2009 NCN@Purdue-Intel Summer School
Notes on Percolation and Reliability Theory

Lecture 5
2D nets in 3D world: Fractal Biosensors

Muhammad A. Alam
Electrical and Computer Engineering
Purdue University
West Lafayette, IN USA

NCN

www.nanohub.org

PURDUE UNIVERSITY

outline of lecture 5

1) Background: A different type of transport problem

2) Example: Classical biosensors

3) Fractal dimension and cantor transform

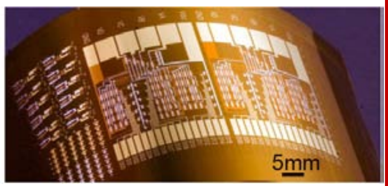
4) Example: fractal nanobiosensors

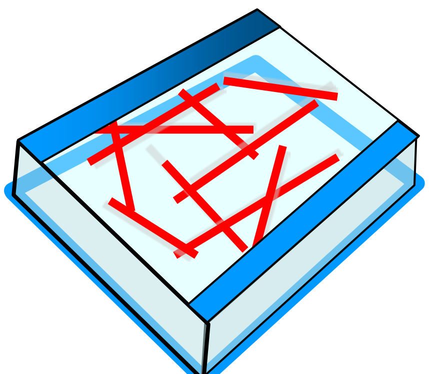
4) Conclusions

5) Appendix: Transparent Electrodes and Antenna

lectures 4, 5 and 6

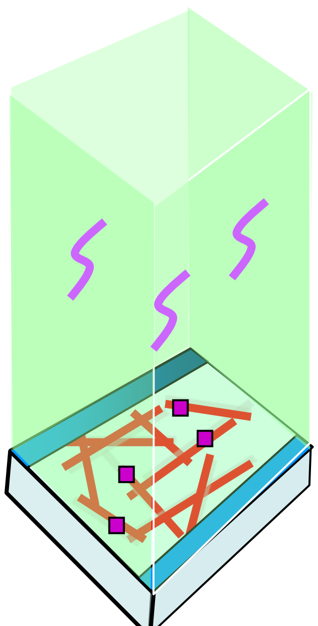
2D transport
in 2D Network



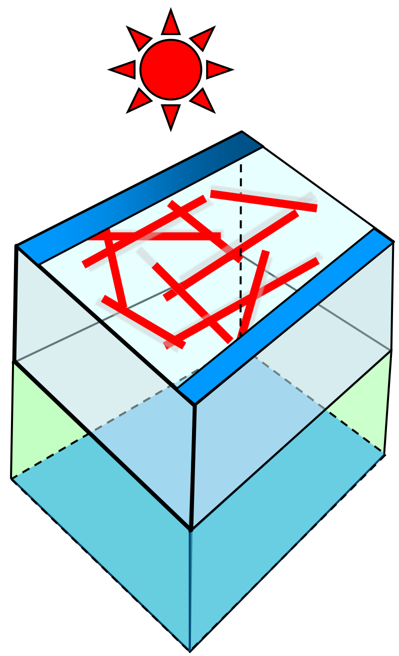


3D transport
towards 2D network

Nanobiosensors

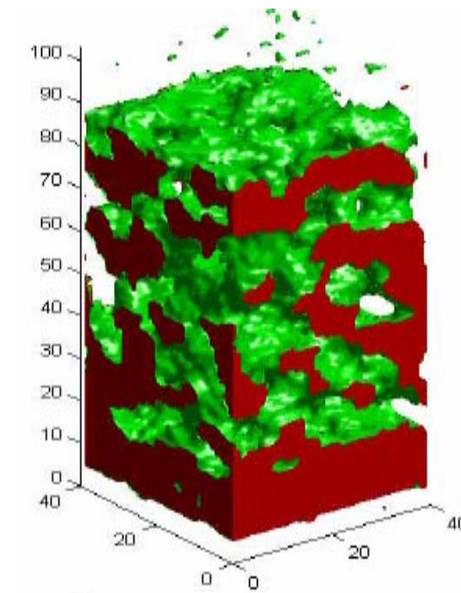


Fractal
electrodes



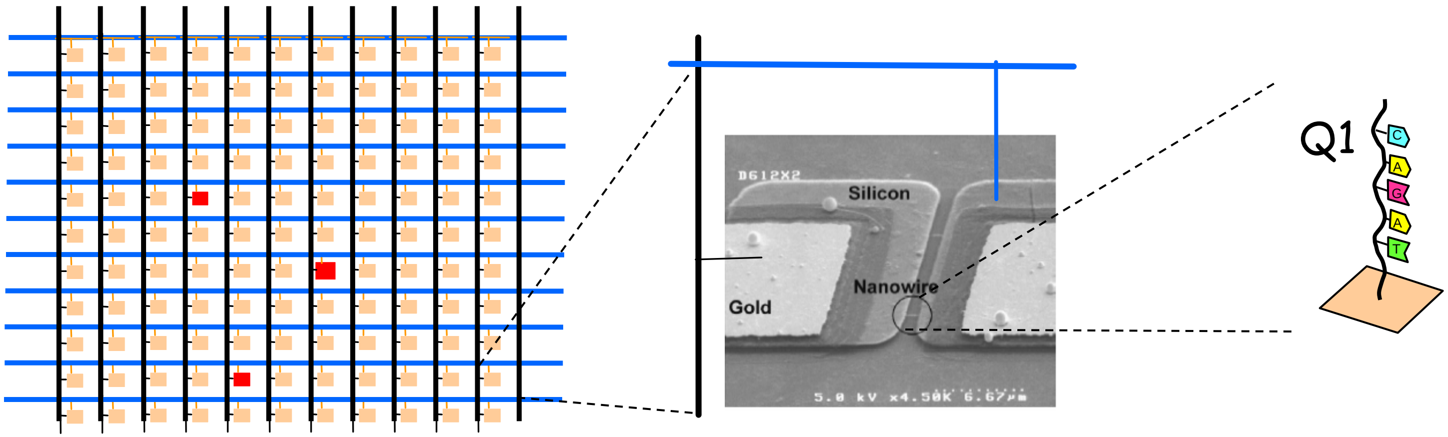
3D transport
in 3D network

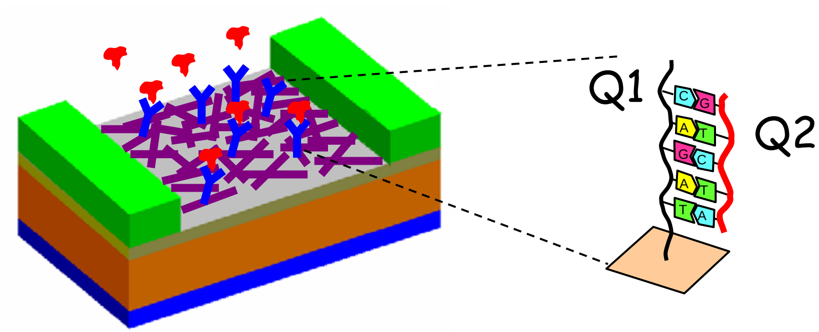
Excitonic solar cell



Basics of Nanobiosensing

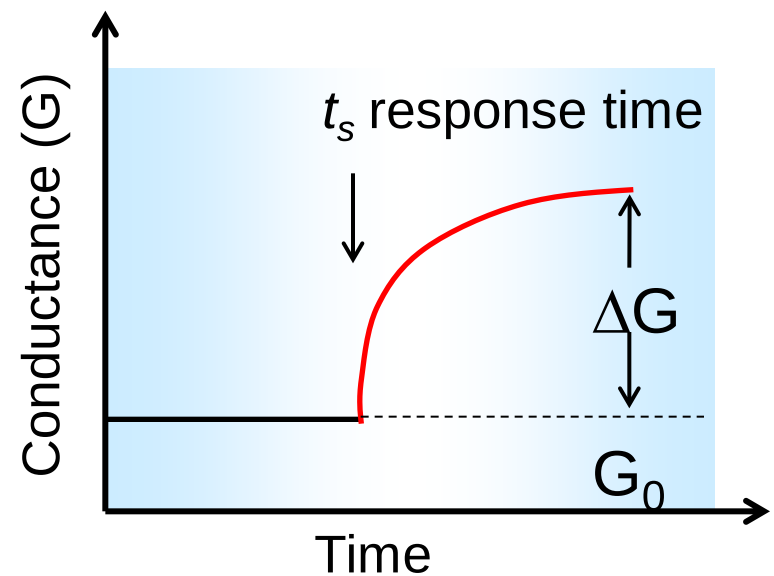
An array … Individual sensor Capture Probe





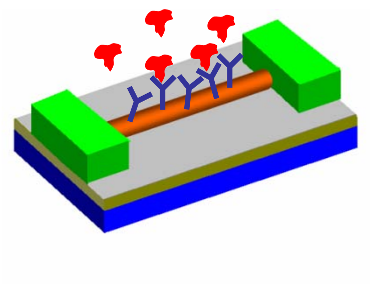
Optical detection schemes

Surface Plasmon Resonance

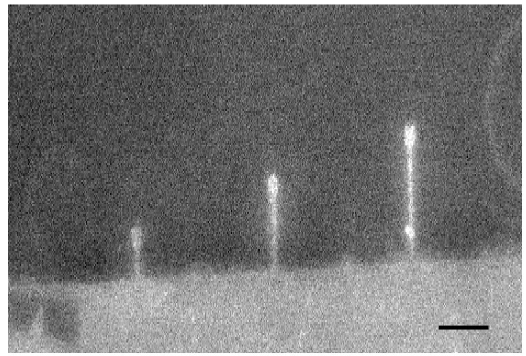


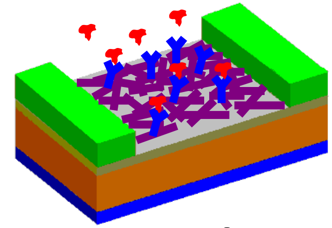
Zoo of biosensors!

Si-NW/CNT (nM-aM)



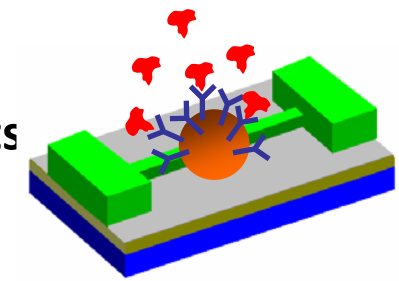
Nano Cantilever (~pM)



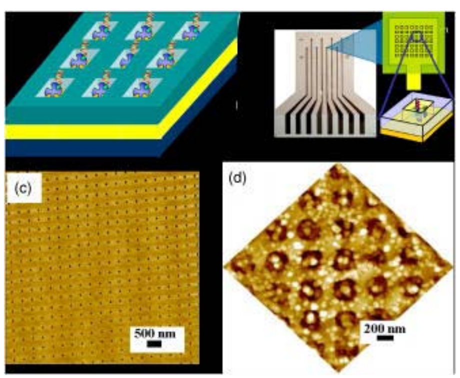


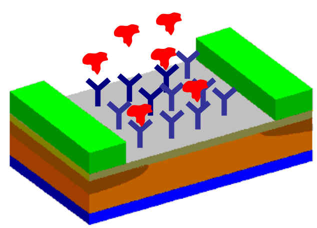
Nano-Net (nM-pM)

Nanodots



Array sensors (~pM)

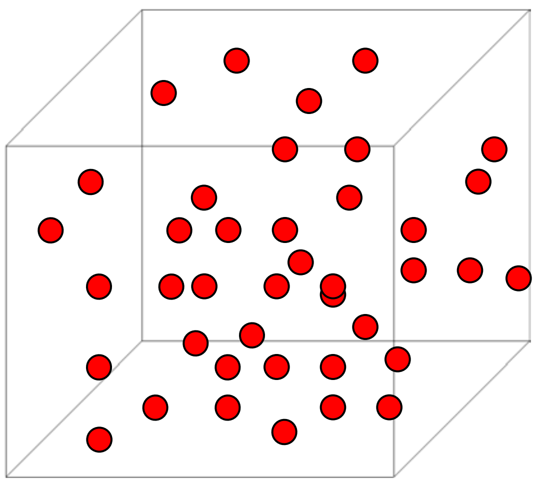




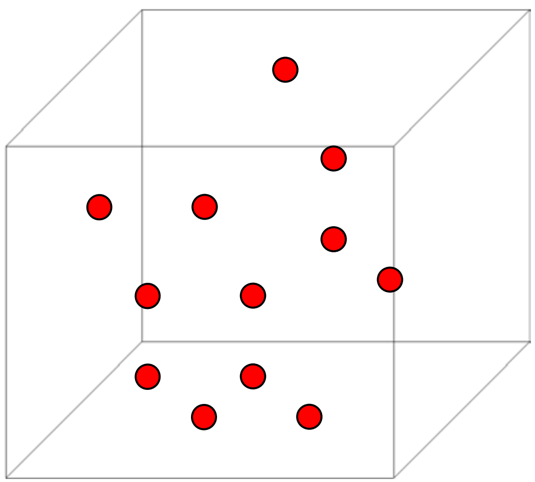
ChemFET/IsFET (~mM)

Micro-Molar, nano Molar, pico Molar ?!

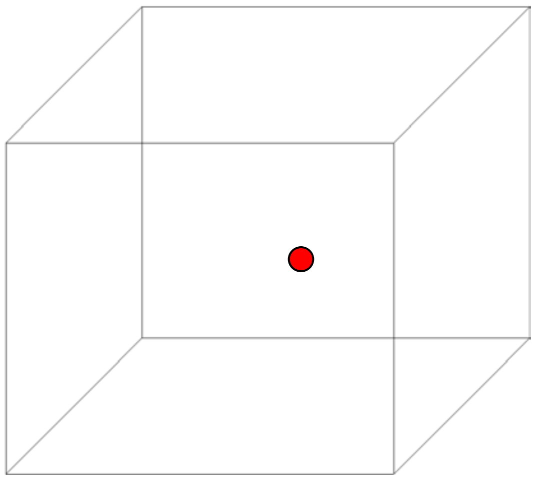
1M = 6×10^{23} molecules/liter ~ $1 \times 10^{15}/(100\ \text{um})^3$ box



1 uM ~ 1 billion



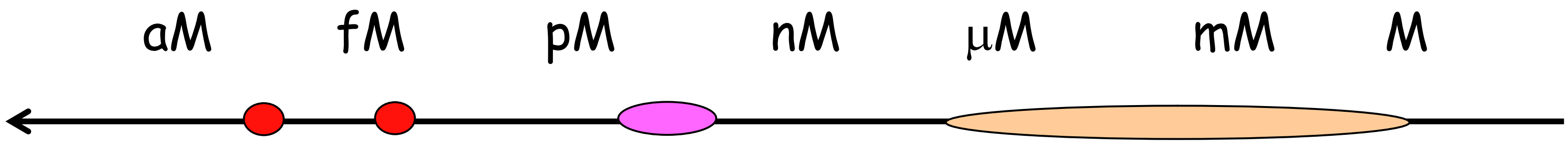
1 pM ~ 1000



1 fM ~ 1

Like a proton in H atom

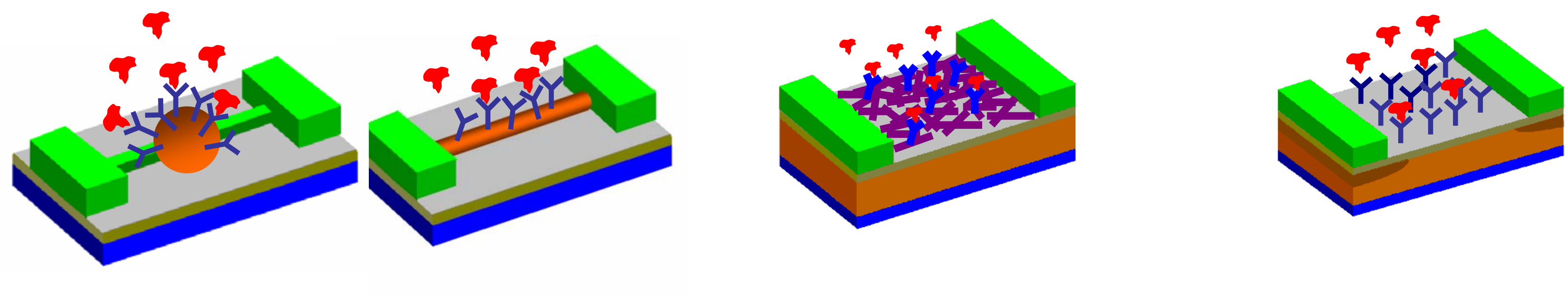
sensitivity @given settling Time



Recently …

Historically …

Nanodots **Si-NW/CNT** **Nano-Net** **ChemFET/IsFET**



Related to the shape of the sensor?

How can I think about the shape-related response?

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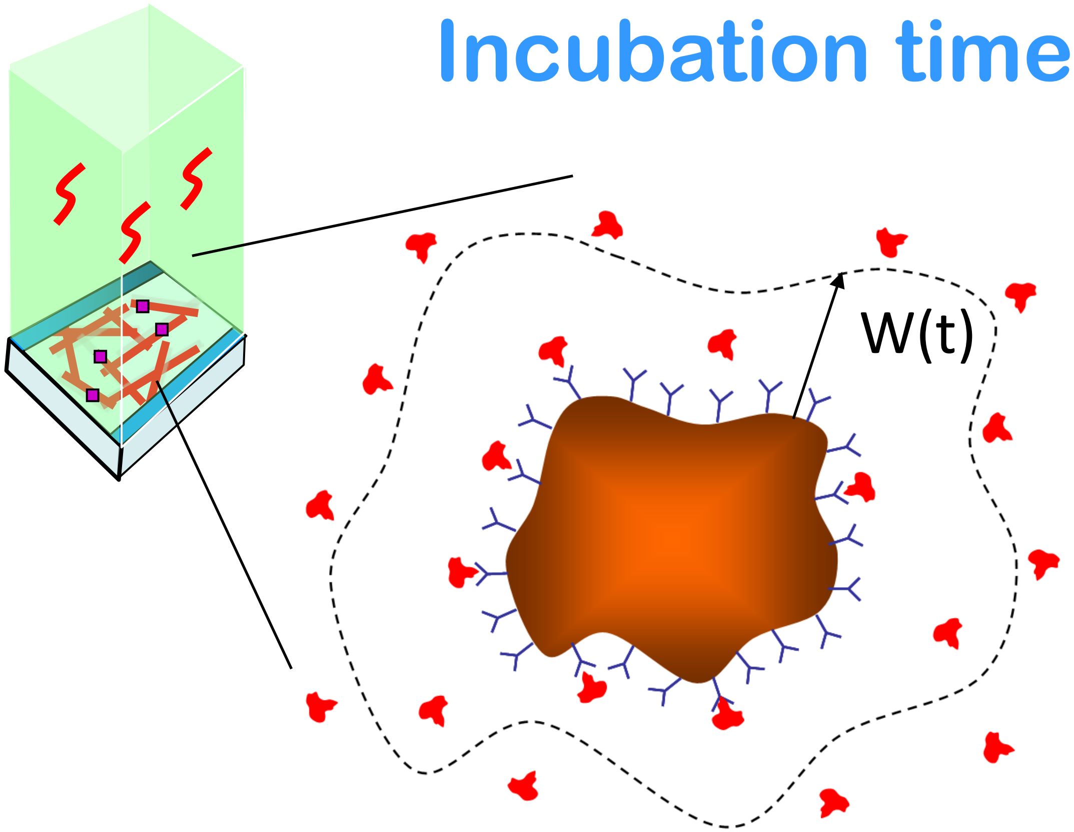
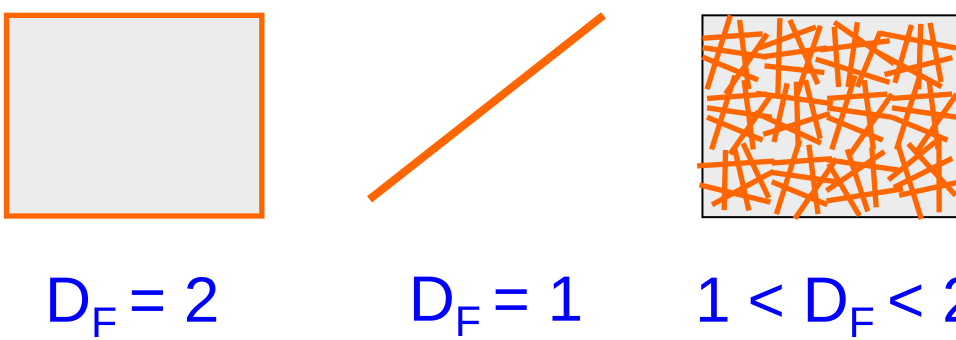
Incubation time for a biosensors



$$\frac{d\rho}{dt} = D\nabla^2 \rho$$

$$\frac{dN}{dt} = k_F (N_0 - N)\rho_s$$

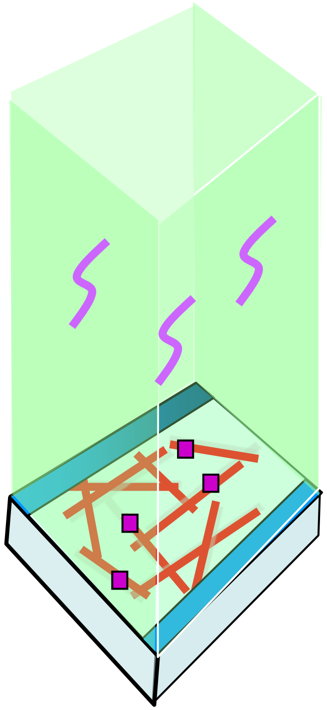
$$N(t) = \rho_0 t_s^{g(D_F)}$$



presumption of fast capture

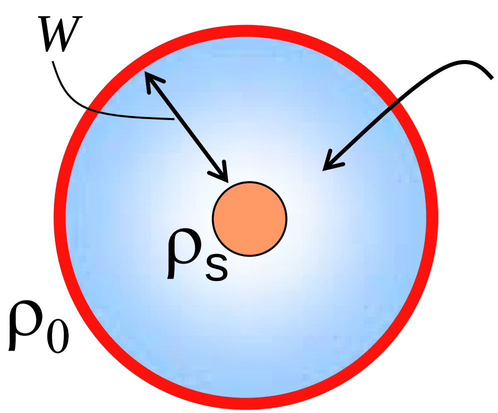
$$\frac{dN}{dt} = k_F (N_0 - N) \rho_s$$

$$\approx k_F N_0 \times \rho_s = \upsilon_s \rho_s$$



If the capture velocity is very large, then ρ_s=0 for finite capture rates.

steady state: analogy to electrostatics



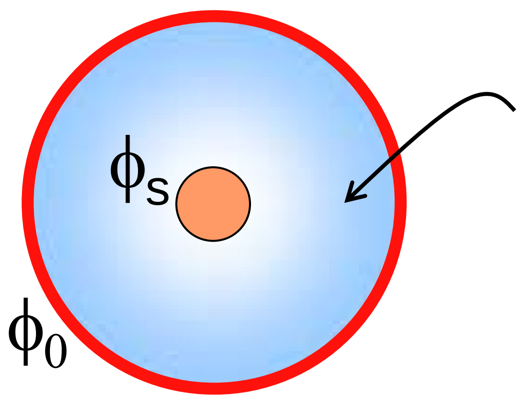
$$D\nabla^2 n = 0$$

$$I = C_0\left(\rho_0 - \rho_s\right)$$

$$C_0 = \frac{D}{W} \quad \text{(Planar)}$$

$$= \frac{2\pi D}{\log\left(W + a_0/a_0\right)} \quad \text{(NW)}$$

$$= \frac{4\pi D}{a_0^{-1} - \left(W + a_0\right)^{-1}} \quad \text{(ND)}$$



$$\varepsilon\nabla^2\phi = 0$$

$$Q = C_0\left(\phi_0 - \phi_s\right)$$

$$C_0 = \frac{\varepsilon}{W} \quad \text{(Planar)}$$

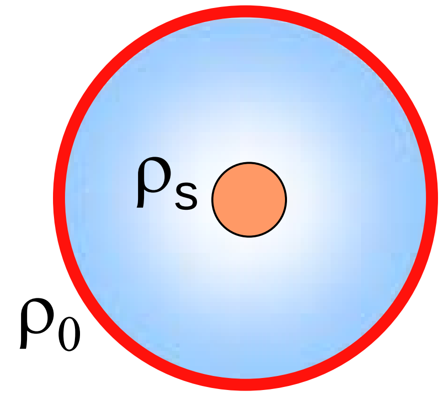
$$= \frac{2\pi\varepsilon}{\log\left(W + a_0/a_0\right)} \quad \text{(Cylinder)}$$

$$= \frac{4\pi\varepsilon}{a_0^{-1} - \left(W + a_0\right)^{-1}} \quad \text{(Sphere)}$$

diffusion and capture in steady-state

$$I = C_0\left(\rho_o - \rho_s\right)$$

$$I = A\frac{dN}{dt} = Ak_F N_0 \rho_s$$



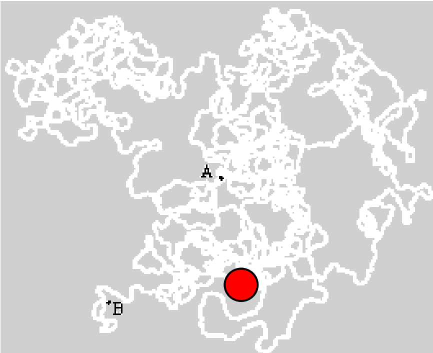
$$N(t) = \rho_0 t\left[\frac{A}{C_0} + \frac{1}{k_F N_0}\right]^{-1}$$

$$C_0 = \frac{D}{W}$$

$$= \frac{2\pi D}{\log\left(W + a_0/a_0\right)}$$

$$= \frac{4\pi D}{a_0^{-1} - \left(W + a_0\right)^{-1}}$$

… but I am not interested in steady-state response !

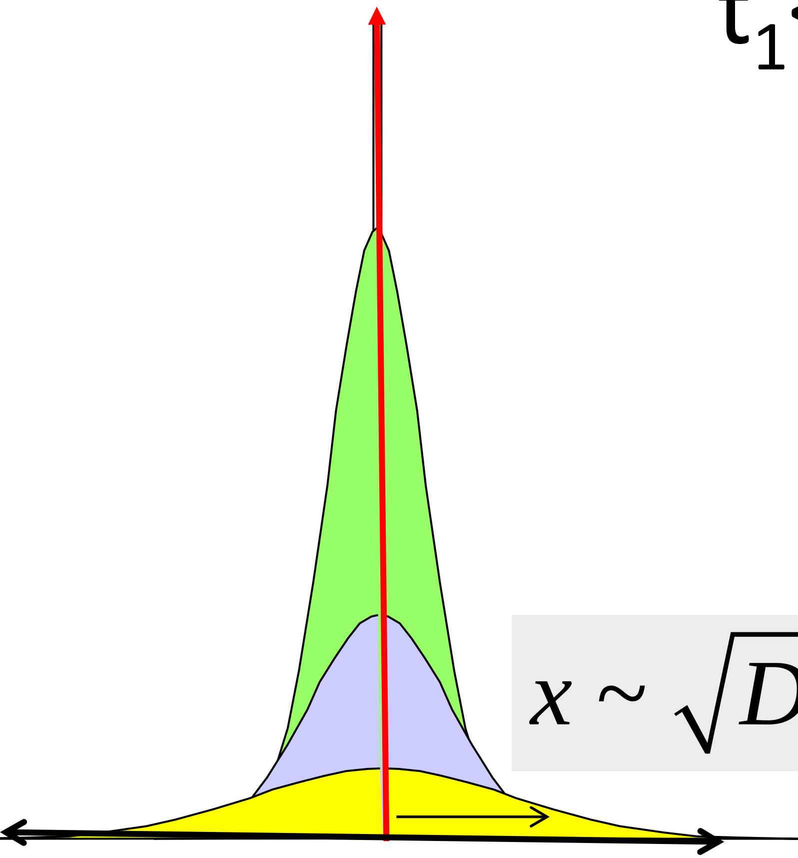


$$\frac{d\rho}{dt} = D\nabla^2\rho$$

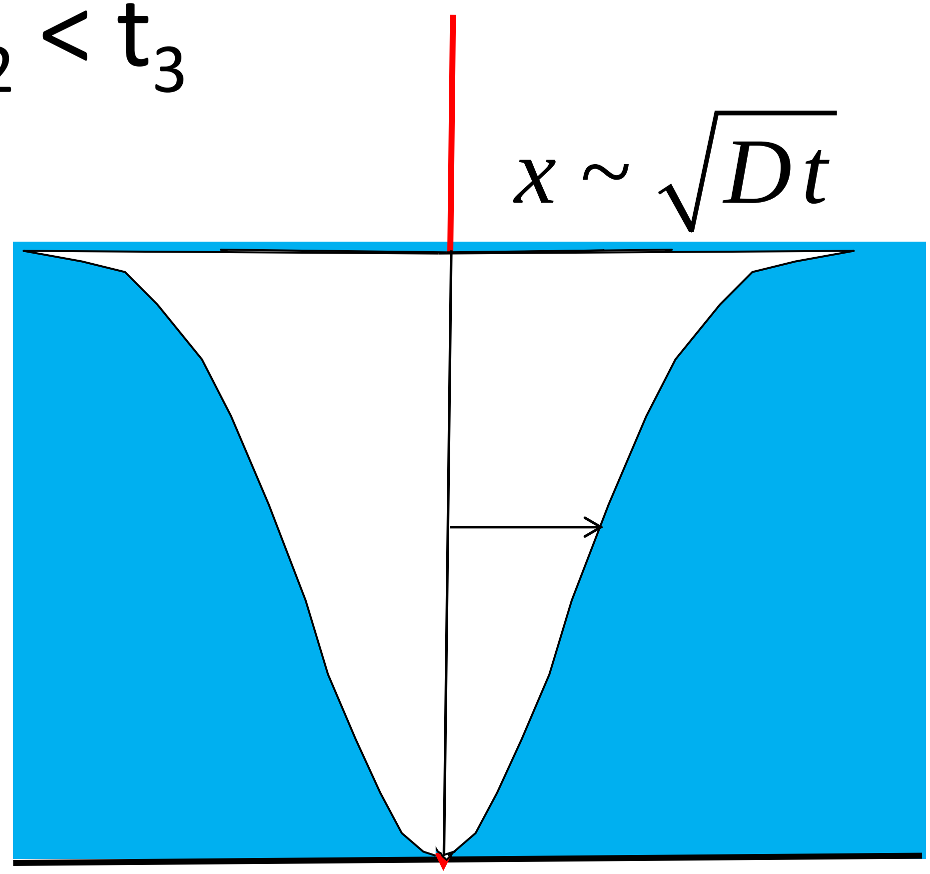
diffusion distance

$t_1 < t_2 < t_3$

$x \sim \sqrt{Dt}$

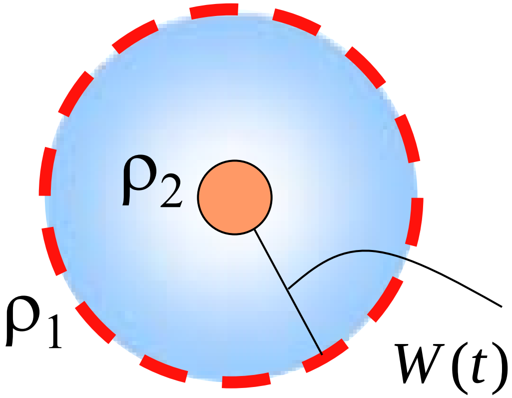


$x \sim \sqrt{Dt}$



time-dependent capture dynamics

$$N(t) = \rho_2 t \left[\frac{A}{C_t} + \frac{1}{k_F N_0} \right]^{-1}$$



$$W(t) = \sqrt{2nDT}$$

$$C_0 = \frac{D}{W} = \frac{2\pi D}{\log\left(W + a_0/a_0\right)} = \frac{4\pi D}{a_0^{-1} - \left(W + a_0\right)^{-1}}$$

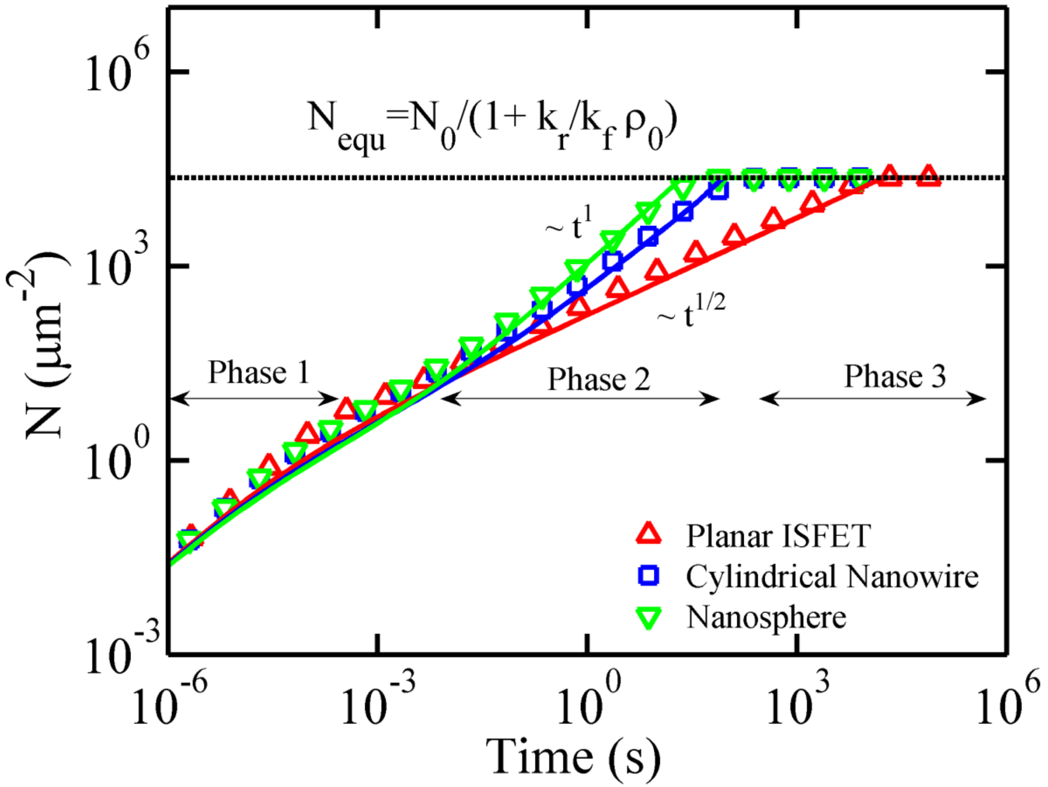
$$\Rightarrow$$

$$C_t = \frac{D}{\sqrt{2Dt}} = \frac{2\pi D}{\log\left(\sqrt{4Dt} + a_0/a_0\right)} = \frac{4\pi D}{a_0^{-1} - \left(\sqrt{6Dt} + a_0\right)^{-1}}$$

pretty good approximation ..

Nair et. al, APL, 88, 233120, 2006

Bishop et. al., Nanotechnology, 2006.



Geometry matters, but the result is not very intuitive …

… simplified diffusion-capture model

Exact Equations …

$$\frac{dN}{dt} = k_F (N_0 - N)\rho_s - k_R N$$

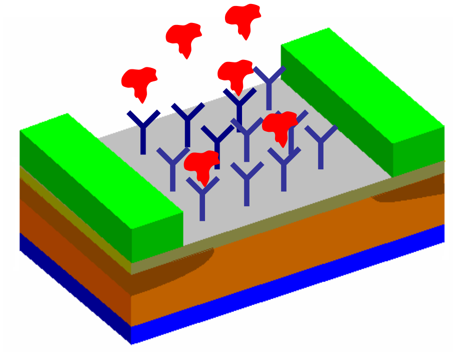
$$\frac{d\rho}{dt} = D\nabla^2 \rho$$

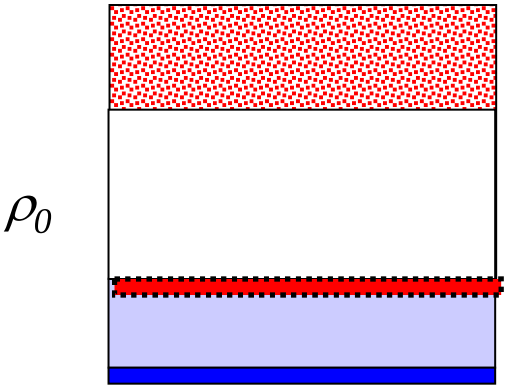
Simplified Equations …

$$\rho_s \approx 0$$

$$l_D \sim \sqrt{Dt}$$

planar sensor (DF=2)





$$R \sim \sqrt{2Dt}$$

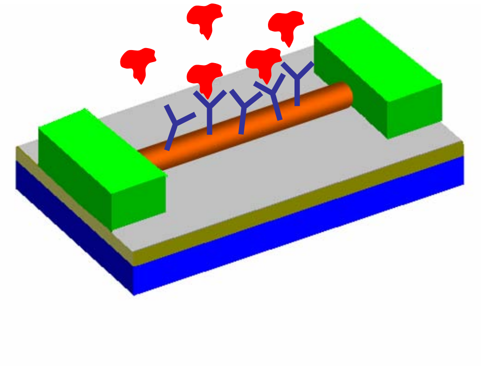
$D_F=2$

$$t_s \sim \frac{N_s^2}{D}\frac{1}{\rho_0^{2}} \leftarrow m$$

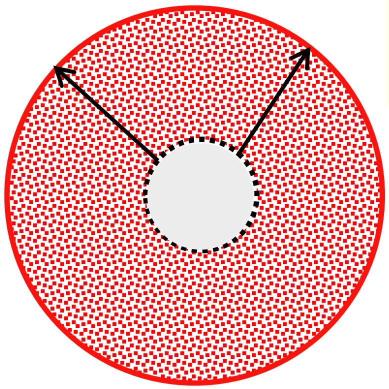
$$N(t) \times A \sim \rho_0 \times R \times A$$

$$\sim \rho_0 \times \sqrt{Dt} \times A$$

cylindrical NW sensor (DF=1)



$D_F=1$

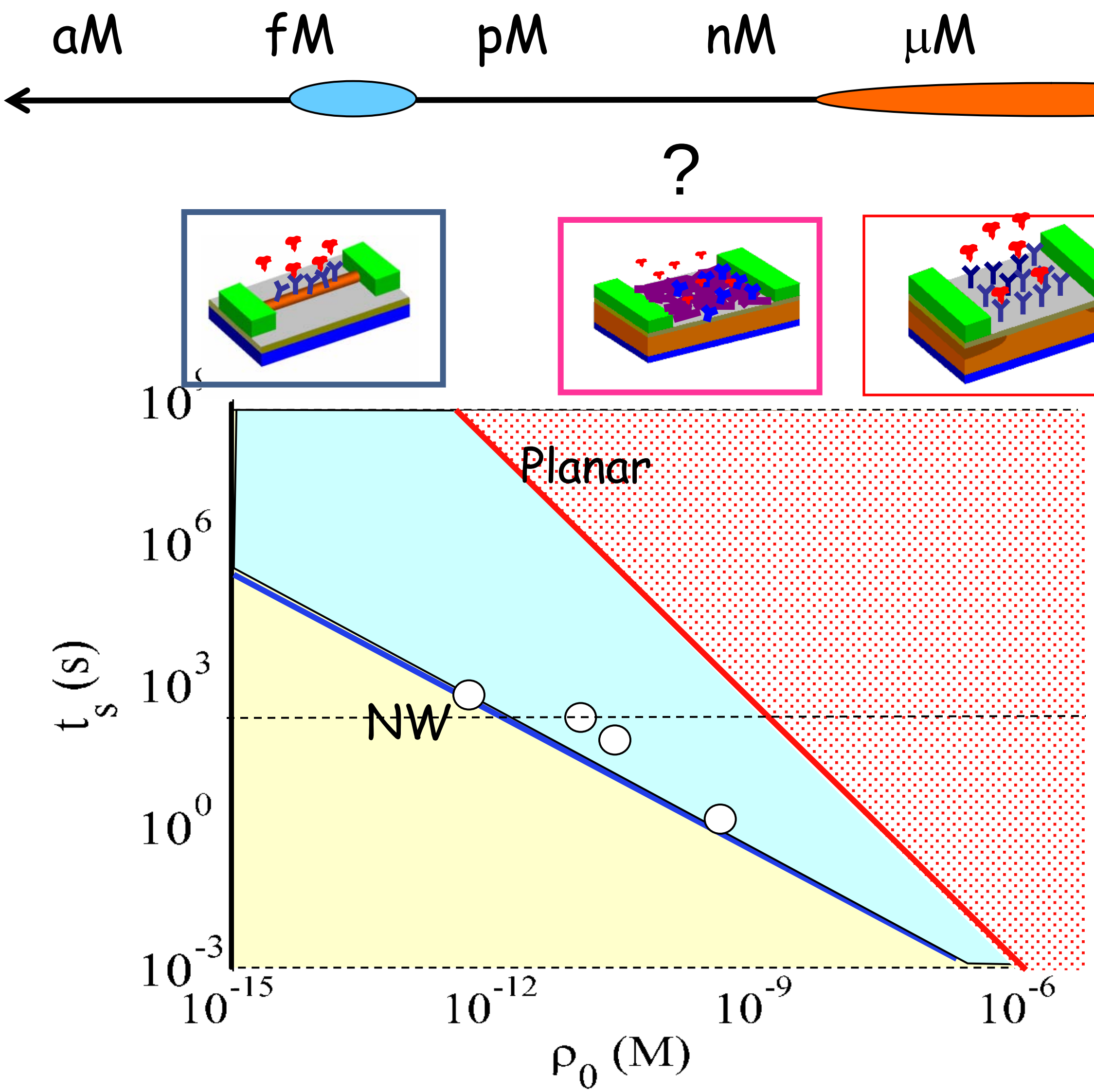


$$R \sim \sqrt{4Dt}$$

$$t_s \sim \frac{N_s a}{D}\frac{1}{\rho_0} \leftarrow m$$

$$N(t) \times 2\pi a \sim \rho_0 \times \pi R^2 \sim \rho_0 \times \pi\left[\sqrt{Dt}\right]^2$$

geometry of diffusion/sensor response



(Planar)

$$t_s \sim \frac{2N_S^2}{D}\frac{1}{\rho_0^2}$$

(SiNW)

$$t_s \sim \frac{N_S a_0}{D}\frac{1}{\rho_0}$$

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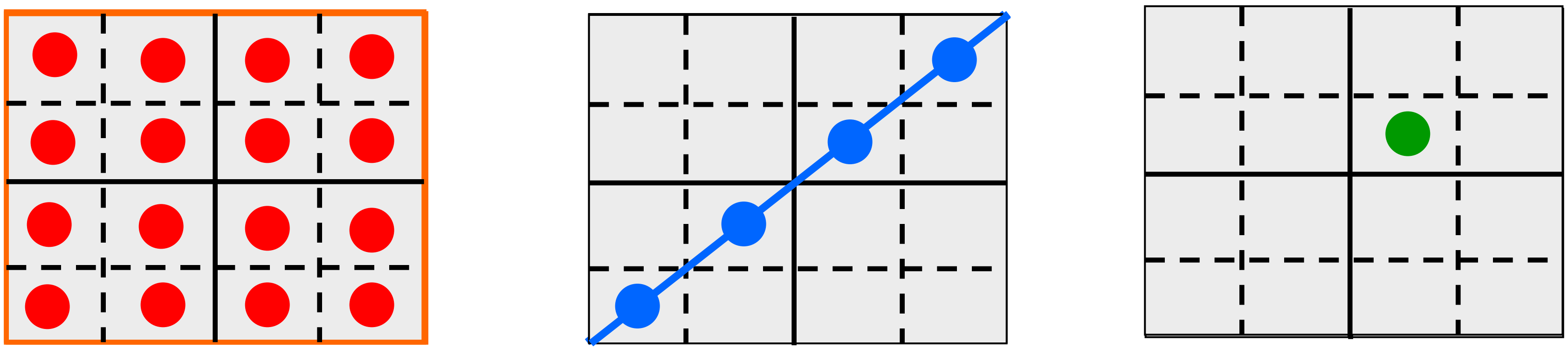
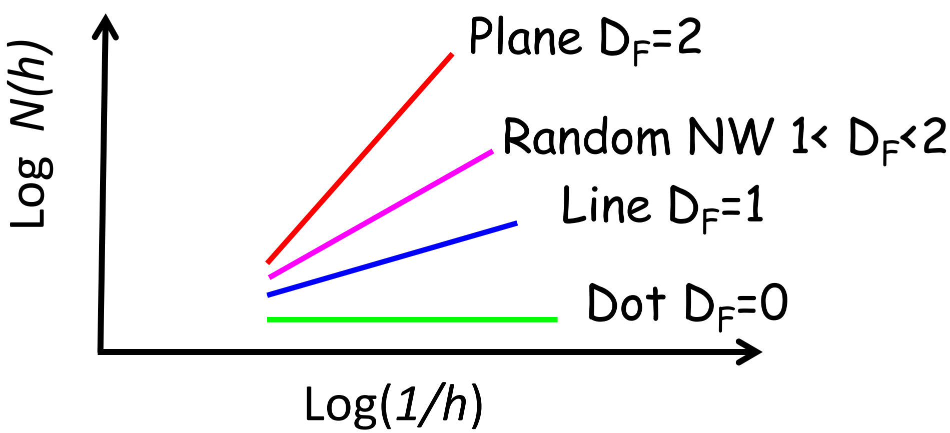
classification of surfaces…

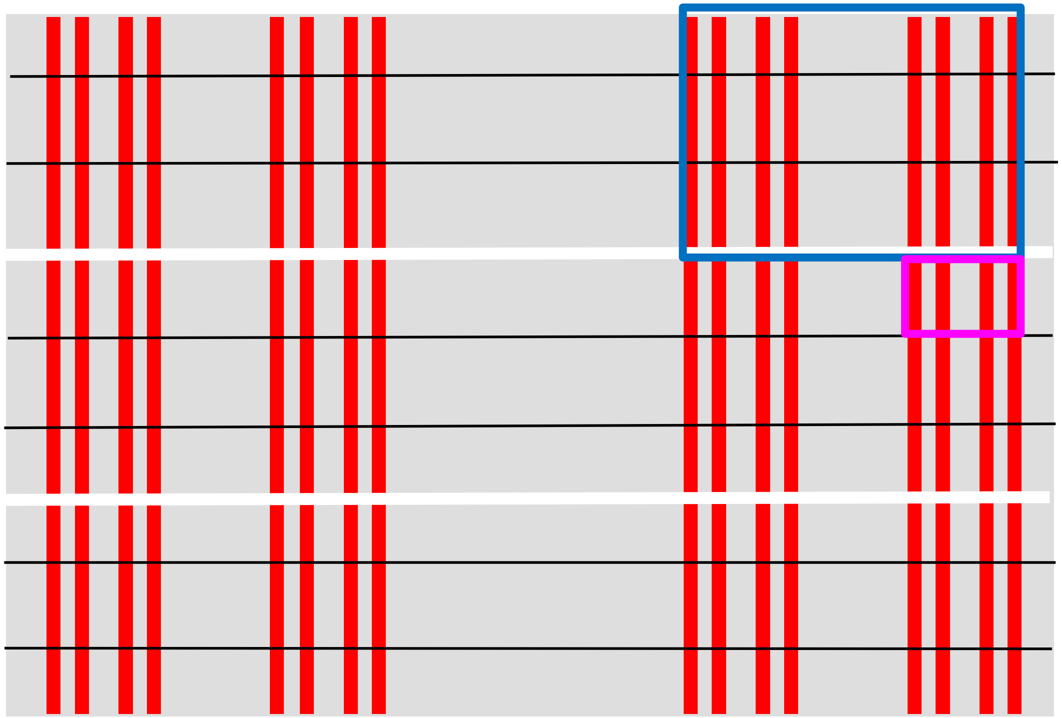
Fractal Dimension (D_F)- Box counting technique



$N(h) \sim h^{2}$ $N(h) \sim h^{1}$ $N(h) \sim h^{0}$



dimension of quasi-2D cantor stripes



h	1/3
N	6

h	1/9
N	36

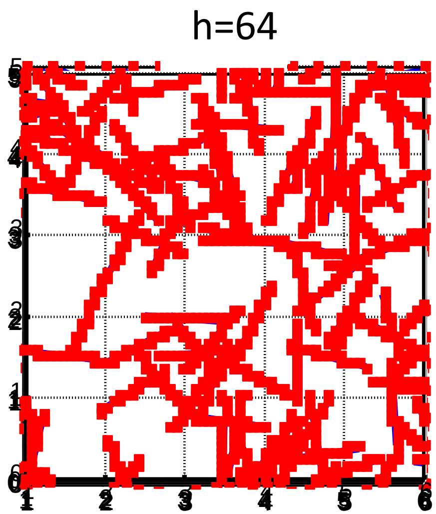
h	1/27
N	216

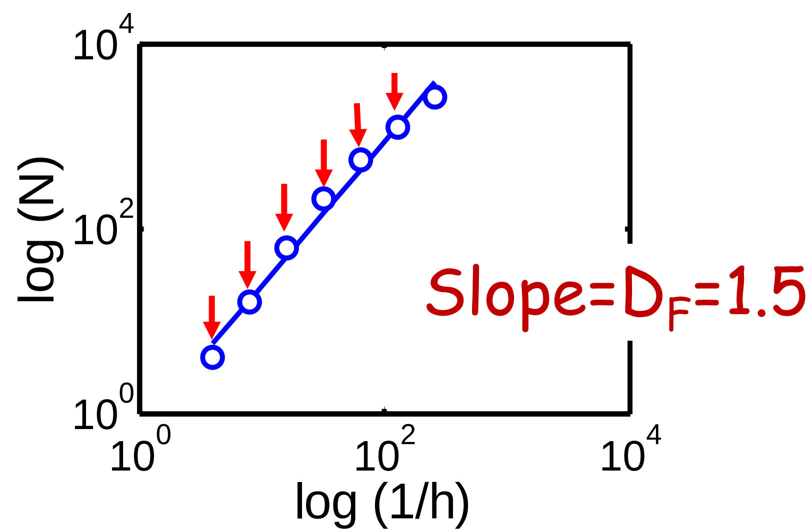
h	$1/3^n$
N	$3^n 2^n$

$$D_{F,2} = \frac{\log(N)}{\log(1/h)} = \frac{\log(3^n) + \log(2^n)}{\log(3^n)} = 1 + \frac{\log(2)}{\log(3)} = 1 + DF_x$$

In general, $D_{F,3} = DF_x + DF_y + DF_z$

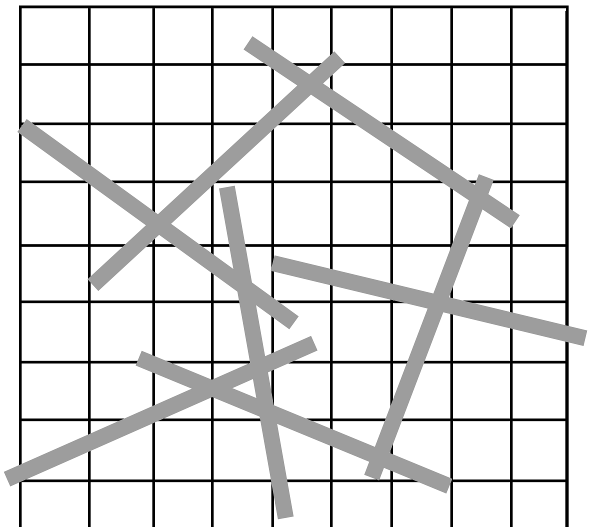
fractal dimension of a stick network



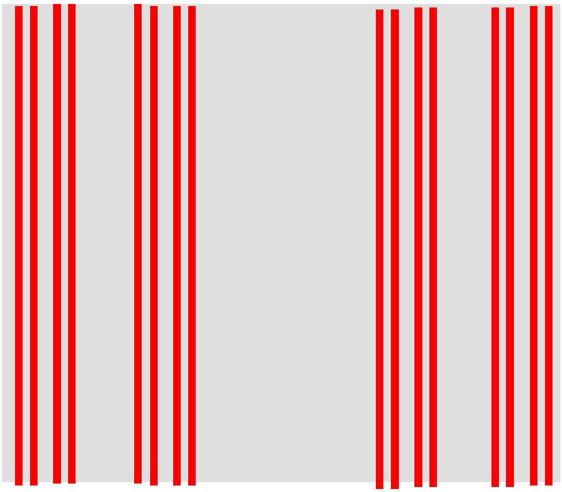


Dimension depends on stick density …

2D to 2D Cantor transform



$D_{F,CT} = D_{F,stick}$



$D_{F,CT}=1+\log(m)/\log(n)$

For $D_{F,stick}=1.5$

Let m=2, solve for n:

$\log(n)=\log(2)/(D_{F,stick}-1)$

Result: n=4

Generation algorithm:

Take a line segment

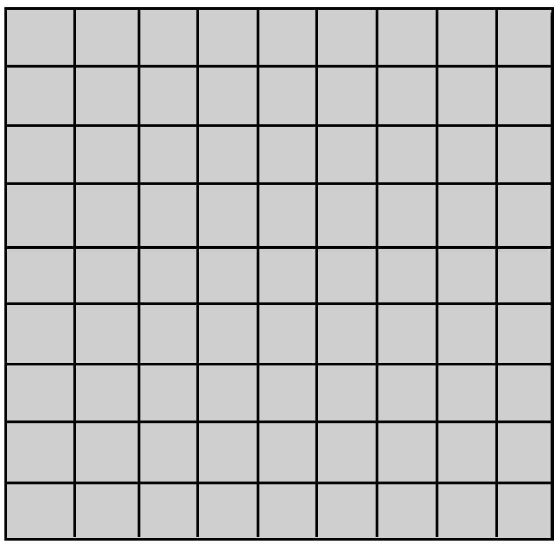
Remove the fraction (n-2)/n

from its centre (result: ½)

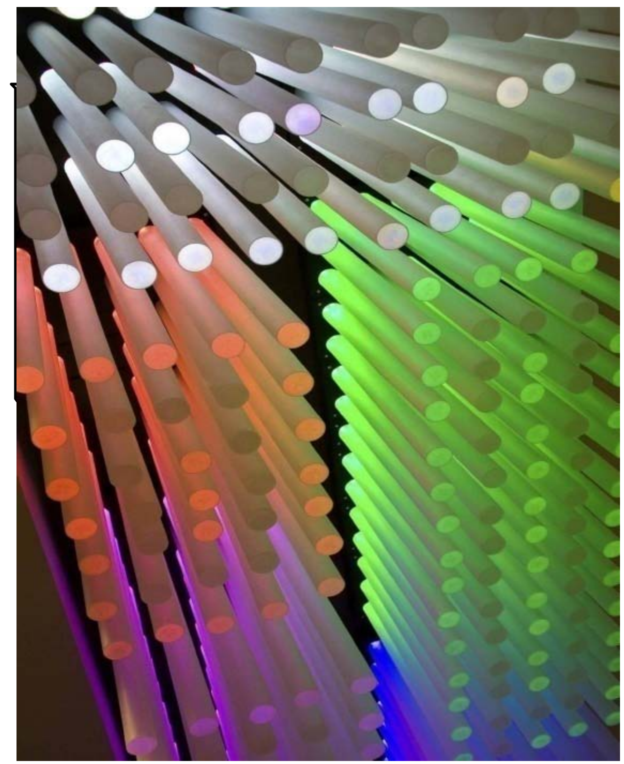
repeat …

2D to 3D Cantor transform

$D_{2D} = 2$

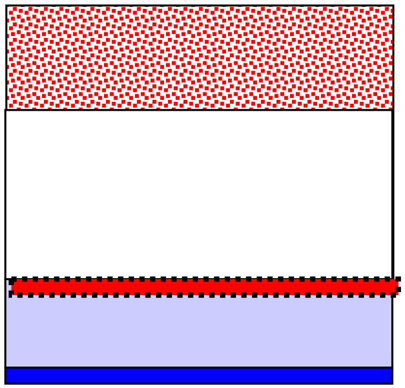


$D_{F,CT} = 1 + 2\log(m)/\log(n)$

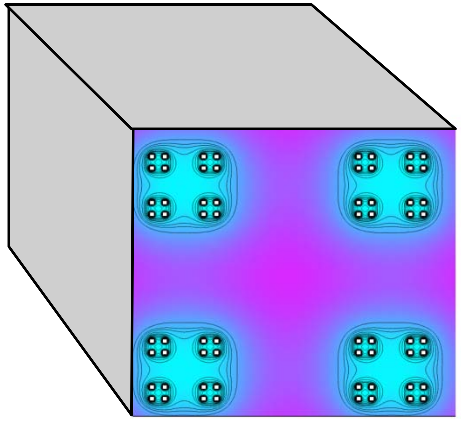


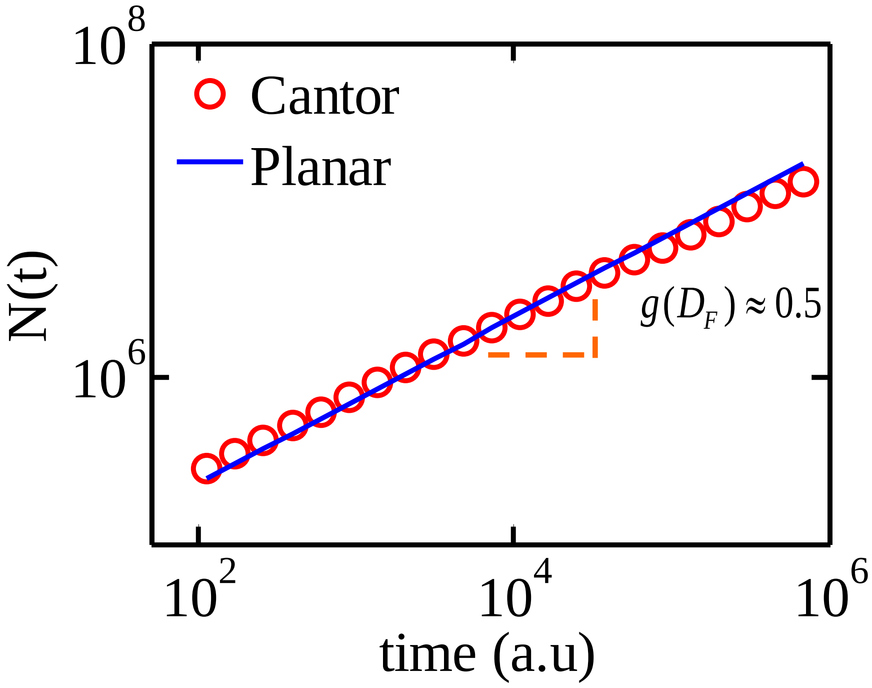
$D_F = 2 = D_{F,CT}$ if $m=2$ and $n=4$

practice run: 2D surface in two different ways



$$D_F = 2 \qquad N(t) = k\rho_0\, t_s^{1/D_F}$$





Surfaces with same D_F have same time response!

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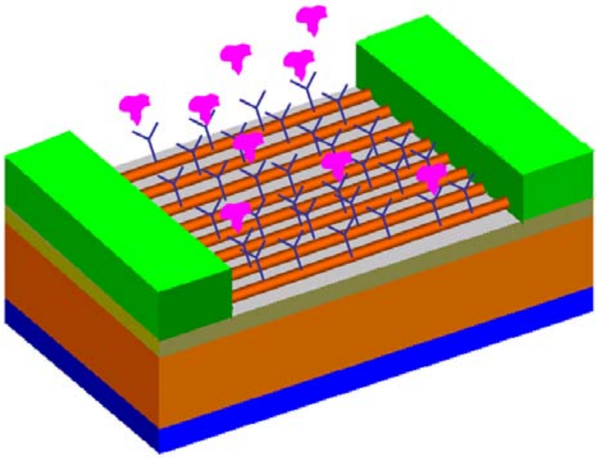
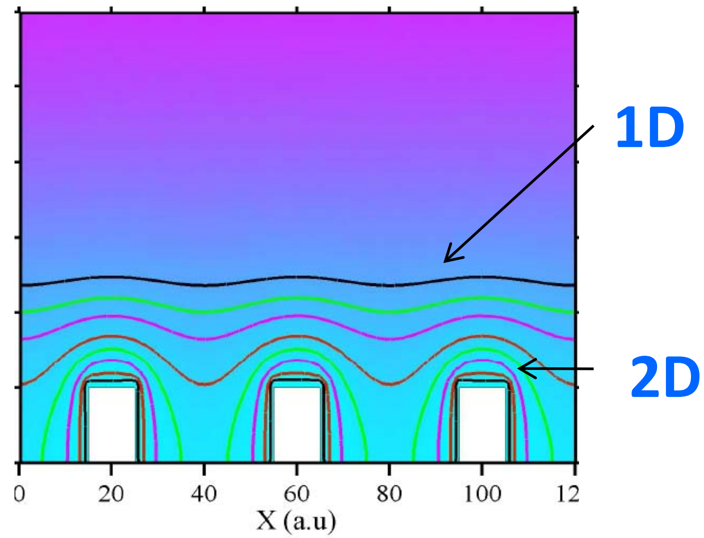
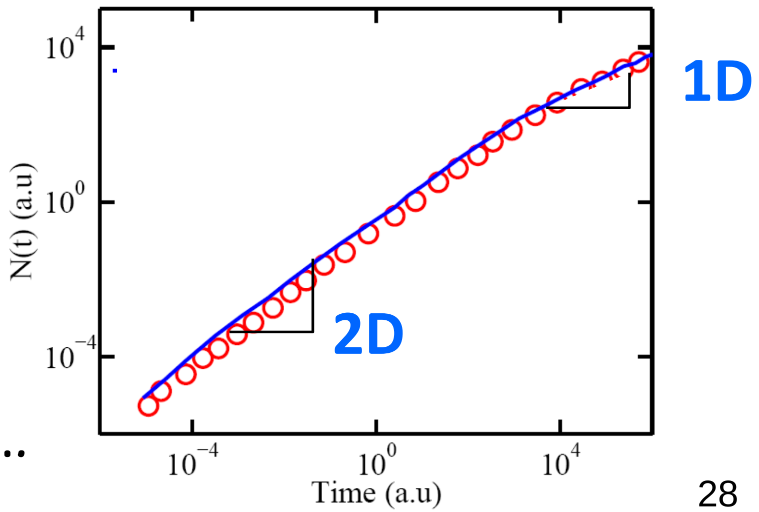
time-dependent DF: from 1D to 2D





$$N(t) = C_{D(t)} \rho_0 \, t$$

$$C_{D(t)} = \frac{2\pi D}{\log\left[\sinh\left(2\pi\sqrt{Dt}P\right) / \pi a_0 P\right]}$$

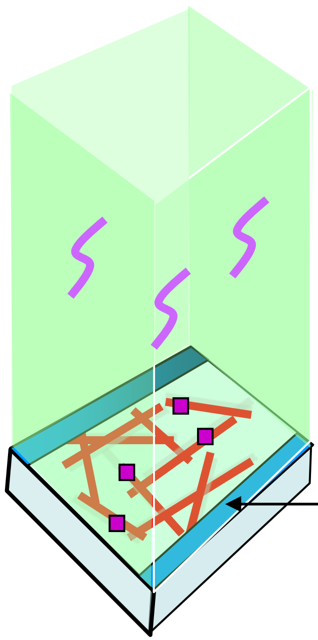


DF of the response of
an object can morph with time …

ready to solve the real problem ?

$$\frac{d\rho}{dt} = D\nabla^2\rho$$

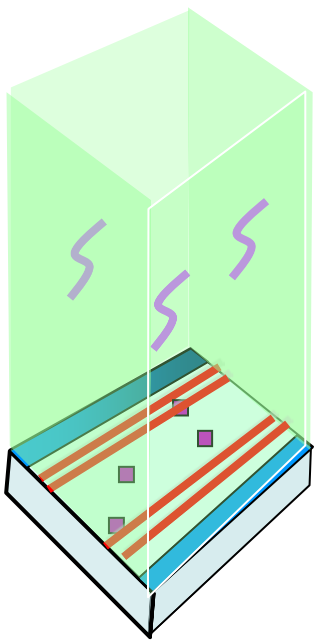
$$\frac{dN}{dt} = k_F(N_0 - N)\rho_s - k_R N$$



Boundary condition

$$\int_{A_D} \frac{dN}{dt} ds = D\int_{A_D} \nabla\rho \, ds$$

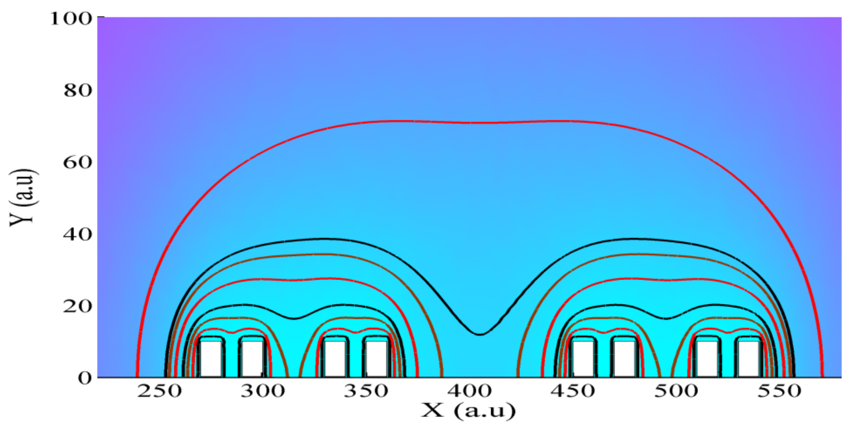
Reaction flux Diffusion flux

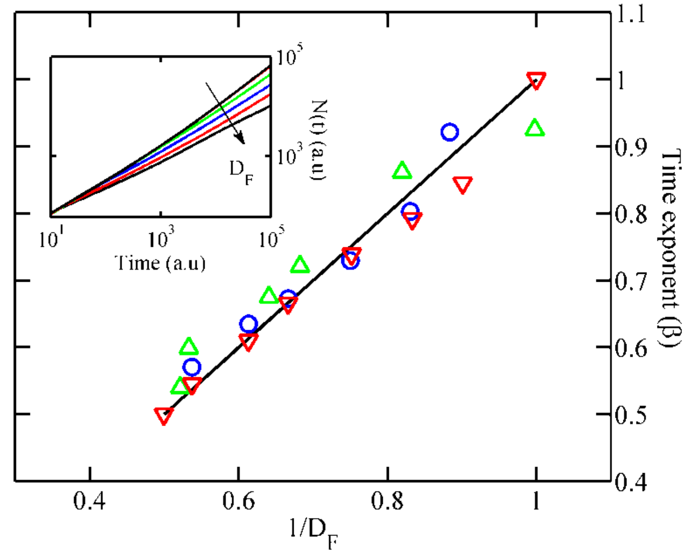


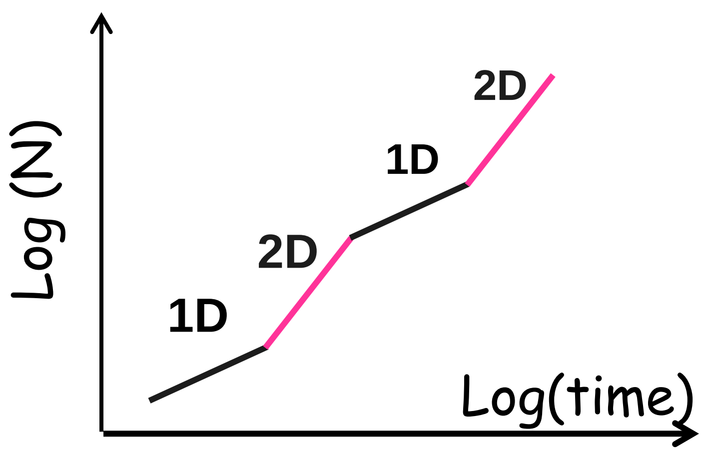
analogous to Fourier transform …

diffusion towards fractal surfaces

Nair/Alam, PRL, (2007).







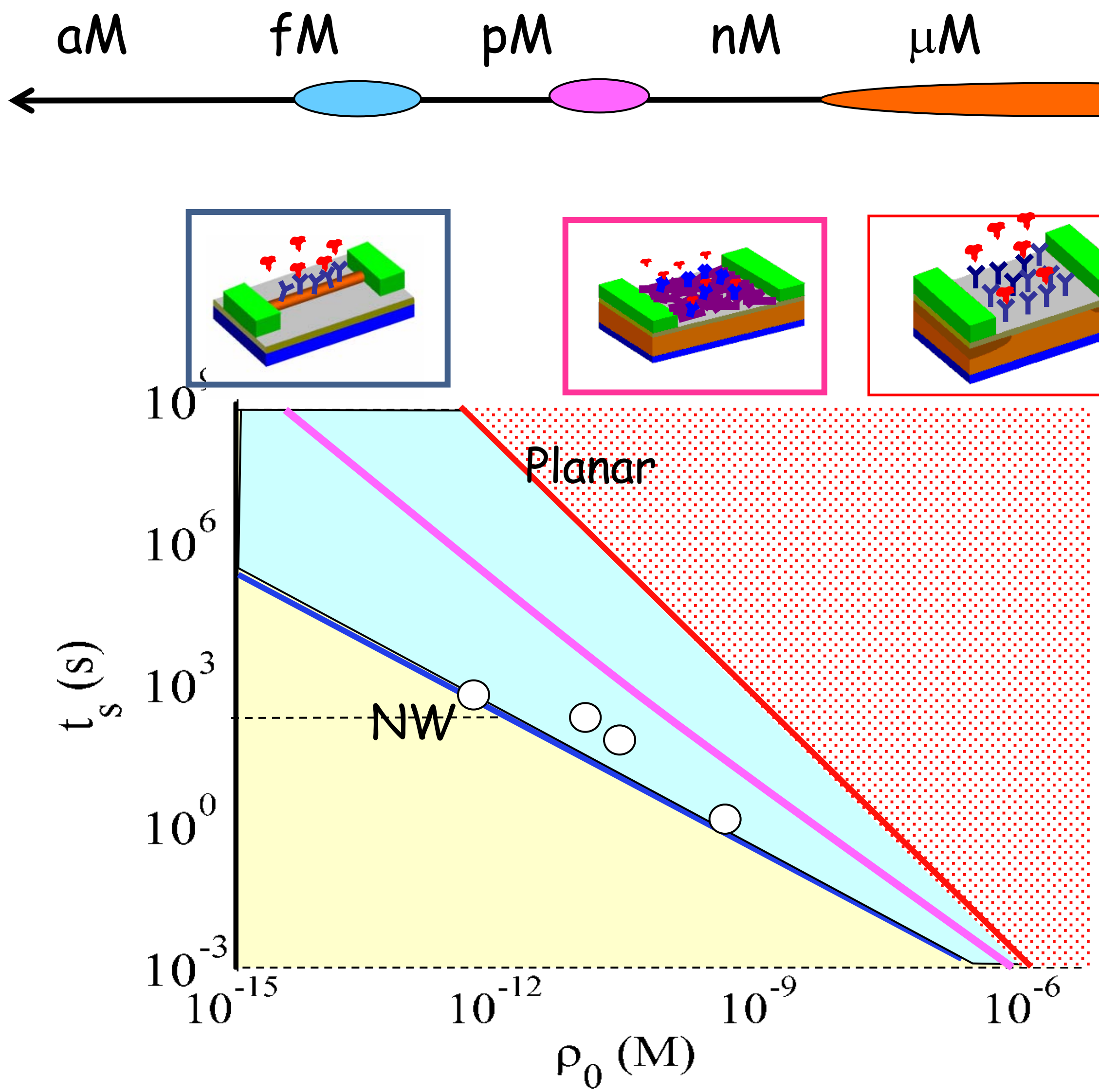
Dimensionally frustrated diffusion!

$$N(t) = \rho_0 \; t_s^{1/D_F}$$

$1 < D_F < 2$

s~ small

geometry of diffusion/sensor response



(Planar)

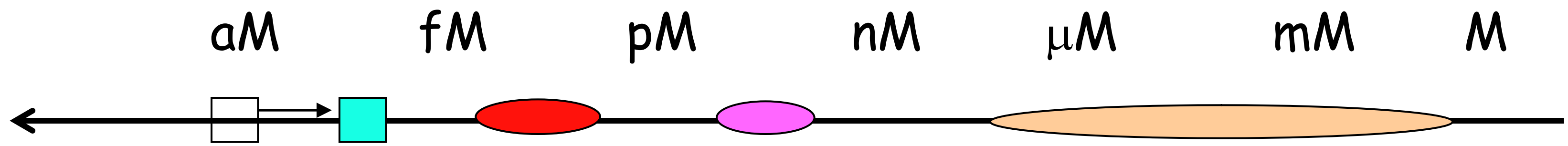
$$t_s \sim \frac{2N_S^2}{D}\frac{1}{\rho_0^2}$$

$$t_s \sim \frac{k_D}{\rho_0^{D_F}}$$

$$t_s \sim \frac{N_S a_0}{D}\frac{1}{\rho_0}$$

(SiNW)

geometry of diffusion & sensor response



In the future ..

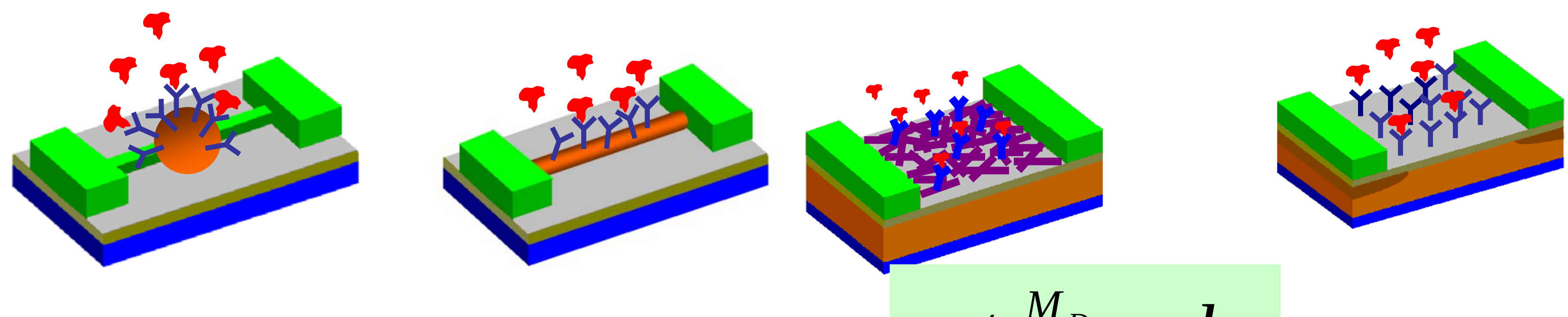
Recently …

Historically …

Nanodots

Si-NW/CNT

ChemFET/IsFET



$$\rho_0 t_s^{M_D} \sim k_D$$

Composites more sensitive than planar, but less than NW

(A. Star, PNAS,103, 4,921, 2006)

conclusions

- One special class of percolation problem involves diffusion towards fractal surfaces (e.g. biosensors, fractal electrodes, fractal antenna, solar cells and super-capacitors, etc.)

- Cantor transform of the irregular surfaces into regular ones allows simple yet accurate calculation of their time-responses both in 2D and 3D systems.

- We found that it is the geometry of diffusion, rather than geometry of electrostatics that defines sensor response.

- The fractal antenna utilizes its self-similarity to broaden its reception and transmission bands.