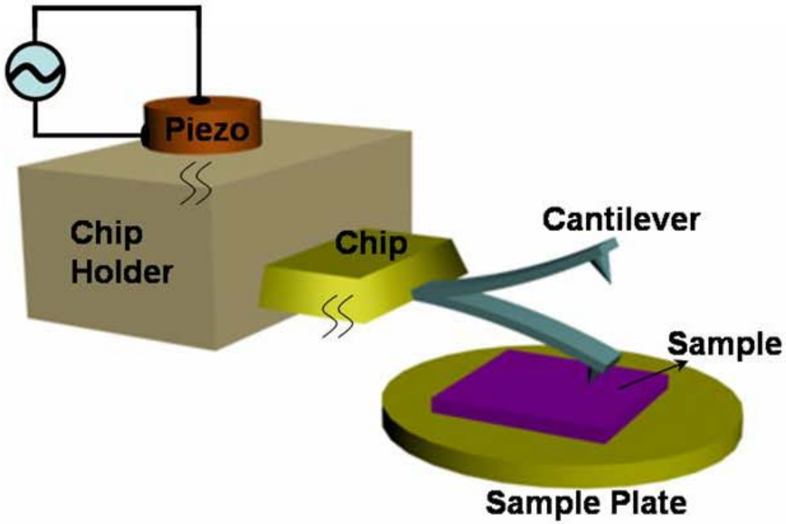
$$x(t) = c_1 e^{\left(-\frac{\omega_0'}{2Q} + \omega_0\sqrt{\frac{1}{4Q^2} - 1}\right)t} + c_2 e^{\left(-\frac{\omega_0'}{2Q} - \omega_0\sqrt{\frac{1}{4Q^2} - 1}\right)t}$$

$if \ Q < \frac{1}{2}$ *Over***damped oscillation**

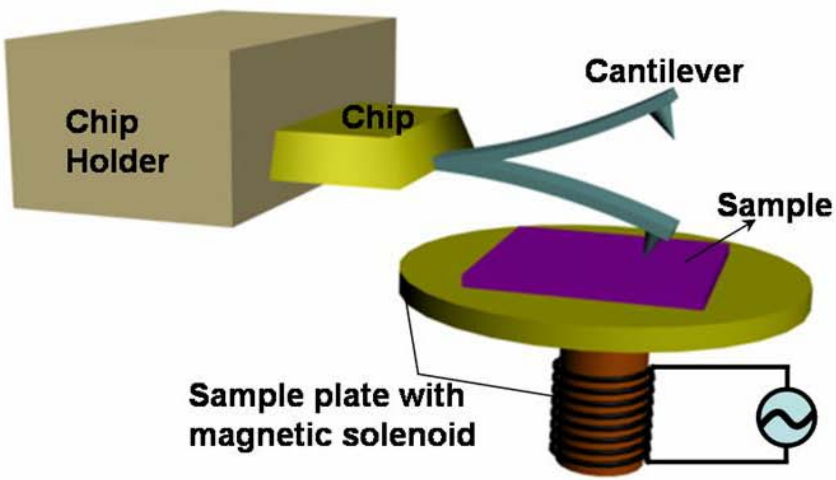
- Damped natural frequency is different from natural frequency
- Q can be regarded as number of oscillation cycles before transients become small

PURDUE
UNIVERSITY

# Forced vibrations




a. Acoustic excitation

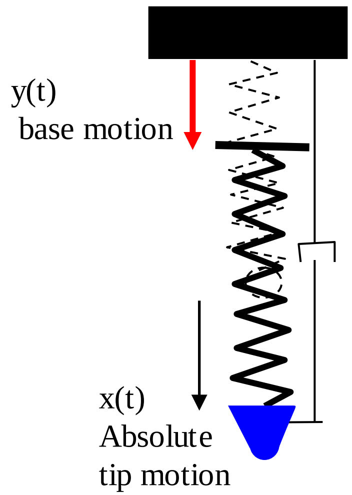



b. Magnetic excitation

- Mechanical (acoustic or piezo excitation)
- Magnetic excitation
- Magnetostrictive excitation
- Photothermal excitation
- Lorentz force excitation
- Ultrasound excitation
- Direct piezoelectric excitation

# Response of acoustically excited levers

$$m\ddot{x} = -k(x-y) - c\dot{x}$$




***Acoustic (inertial or piezo)***

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = y(t);\ with \quad \omega_0 = \sqrt{\frac{k}{m}},\ Q = \frac{m\omega_0}{c}$$

$$Measured\ motion\ z(t) = x(t) - y(t)$$

$$\frac{\ddot{z}}{\omega_0^2} + z + \frac{1}{\omega_0 Q}\dot{z} = -\frac{\ddot{y}}{\omega_0^2} - \frac{1}{\omega_0 Q}\dot{y}$$

$$y(t) = Y_0 \ \sin(\omega t)$$

$$z^p(t) = A\sin(\omega t - \phi_{inertial})$$

$$|H_{inertial}(\omega)| = \frac{A}{Y_0} = \left(\frac{r^4 + (r/Q)^2}{(1-r^2)^2 + (r/Q)^2}\right)^{1/2}$$

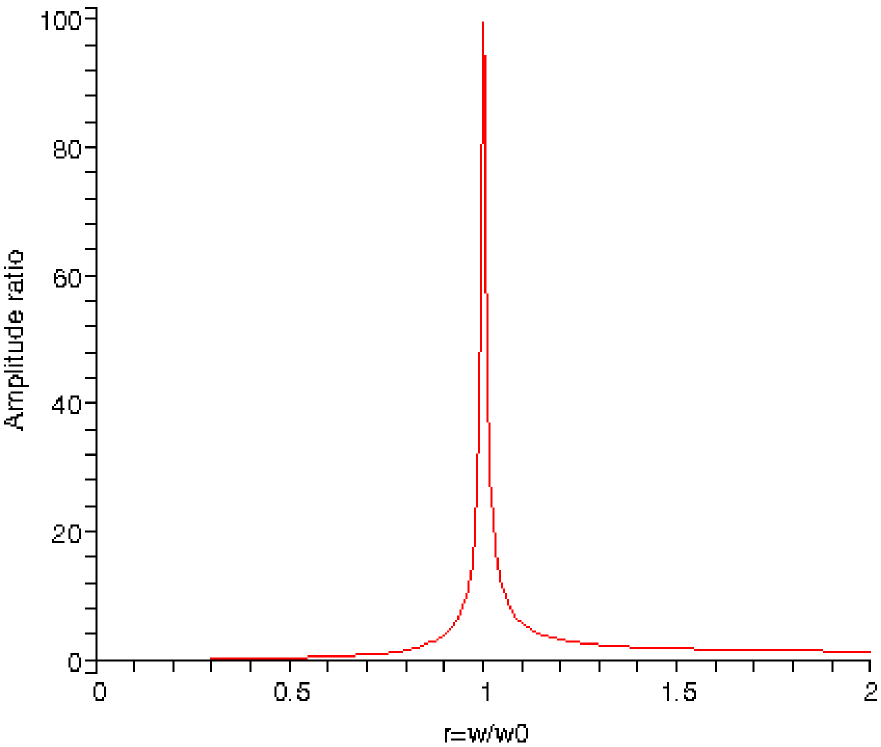
$$\phi_{inertial}(\omega) = \tan^{-1}\left(\frac{Q}{r(1+Q^2 r^2 - Q^2)}\right)$$

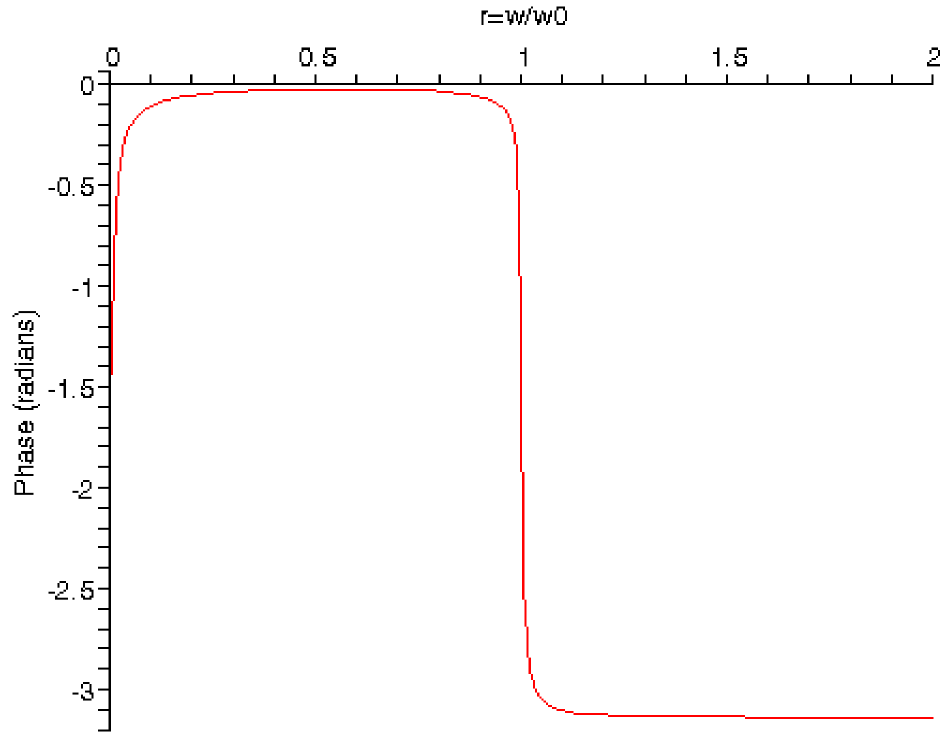
$$where\ r = \frac{\omega}{\omega_0}$$

- $\omega_0$ is the natural freq, $\omega$ is the drive freq
- Maximum amplitude occurs when $\omega > \omega_0$!
- Base motion amplitude at r=1 is *A*/Q!

PURDUE UNIVERSITY

# Response of acoustically excited levers

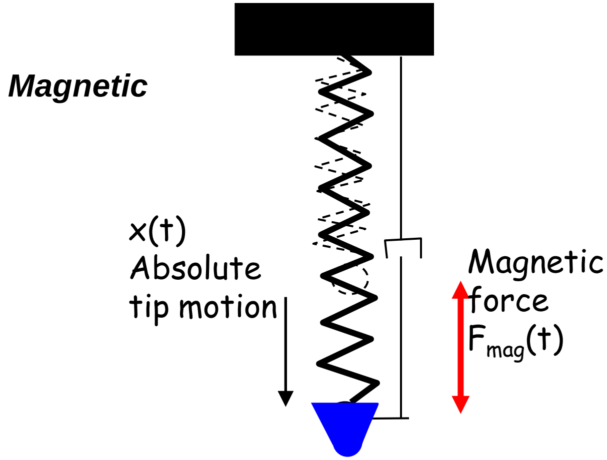



- For Q=100, see response above
- Asymmetric peak, amplitude greater when $\omega>\omega_0$

PURDUE UNIVERSITY

# Response of directly excited AFM levers




$$m\ddot{x} = -kx - c\dot{x} + F_{mag}(t)$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = \frac{1}{k}F_{mag}(t);\ with \quad \omega_0 = \sqrt{\frac{k}{m}},\ Q = \frac{m\omega_0}{c}$$

$$Measured\ motion = x(t)$$

$$F_{mag}(t) = F_0\ \sin(\omega t)$$

$$x^p(t) = A\sin(\omega t - \phi_{magnetic})$$

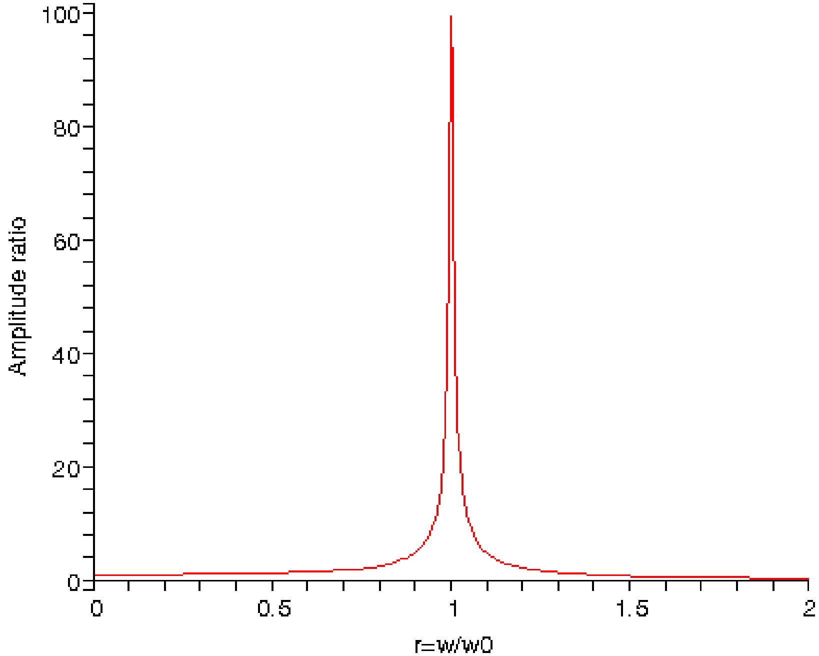
$$|H_{mag}(\omega)| = \frac{A}{F_0 / k} = \left(\frac{1}{(1-r^2)^2 + (r/Q)^2}\right)^{1/2}$$

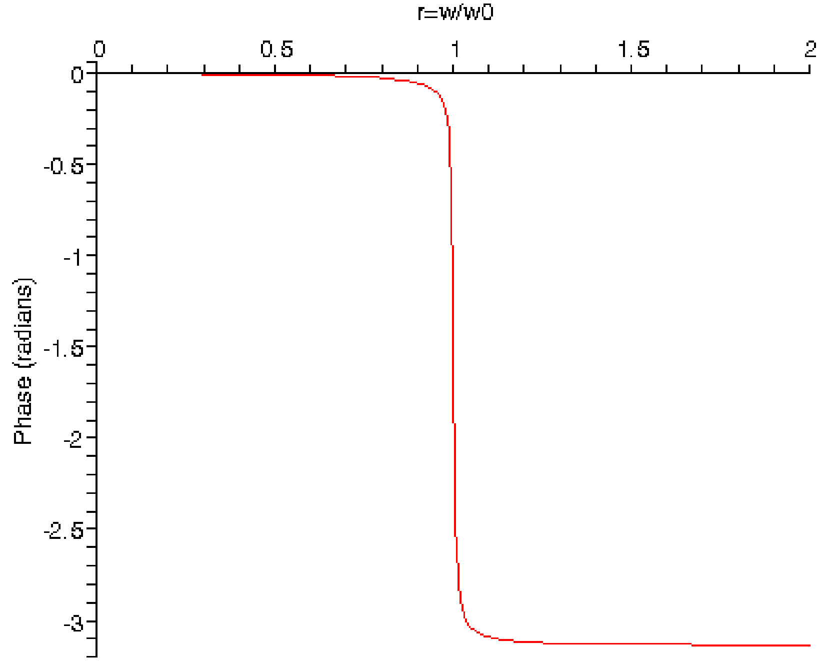
$$\phi_{mag}(\omega) = \tan^{-1}\left(\frac{r}{Q(r^2-1)}\right)$$

$$where\ r = \frac{\omega}{\omega_0}$$

- $\omega_0$ is the natural freq, $\omega$ is the drive freq
- Maximum amplitude occurs when $\omega<\omega_0$!
- For $\omega<<\omega_0$ $A=F_{mag}/k$!

# Response of directly excited AFM levers

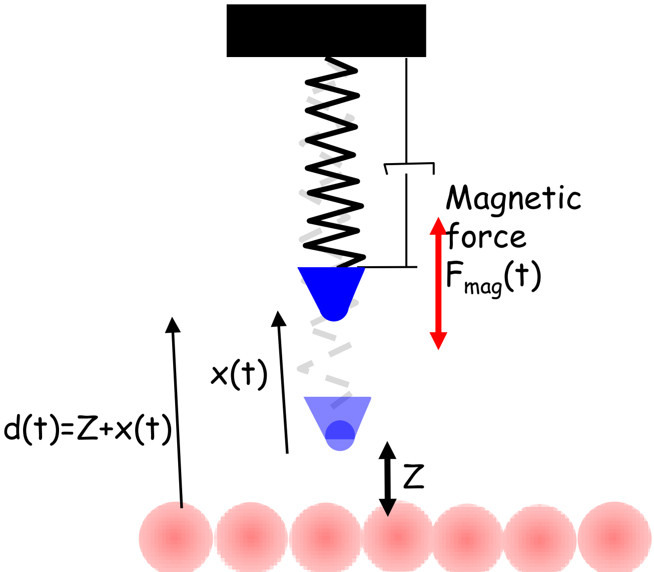



- Asymmetric response with greater amplitude when $\omega<\omega_0$!
- Classical phase response

PURDUE UNIVERSITY

# Driven point mass model with tip-sample interaction




*Magnetic*

$$m\ddot{x} = -kx - c\dot{x} + F_{mag}(t) + F_{ts}(Z + x(t))$$

$$\frac{\ddot{x}}{\omega_0^2} + x + \frac{1}{\omega_0 Q}\dot{x} = \frac{1}{k}\left(F_{mag}(t) + F_{ts}(Z + x(t))\right);$$

$$\text{with} \quad \omega_0 = \sqrt{\frac{k}{m}},\ Q = \frac{m\omega_0}{c}$$

$$\text{Measured motion} = x(t)$$

$$F_{mag}(t) = F_0 \sin(\omega t)$$

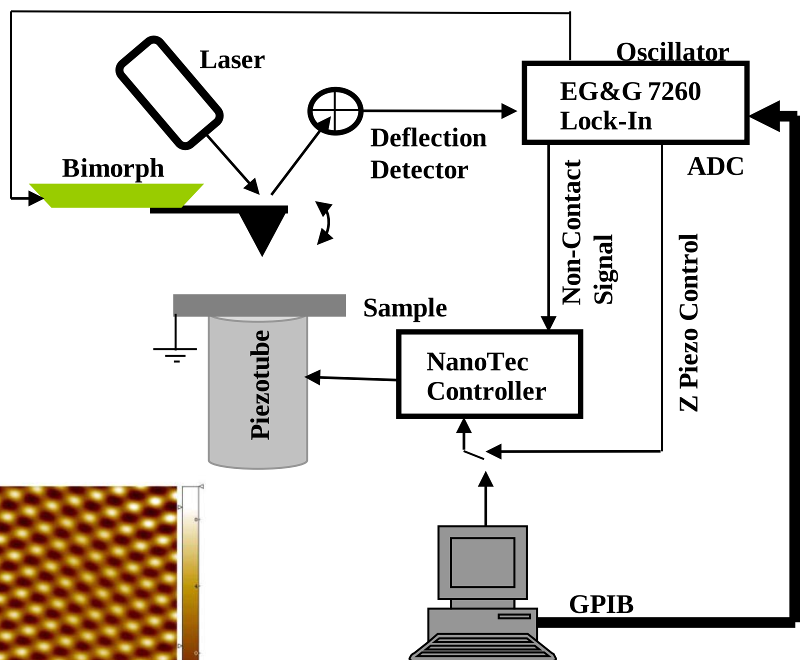
*Acoustic excitation*

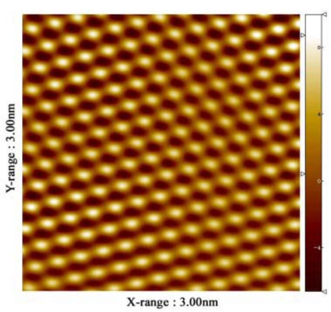
$$\frac{\ddot{z}}{\omega_0^2} + z + \frac{1}{\omega_0 Q}\dot{z} = -\frac{\ddot{y}}{\omega_0^2} - \frac{1}{\omega_0 Q}\dot{y} + \frac{F_{ts}(Z + y(t) + x(t))}{\omega_0^2}$$

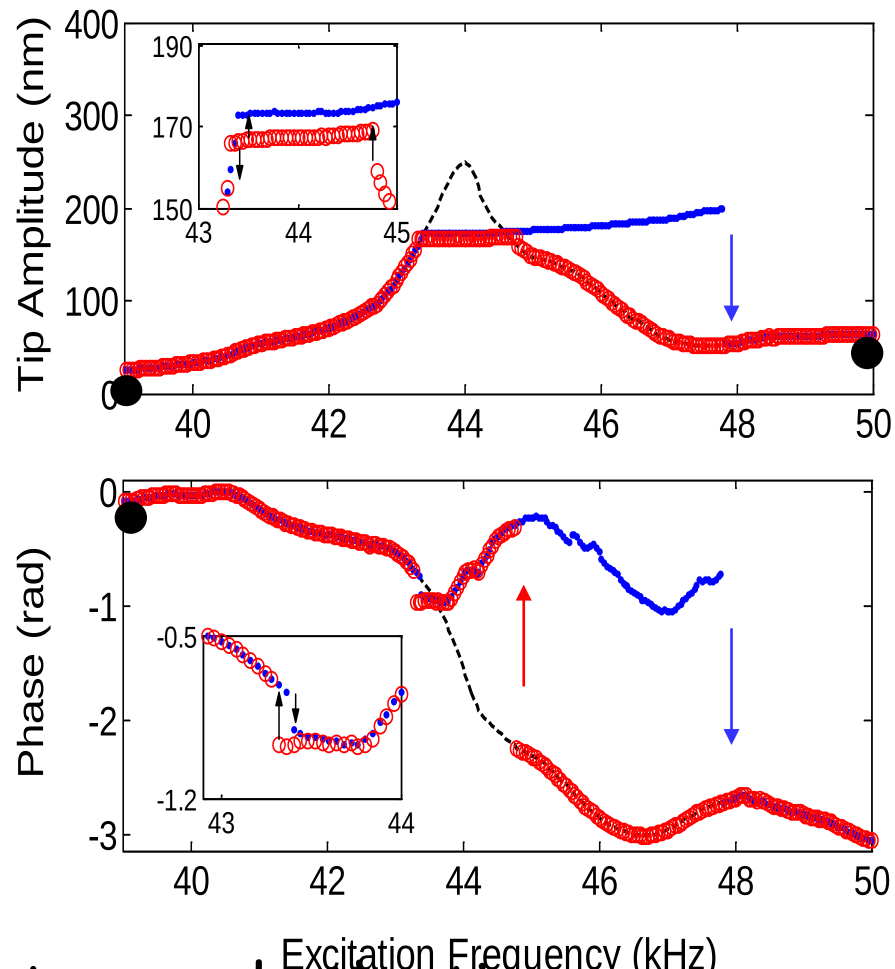
- Highly nonlinear ordinary differential equation
- What happens to frequency response when probe is brought close to sample?

# Experiments with conventional tips

- **Si tip / HOPG sample z=90 nm, frequency sweep**




- When brought closer to sample the tip sometimes sticks to the sample

*Lee et al, Phys Rev B (2002)*

PURDUE
UNIVERSITY