2009 NCN@Purdue-Intel Summer School
Notes on Percolation and Reliability Theory

Lecture 9
Breakdown in Thick Dielectrics

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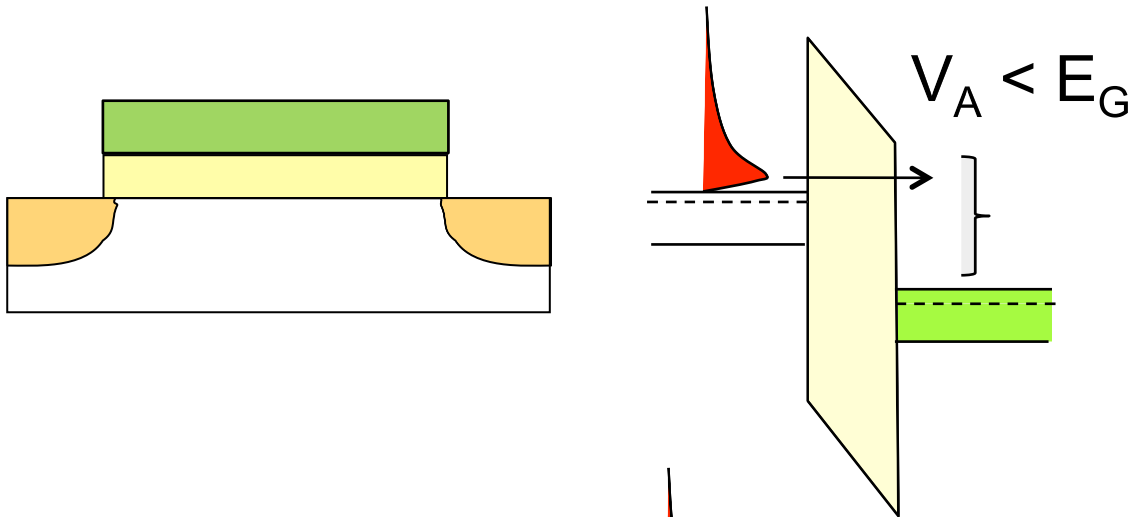
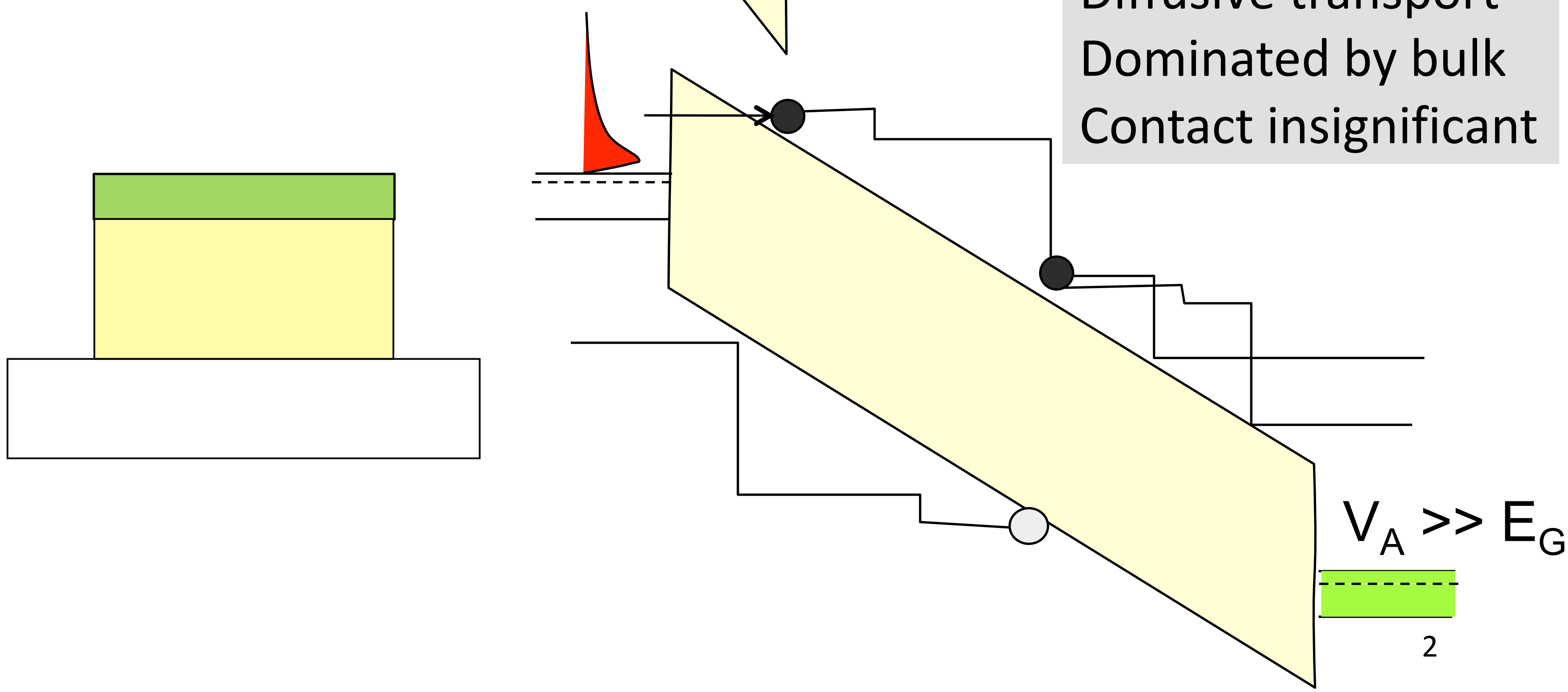
breakdown in thick vs. thin oxides



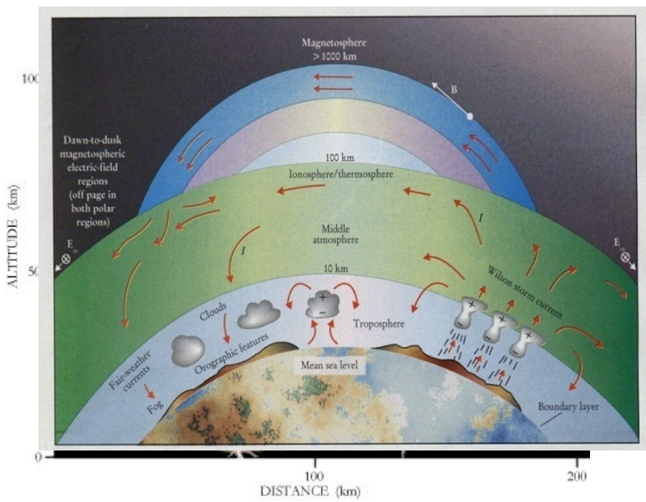
Ballistic transport
Hot contact
Contact dictates BD

Diffusive transport
Dominated by bulk
Contact insignificant



dielectric breakdown in everyday life

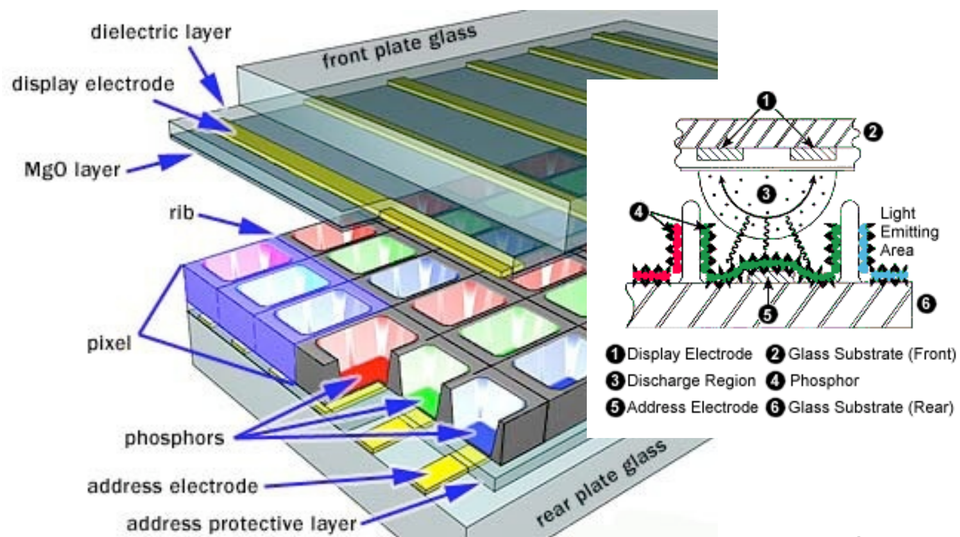
Lightning



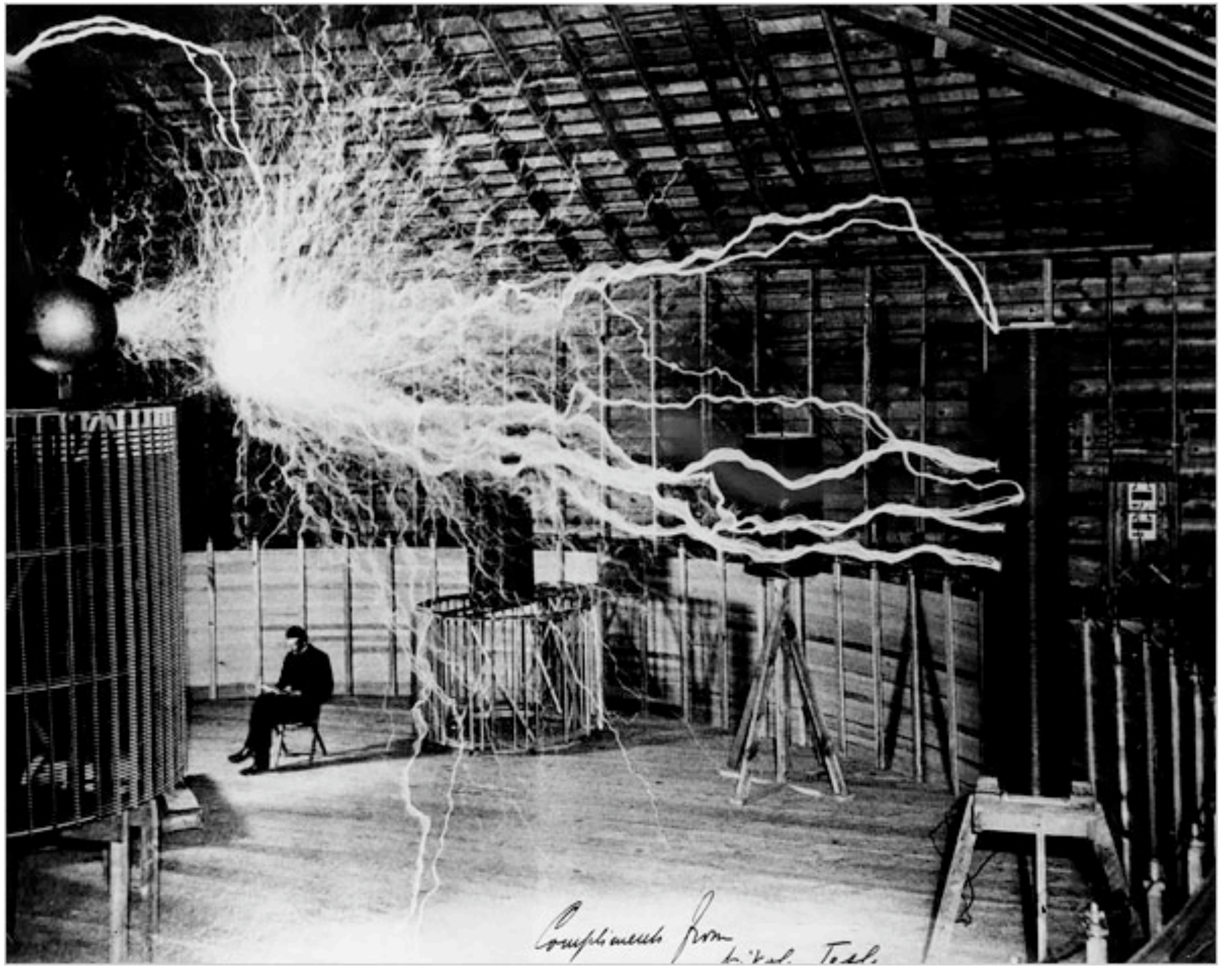
Unrolling a scotch tape

Putterman, Nature, 2008

Plasma TV



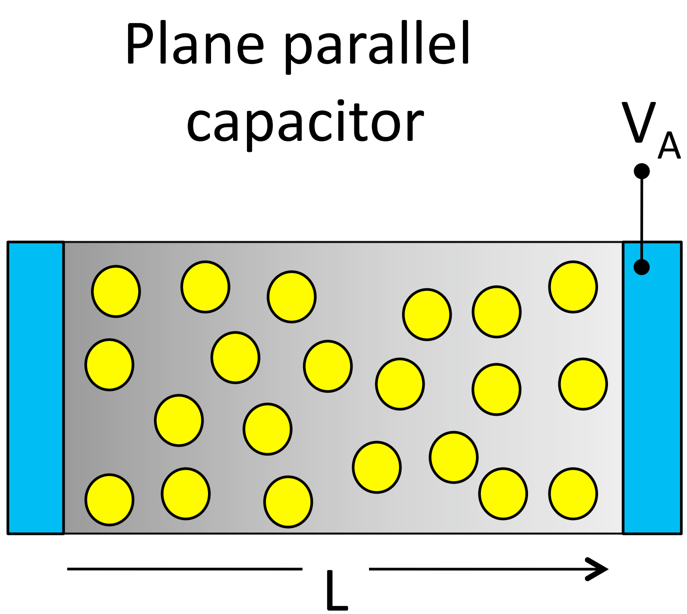
Tesla coils ....



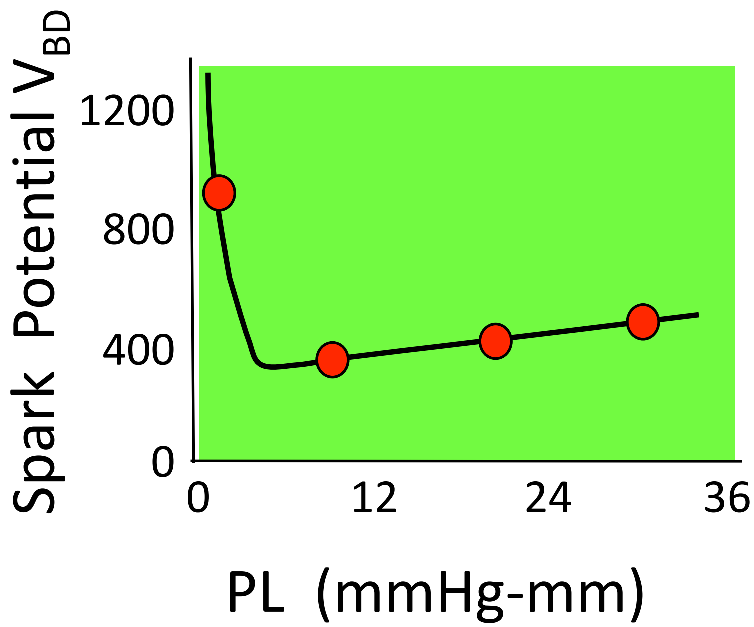
outline of lecture 9

1) **Breakdown in gas dielectric and Paschen's law**
2) Spatial and temporal dynamics during breakdown
3) Breakdown in bulk oxides: puzzle
4) Theory of pre-existing defects: Thin oxides
5) Theory of pre-existing defects: thick oxides
6) Conclusions

Paschen's law (experiment)



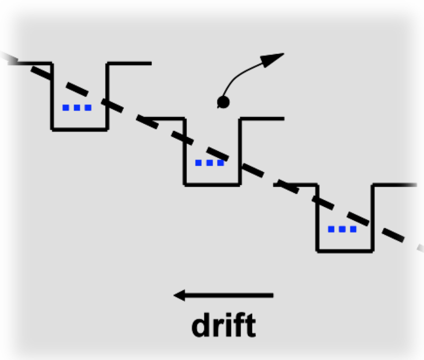
$$V_{BD} = \frac{a(P \times L)}{\ln(P \times L) + c}$$



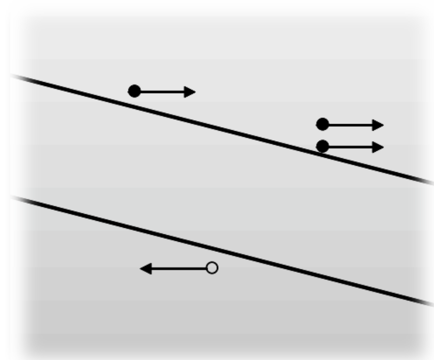
Many gases show this breakdown behavior ....

stages of ionization and breakdown

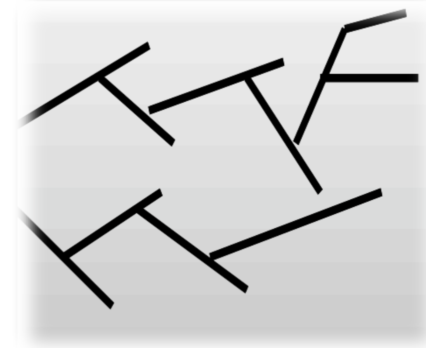
1. Field Ionization

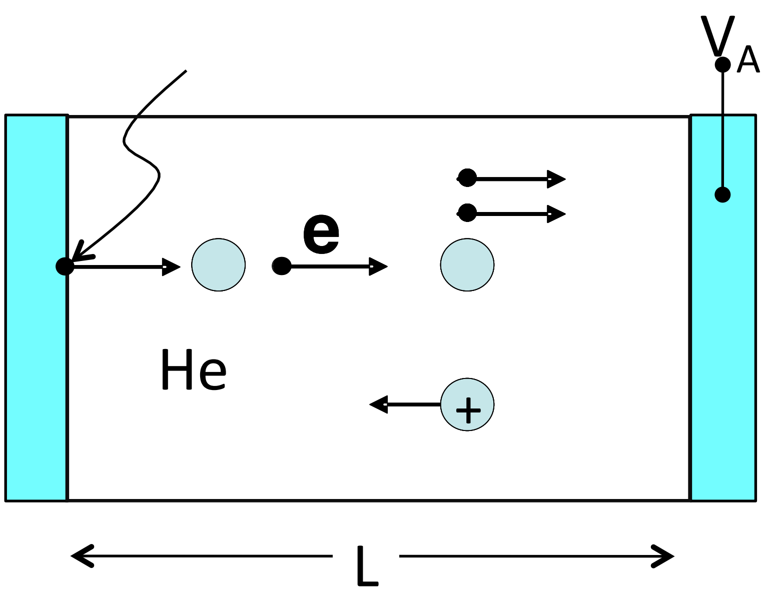


2. Avalanche

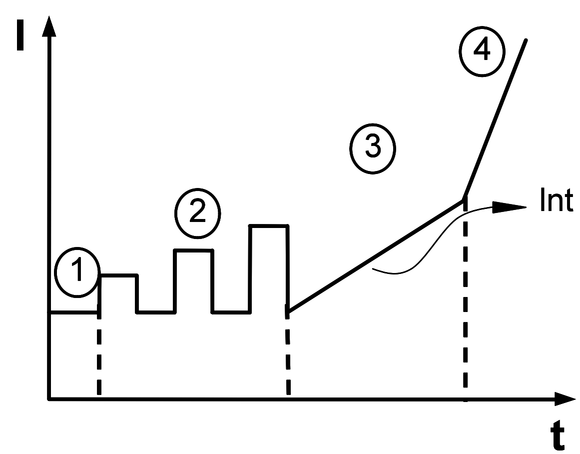


3. Spatial Dynamics





4. Time Signatures



(1) basics of field ionization

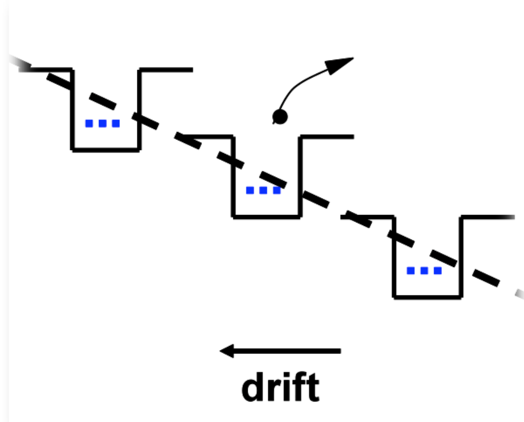
Energy flux balance ...

$$q\mathcal{E}\upsilon = \frac{E - E_0}{\tau} \qquad \langle E - E_0 \rangle \approx \langle E \rangle = q\mathcal{E} \times \upsilon\tau \equiv q\mathcal{E}\lambda$$

Energy balance ...

$$\langle E \rangle = \frac{3}{2}k_B T_e + \frac{1}{2}m^2\upsilon^2 \approx \frac{3}{2}k_B T_e$$

$$\langle E \rangle = q\mathcal{E}\upsilon\tau = 3k_B T_e/2$$



Ionization/Length ....

$$\alpha \sim \overbrace{N}^{p_c} \times \overbrace{e^{-E_0/kT_e}}^{p_e} \sim N \times e^{-3E_0/2q\mathcal{E}\lambda}$$

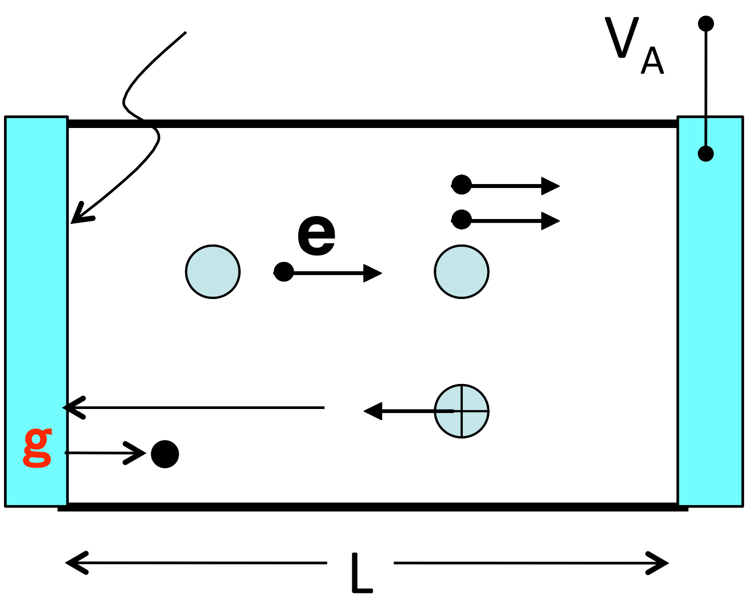
(2) avalanche: initiation

$$\frac{dn}{dx} = \alpha n \quad n(x) = n_0 e^{\alpha d} + \gamma\left[n_0\left(e^{\alpha d} - 1\right)\right]e^{\alpha d} + \ldots$$

+ve ion

Cathode factor

$$= \frac{n_0 e^{\alpha d}}{1 - \gamma\left(e^{\alpha d} - 1\right)}$$



At breakdown, even noise is amplified to runaway condition ….
The numerator diverges …

$$\ln\left(1 + \frac{1}{\gamma}\right) \sim \alpha_{BD} L$$

(2) avalanche: breakdown voltage

Ionization coefficient and pressure ....

$$\alpha_{BD} \sim \overbrace{N}^{p_c} \times e^{\overbrace{-3E_{0,i}/2q\mathcal{E}_{BD}\lambda}^{p_e}} \sim B_i P \times e^{-A_i P (L/V_{BD})}$$

and breakdown condition ...

$$\ln\left(1+\frac{1}{\gamma}\right) \sim \alpha_{BD} L$$

Implies Pachen's law ...

$$V_{BD} = \frac{A_i P \times L}{\ln(B_i P \times L) - \ln\left\{\ln\left(1+\gamma^{-1}\right)\right\}}$$

$$V_{BD} = \frac{a(P \times L)}{\ln(P \times L) + c}$$

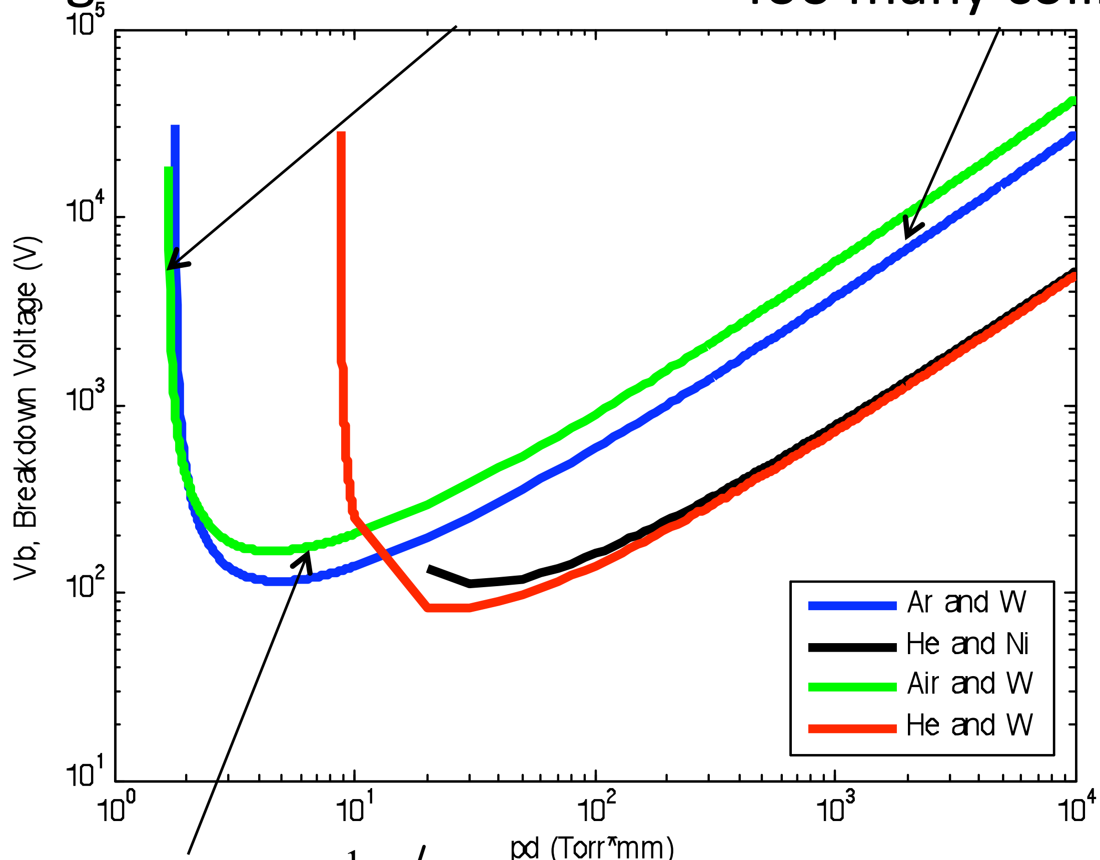
(2) paschen’s law (experiment)

$$V_{BD} = \frac{a(P \times L)}{\ln(P \times L) + c}$$

a and c material constants

Insignificant ionization

Too many collisions



$$(P \times L)_{\min} = e \ln(1 + \gamma^{-1}) / B$$

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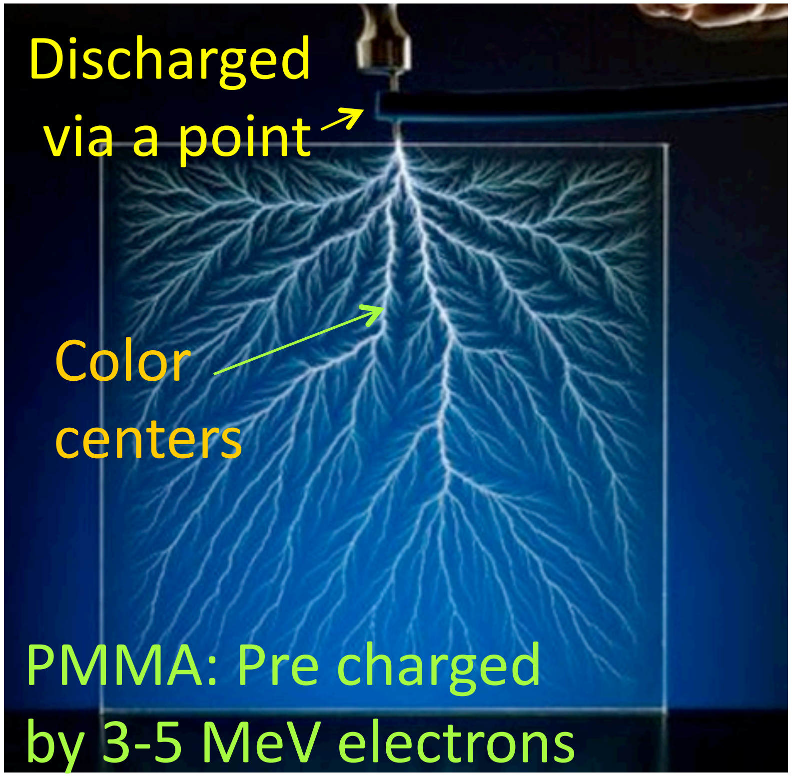
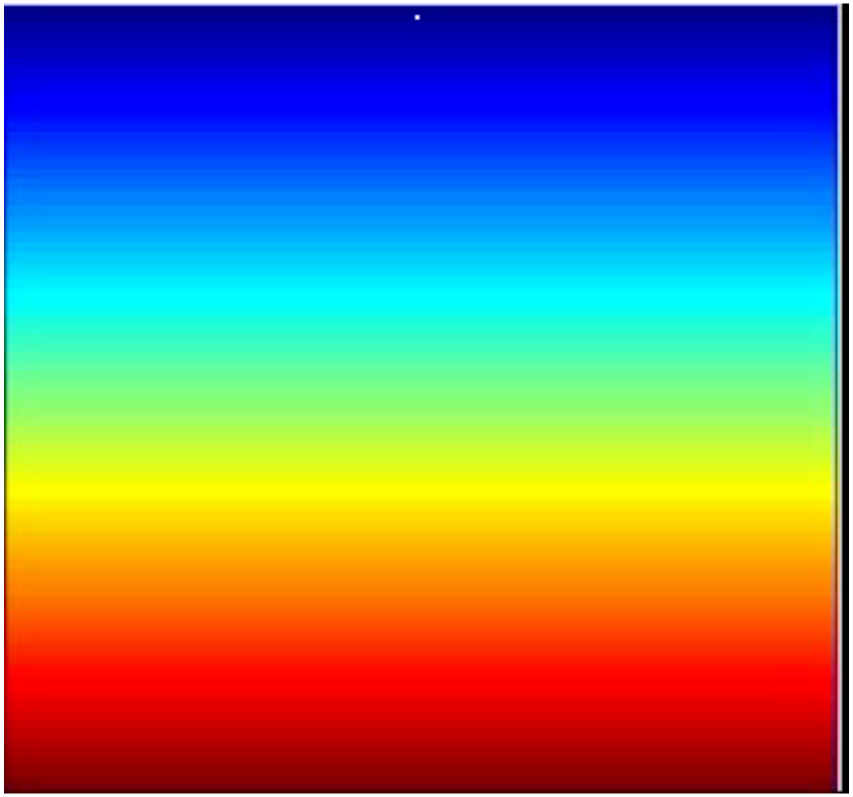
(3) spatial dynamics at breakdown

Lichtenberg figures



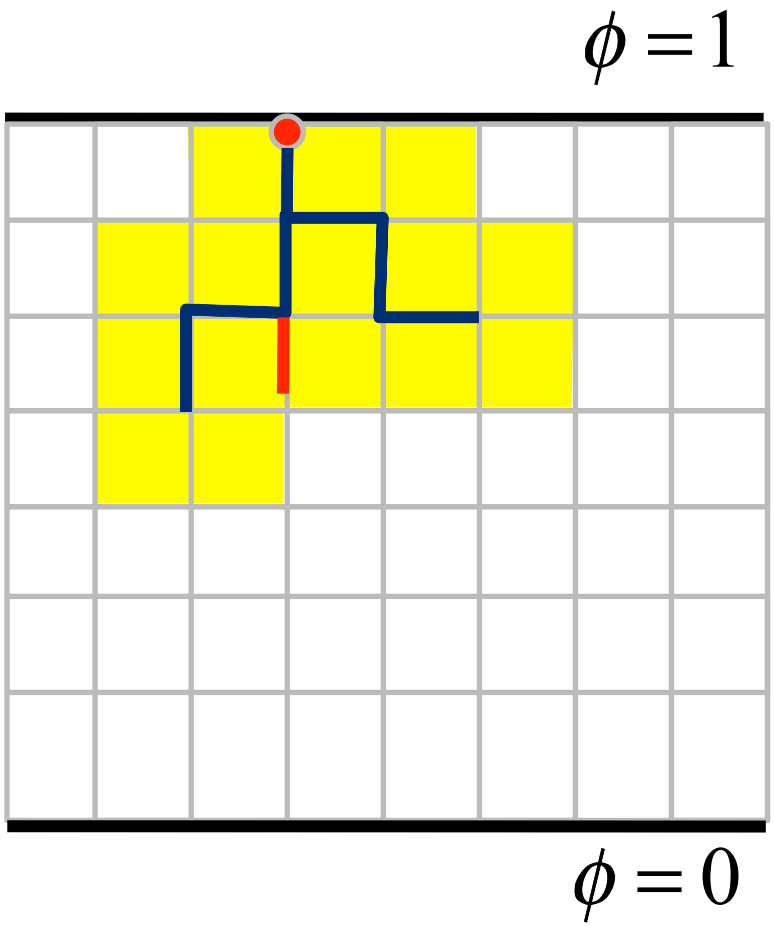
Niemeyer, PRL,52(12), 1984.



Fractal dimension ~ 1.6-1.7

One of the great successes in understanding BD in 1980s …

Algorithm for Fractal Model of Breakdown

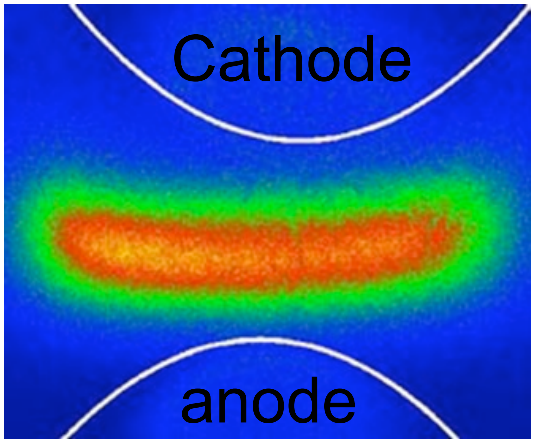


- Pattern grows stepwise
- f=0 for all discharged points
- Solve Poisson equation at every time-step
- One step is selected at each step

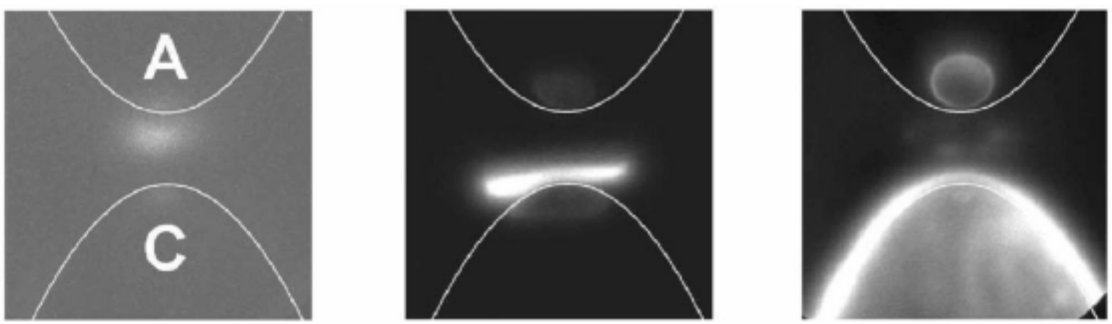
  P(i,k -> I',k')= E (I',k')/sum(E)
- Repeat

Niemeyer, PRL,52(12), 1984.

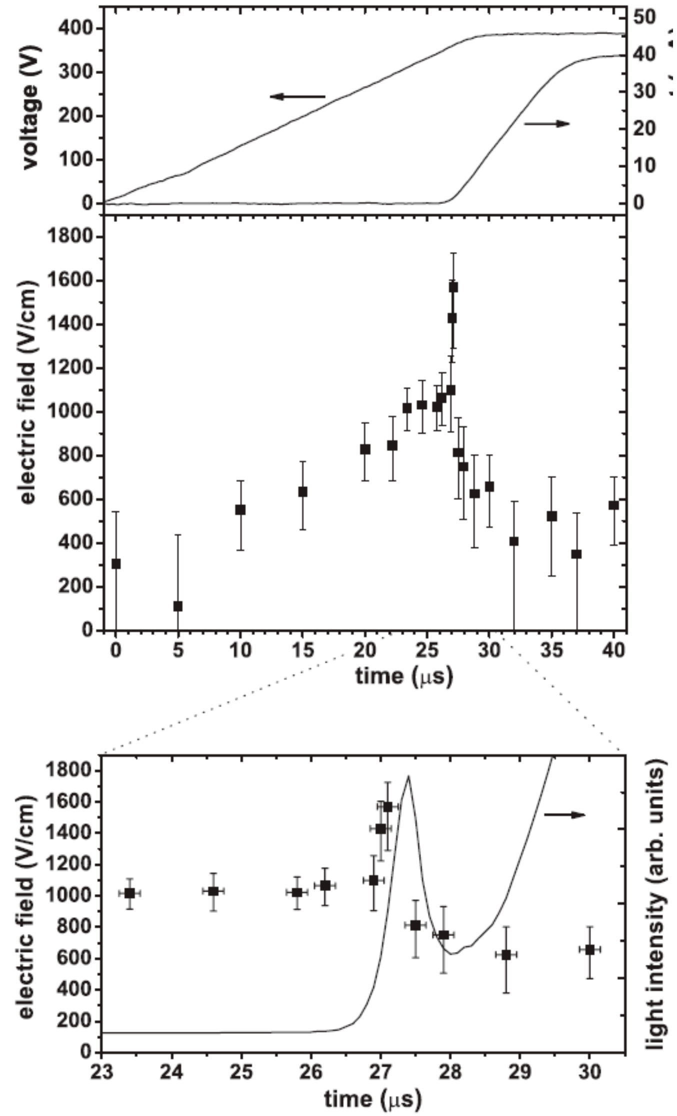
(4) temporal dynamics of breakdown



Xenon gas discharge

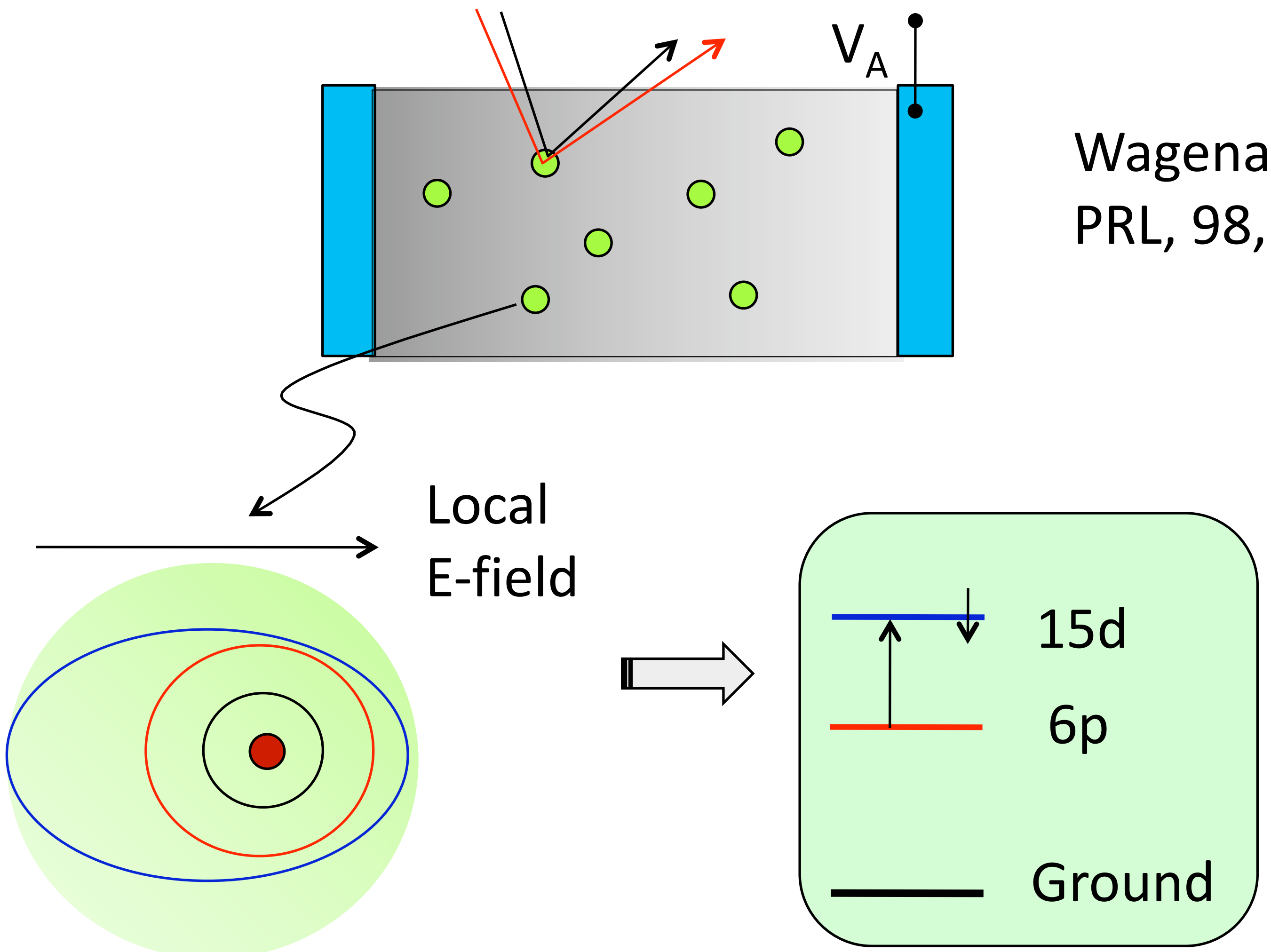


t = 26.0 μs t = 27.5 μs t = 40.0 μs



Wagenaars et al., PRL, 98, 075002, 2007.

(4) temporal dynamics by stark spectroscopy



Wagenaars et al.,
PRL, 98, 075002, 2007.

Stark Shifts define E-field

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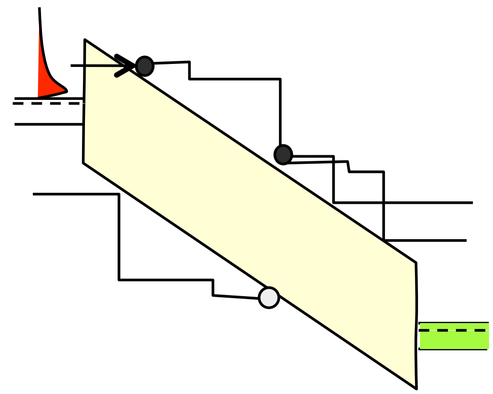
breakdown in bulk solid dielectric

Momentum balance ...

$$\frac{m^* \upsilon}{\tau} = q\mathcal{E} \Rightarrow \upsilon = \frac{q\mathcal{E}\tau}{m^*}$$

$$\upsilon \approx \sqrt{\frac{\hbar\omega_0}{m^*}}$$



Energy balance ...

$$q\mathcal{E}\upsilon = \frac{E - E_0}{\tau} \approx \frac{\hbar\omega_0}{\tau}$$

$$\mathcal{E} \approx \frac{\hbar\omega_0}{q\upsilon\tau} = \frac{\sqrt{\hbar\omega_0 m^*}}{q\tau}$$

$$\frac{1}{\tau} \sim m^{*3/2}\sqrt{E}$$

Ionization/per atom/length ....

$$\frac{V_{BD}}{L} \equiv \mathcal{E}_{BD}^{II} = \frac{\sqrt{\hbar\omega_0}\, m^{*2}}{q} \bullet \sqrt{E_G}$$

Hole generation condition ...

theory vs. experiment: small bandgap solids

1) J.M. Meek and D.J. Craggs, *Electrical Breakdown of Gases* (Oxford U.P., London, 1953)
2) S.M. Sze, *Physics of Semiconducting Devices* (Wiley, New York, 1969)

Materials	ϵ_G (eV)	m^*/m_0	$\hbar\omega_0$ (eV)	E_B (predicted)	E_B (observed)	Reference
InSb	0.17	0.013	0.025	2.5×10^2	4×10^2	(1)
InAs	0.36	0.02	0.03	8.6×10^2	1×10^3	(1)
Ge	0.66	0.22	0.037	…	1×10^5	(2)
Si	1.12	0.32	0.063	3.7×10^5	3×10^5	(2)
GaAs	1.43	0.35	0.035	3.7×10^5	3×10^5 $\sim5\times10^5$	(2) (1)
GaP	2.24	0.35	0.05	5.5×10^5	5×10^5 $\sim10\times10^5$	(2) (1)

Very good correspondence between theory and experiments for low-gap relatively clean materials.

theory vs. experiment: large bandgap solids

1) J.M. Meek and D.J. Craggs, *Electrical Breakdown of Gases* (Oxford U.P., London, 1953)
2) M. Lenzlinger and E.H. Snow, *J. Appl. Phys.* (40, 287) (1969)
3) C.M. Osburn and E.J. Weitzmann, *J. Electrochem. Soc.* (119, 603) (1972)

		Acoustic Phonons			Optical Phonons	
Materials	ϵ_G (eV)	E_B (predicted)	E_B (observed)	Reference	$\hbar\omega_o$ (eV)	E_B (predicted)
CdS	2.5	1.7×10^7	2×10^6	(1)	0.038	4.1×10^6
ZnSe	2.6	1.7×10^7	2×10^6	(1)	0.03	3.6×10^6
ZnO	3.3	2.2×10^7	4×10^6	(1)	0.07	6.4×10^6
SiO_2	9.0	6.1×10^7	9×10^6	(2,3)	0.12	1.4×10^7
NaCl	8.0	5.5×10^7	1.6×10^6	(1)	0.024	6.2×10^6

Poor correspondence between theory and experiments for larger-gap materials ….

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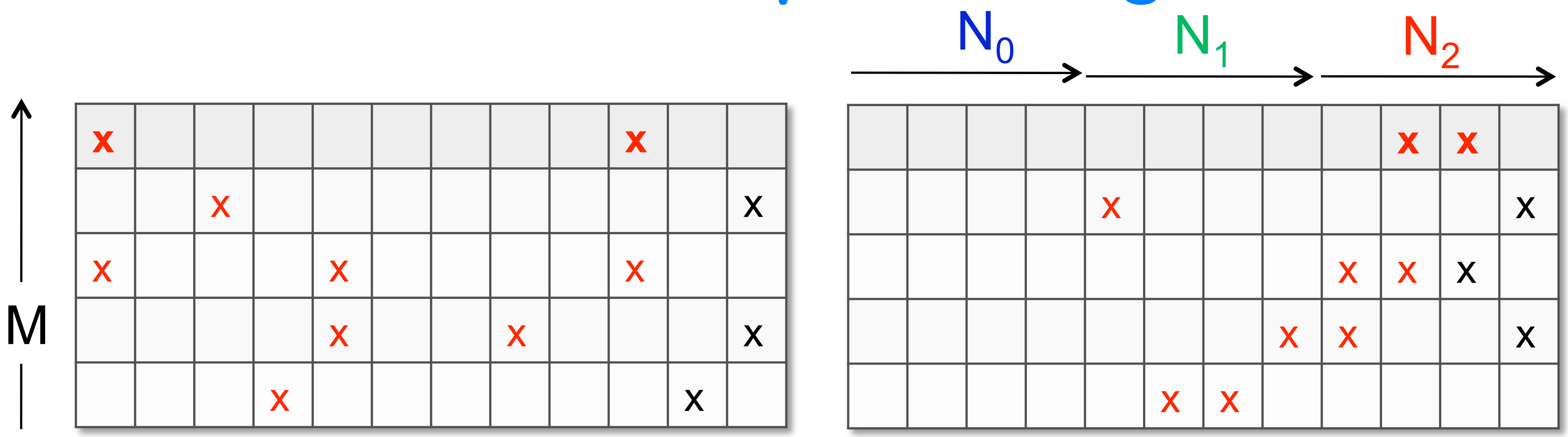
breakdown with preexisting defects



$$1-F=(1-F_{m=0})^{N_0}\,(1-F_{m=1})^{N_1}\,(1-F_{m=2})^{N_2}\,\ldots(1-F_{m=M-1})^{N_{m-1}}$$

$$1-F=(1-q^{M})^{N_0}(1-q^{M-1})^{N_1}(1-q^{M-2})^{N_2}\ldots(1-q^{M-m+1})^{N_{m-1}}$$

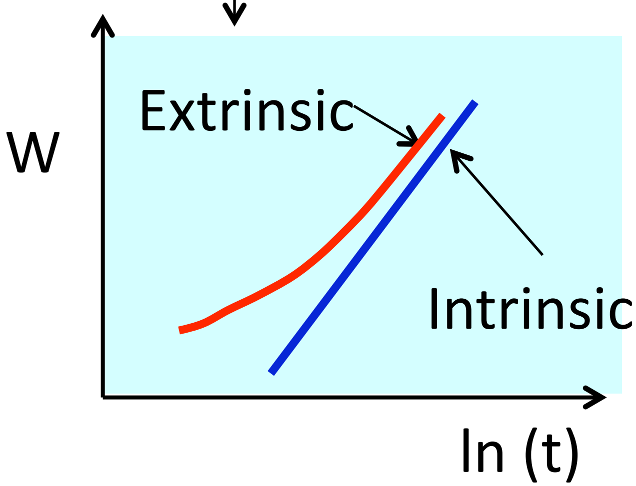
$$-\ln(1-F)=N_0q^{M}+N_1q^{M-1}+N_2q^{M-2}+\ldots.=N_0q^{M}[1+\frac{N_1}{N_0}\frac{1}{q}+\ldots.]$$

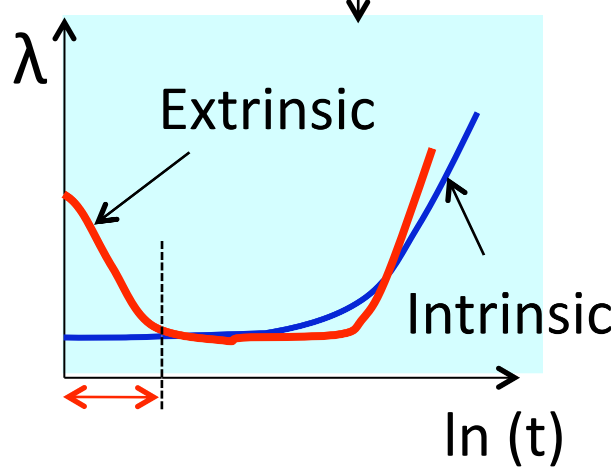
breakdown with preexisting defects

$$\ln(-\ln(1-F)) = \ln N_0 + M \ln q + \ln[1 + \frac{N_1}{N_0}\frac{1}{q} + ...] \qquad q = at^{\alpha} = (\frac{t}{\eta_0})^{\alpha}$$

$$\cong \ln N_0 + \beta \ln(\frac{t}{\eta_0}) + \frac{N_1}{N_0}\frac{1}{q} = \ln N_0 + \beta \ln(\frac{t}{\eta_0}) + \frac{N_1}{N_0}(\frac{\eta_0}{t})^{\alpha}$$

$$\lambda \equiv \frac{dF_n/dt}{1-F_n} = (N_0 \frac{\beta}{\eta_0^{\beta}})t^{\beta-1}[1 + \frac{N_1\eta_0^{\alpha}}{N_0}(\frac{\beta-\alpha}{\beta})\frac{1}{t^{\alpha}}]$$





Infant mortality, burn-in protocol …

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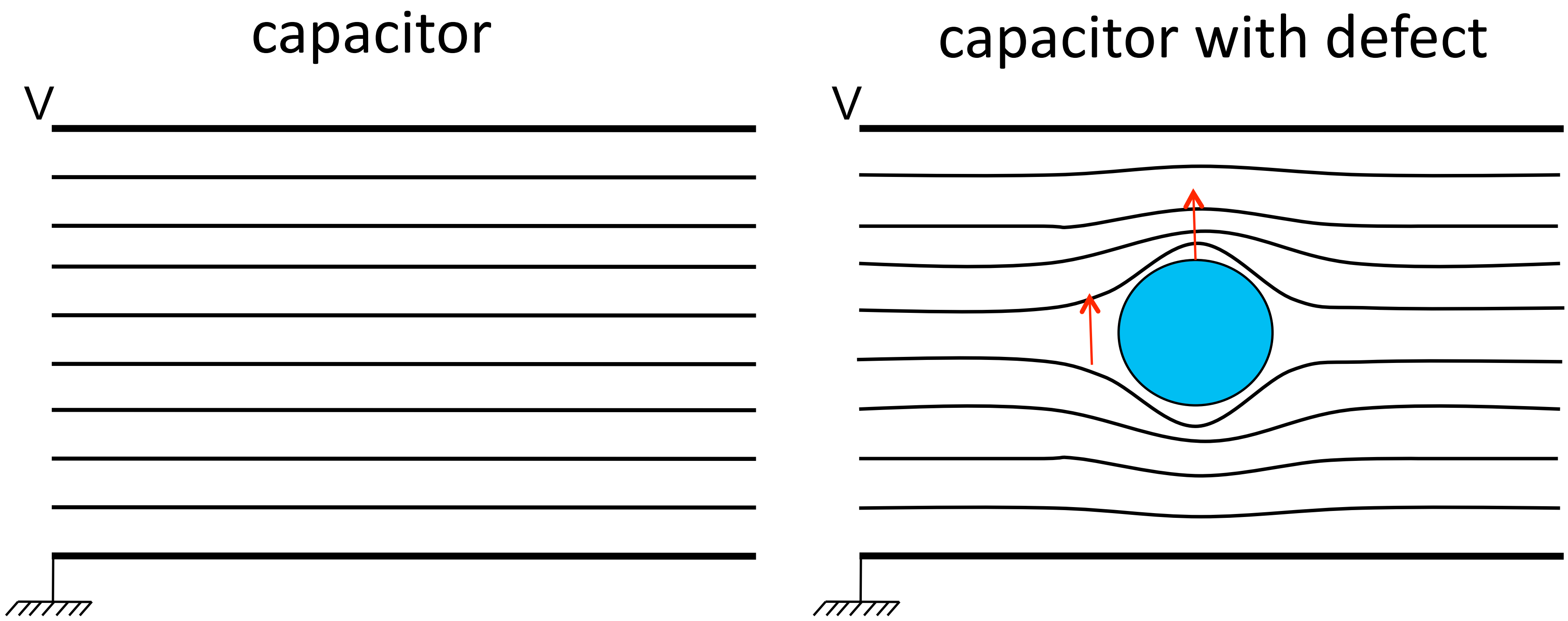
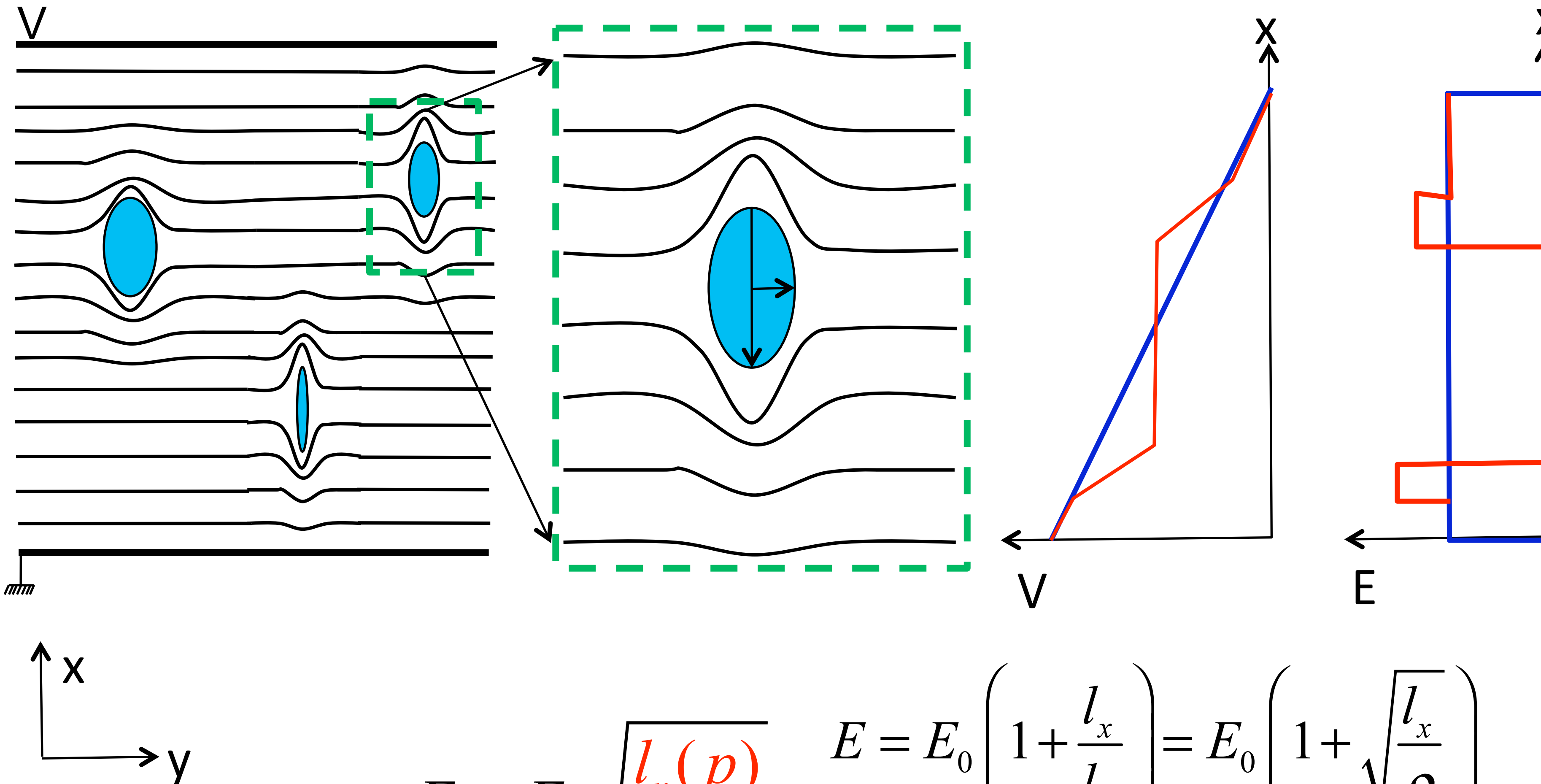
pre-existing defects, field enhancement, and breakdown in thick insulators (e.g. polymers)



Defects enhances local electric field
and reduces breakdown strength

defects and fields …

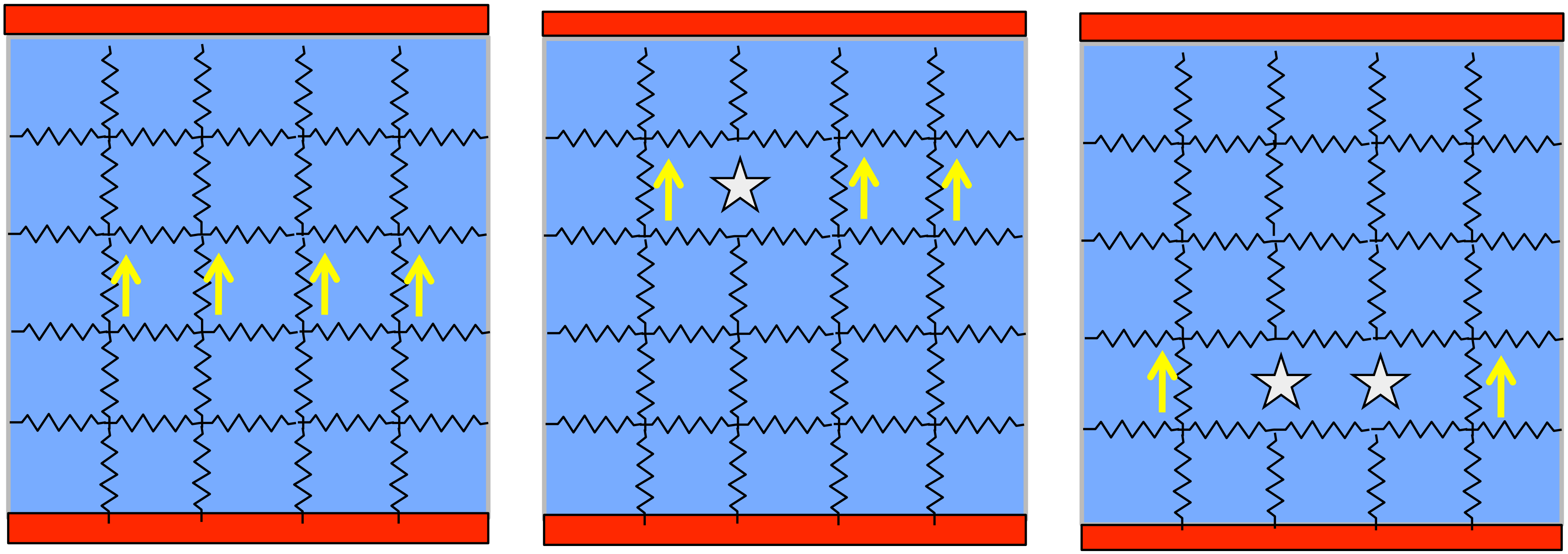


$$E \sim E_0 \sqrt{\frac{l_x(p)}{\rho}}$$

$$E = E_0\left(1+\frac{l_x}{l_y}\right) = E_0\left(1+\sqrt{\frac{l_x}{\rho}}\right)$$

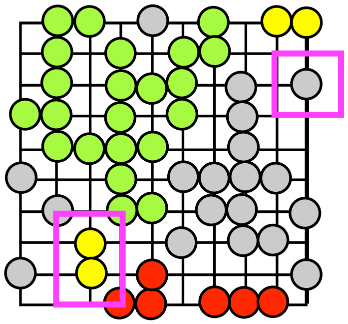
$$\rho \equiv l_y^{\;2} / l_x$$

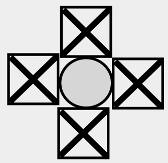
analogy to resistor network



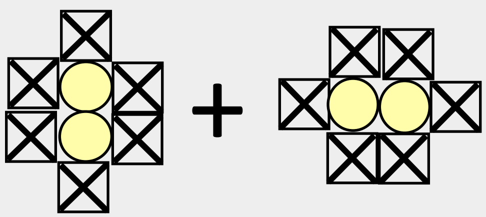
defects reduce breakdown current ....

small-cluster size distribution

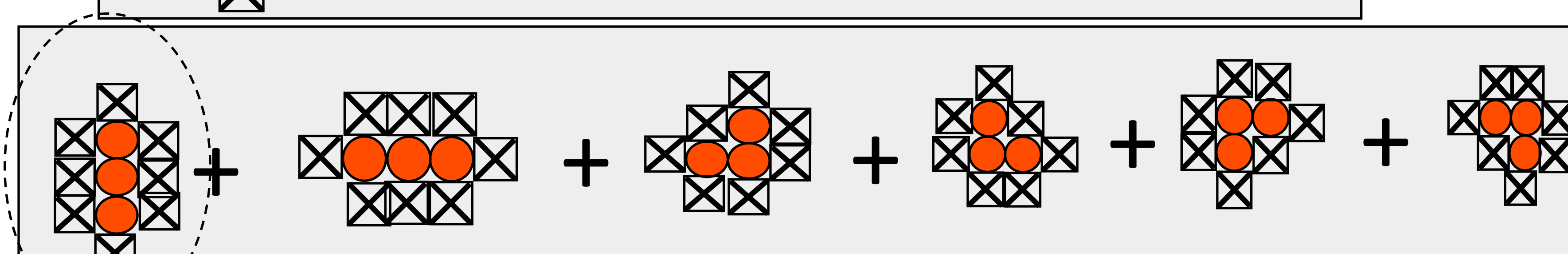




$$n_1(p) = 1 \times p \times (1-p)^4$$



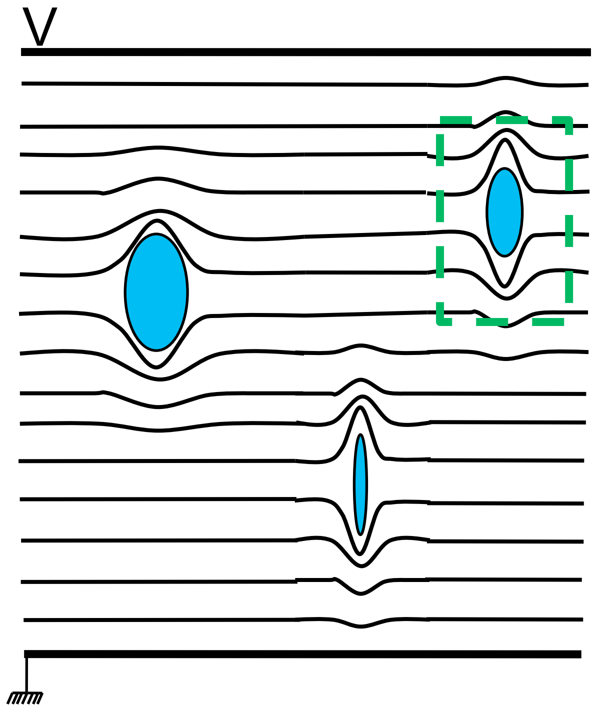
$$n_2(p) = 2 \times p^2 \times (1-p)^6$$

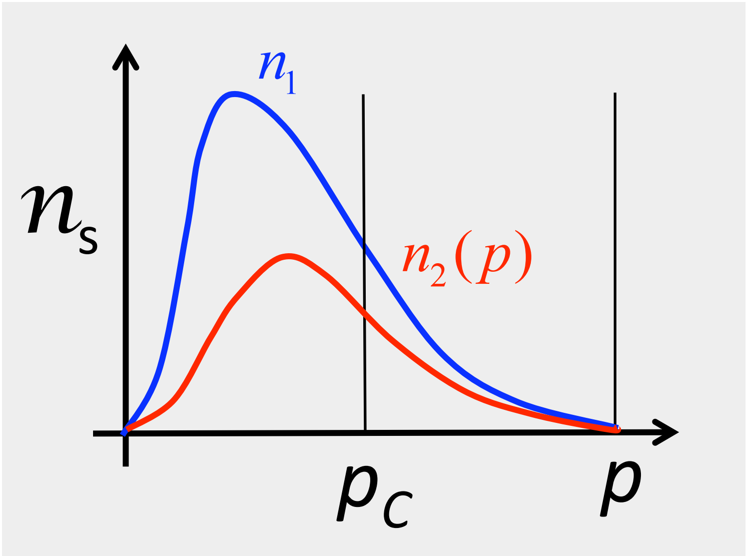


$$n_3(p) = 2 \times p^3 \times (1-p)^8 \quad + 4 \times p^3 \times (1-p)^7$$

$$n_s(p) = \sum_t g_{st} \times p^s \times (1-p)^t$$

size distribution and critical defect size



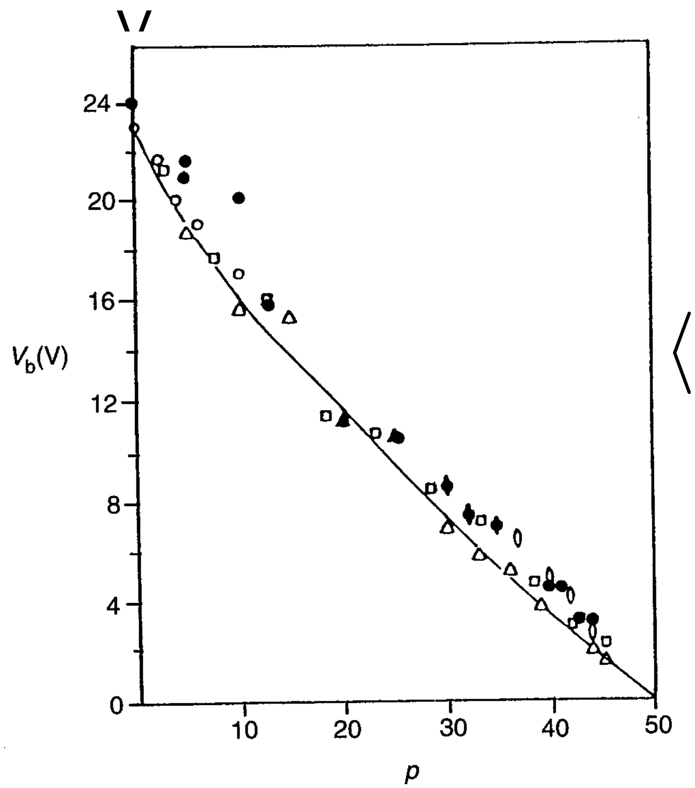


$$n_l(p) = L^2 p^{l_x} (1-p)^{2l_x+2}$$
$$\sim L^2 p^{l_x} \ (p \to 0)$$

$$p^{\langle l_x \rangle} L^2 \sim 1 \Rightarrow$$
$$\langle l_x(p) \rangle = \frac{-2 \ln L}{\ln p}$$

At a defect level p, most probable size of defect is $l_c(p)$

breakdown field for islands of size Lc



$$E_0^{crit}\sqrt{\langle l_x(p)\rangle/\rho} = E_{BD}$$

$$\langle l_x(p)\rangle = \frac{-2\ln L}{\ln p} = \rho\left(\frac{E_{BD}}{E_0}\right)^2 = \rho\left(\frac{LE_{BD}}{V_{app}}\right)^2$$

$$\frac{V_{app}}{LE_{BD}} = \sqrt{\frac{\rho \ln p}{-2\ln L}}$$

Smaller p, larger VBD

VBD reduces to zero at sample size L goes to infinity, because large defects is present with probability 1.

VBD distribution close to percolation threshold

$$1 - F(E_0) = \prod_{i=1}^{n} \left[1 - f_i(E_0)\right]$$

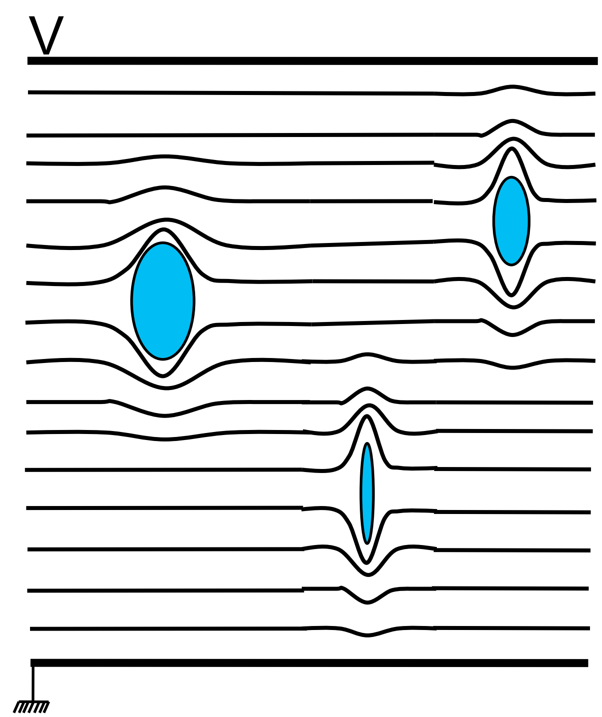
Prob. of failure of i-sized island at E_0

$$\approx 1 - \sum_{i=1}^{n} f_i(E_0) \simeq e^{-\sum_{i=1}^{n} f_i(E_0)}$$

$$\equiv e^{-A g_l(E_0)}$$

Area

Defect density that breaks at E0=Vapp/L



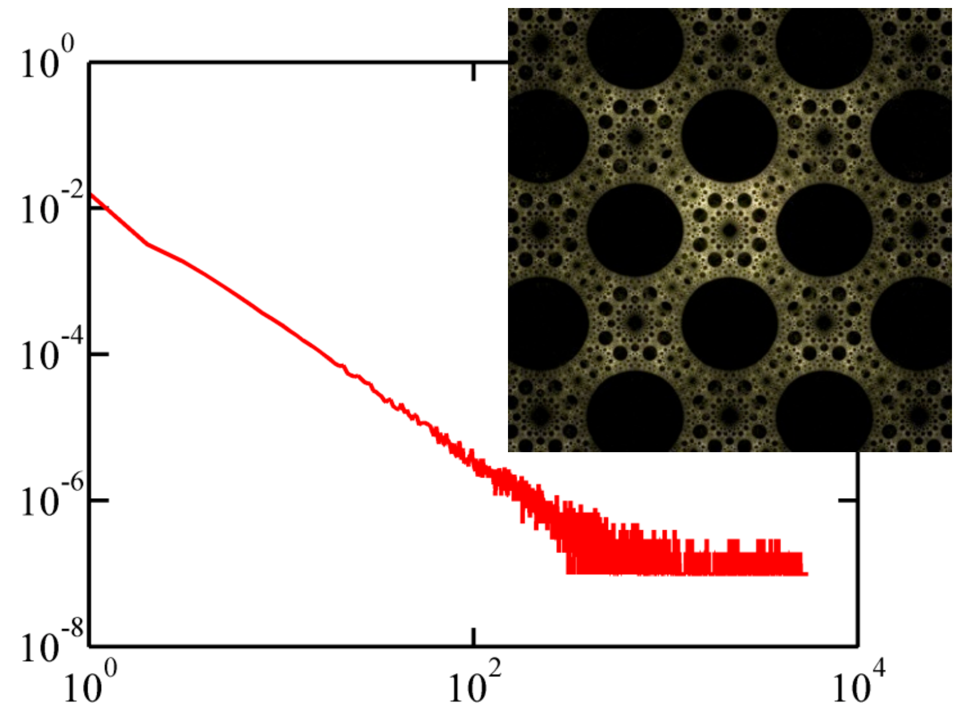
$$\ln(-\ln(1 - F(E)) = \ln A + \ln g_l(E)$$

size distribution at percolation threshold

$$g_l(p) \equiv L^{-2} n_l(p)$$
$$= p^{l_x} \times (1-p)^{2l_x+2}$$
$$\sim l_x^{-\tau} \ @\ p = p_c$$

$$\langle l_x \rangle = \rho \left(\frac{E}{E_0} \right)^2$$

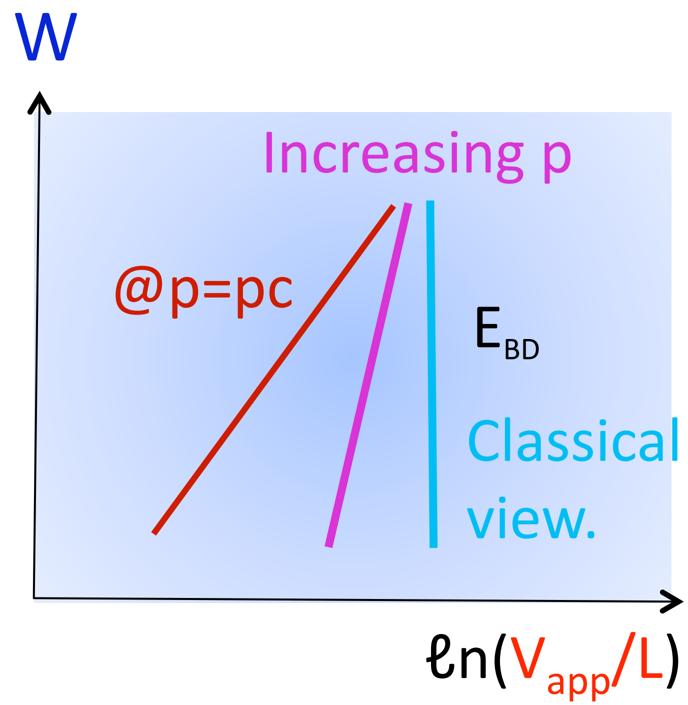
$$g_l(E) \sim \rho^{-\tau} \left(\frac{E}{E_0} \right)^{-2\tau}$$



Recall from Lecture 2

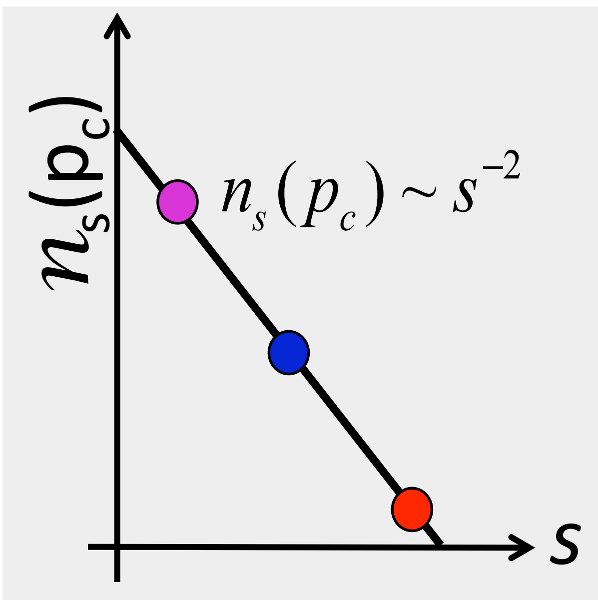
distributed failure probabilities

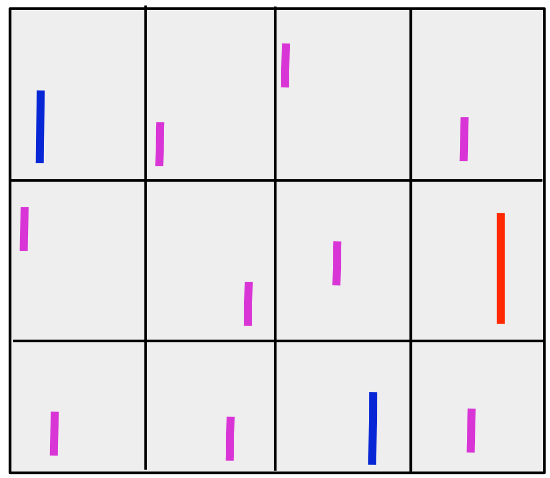
$$W \equiv \ln(-\ln(1-F(E))$$
$$= \ln A + \ln g_l(E)$$
$$= \ln\left(A\rho^{-\tau}\right) + 2\tau \ln\left(E_0 / E_{BD}\right)$$
$$= \ln(A\rho^{-\tau}) + 2\tau \ln\left(\frac{1}{E_{BD}} \frac{V_{app}}{L}\right)$$

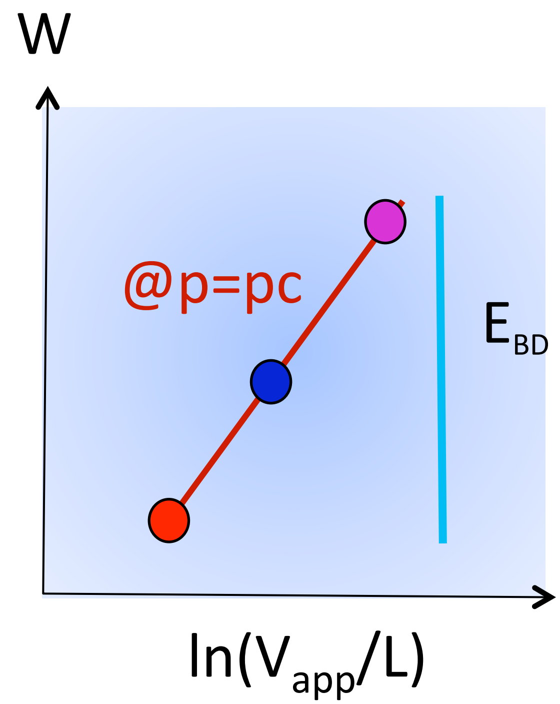


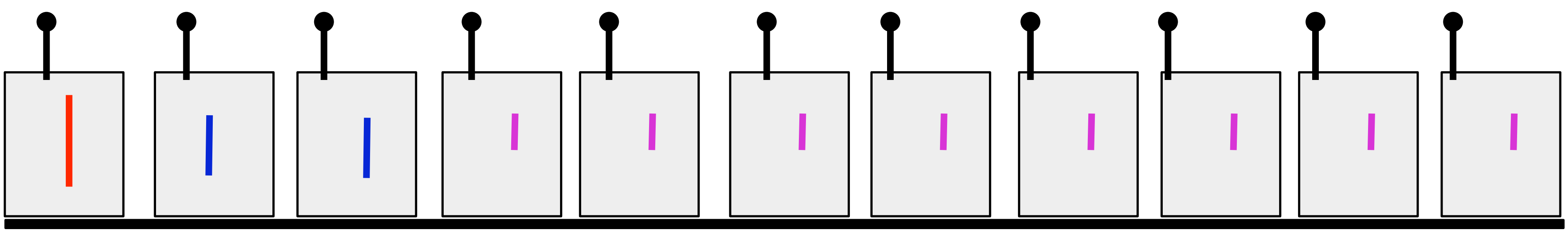
HW: Show that larger area oxides fail at smaller voltages.

what does it all mean (ramp voltage tests) ?









conclusions

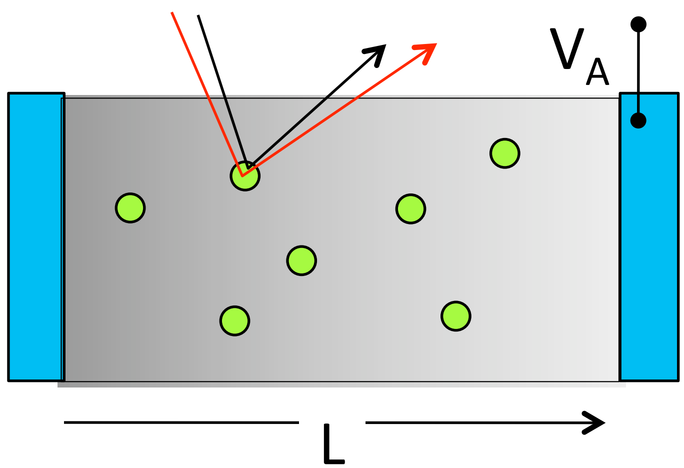
- The our basic steps of breakdown processes are essentially the same for thin and thick oxides; the key differences are ...

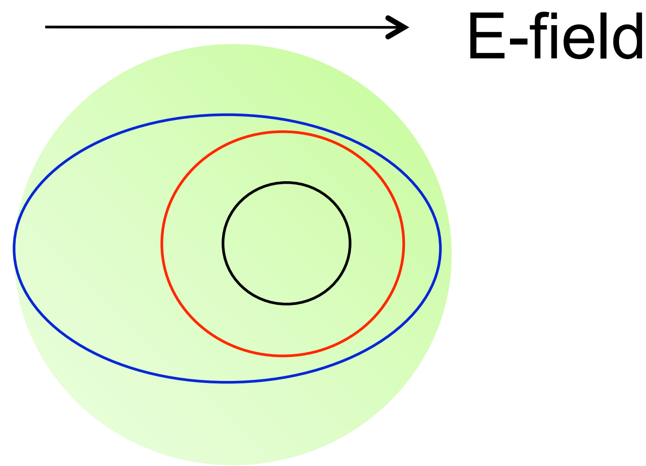
  Breakdown in thick oxides in extrinsic, dominated by defects, while that of thin oxide is intrinsic, dominated by contacts.

  Breakdown in thick oxides is correlated (Lichtenberg figures), while the BD in thin films is uncorrelated.

- Many features of BD in thick dielectric again echo features of spatial fluctuation, i.e. fractal dimension of 1.7, the distribution of defect size given by percolation theory, etc. because they share the same underlying physics.

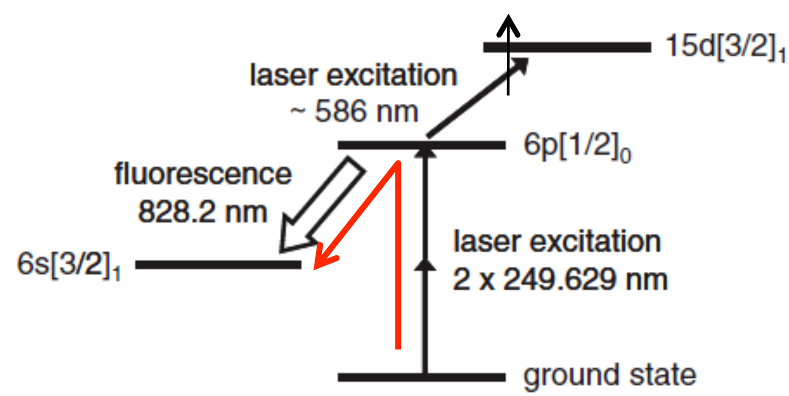
(4) temporal dynamics by stark spectroscopy





Stark Shifts define E-field

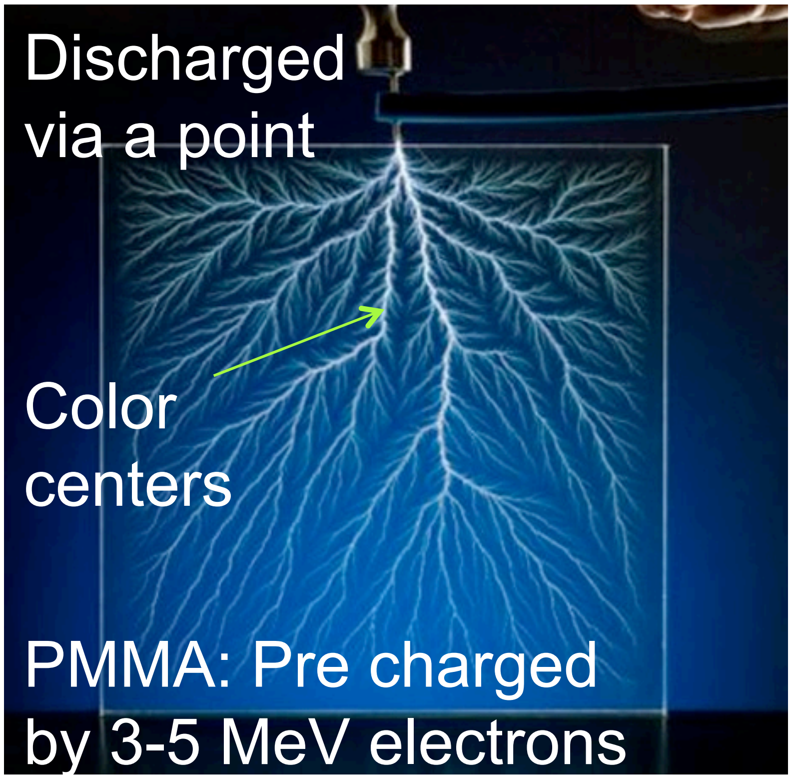
The flourcence dip in 828.2 nm Line suggests that resonance is Achieved. The unknown wavelength (e.g. 586 nm) then defines where The excited level is.



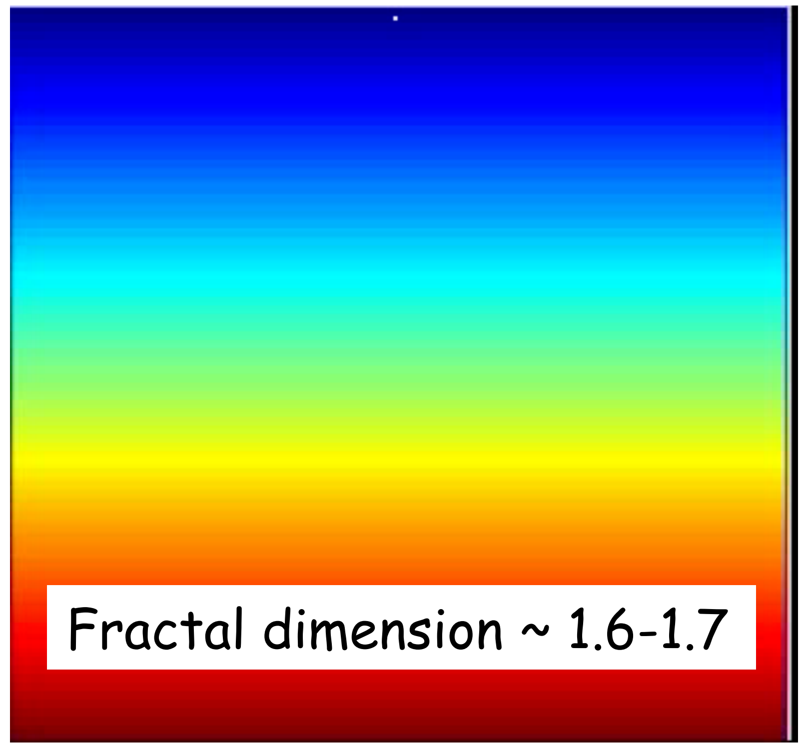
Wagenaars et al., PRL, 98, 075002, 2007.

(3) spatial dynamics at breakdown

Lichtenberg figures



Niemeyer, PRL,52(12), 1984.



One of the great successes in understanding BD in 1980s …