2009 NCN@Purdue-Intel Summer School

Notes on Percolation and Reliability Theory

Lecture 4
Stick Percolation and Nanonet Electronics

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www.nanohub.org

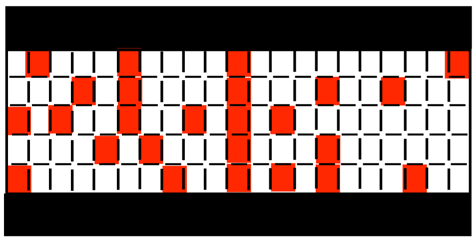
PURDUE UNIVERSITY

outline lecture 4

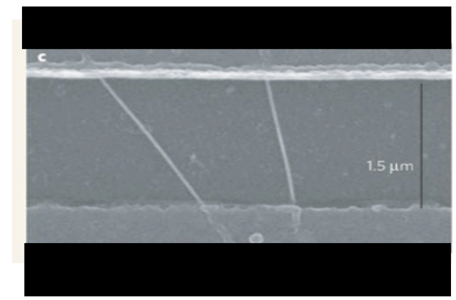
1) **Stick percolation and nanonet transistors**
2) Short channel nanonet transistors
3) Long channel nanonet transistors
4) Transistors at high voltages
5) Conclusions

lecture 3 vs. lecture 4

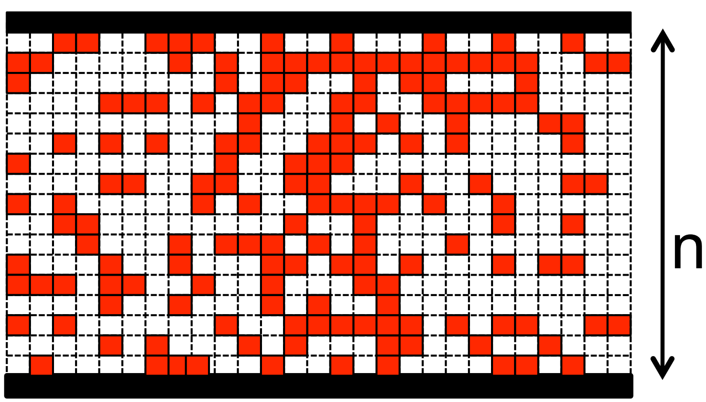
$$G \sim \sigma_{row} p^L \frac{W}{L}$$



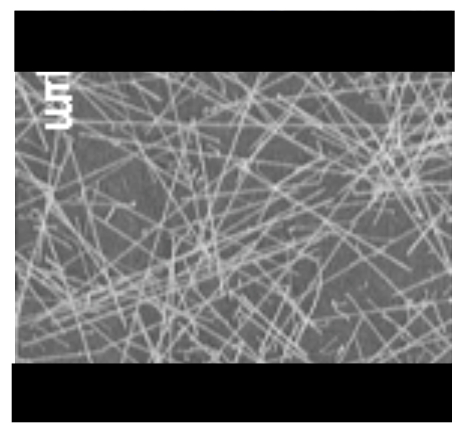
G ?



$$G \sim \sigma_{row} \frac{W}{L^\alpha}$$



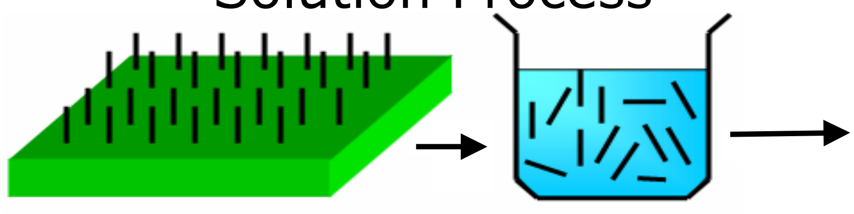
G ?



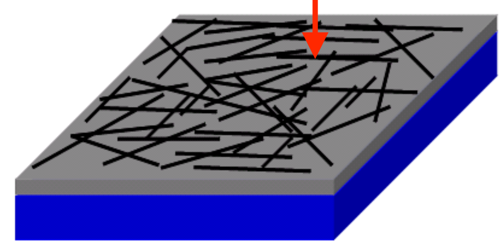
how to make nanonet transistors ?

CNT / SiNW / ZnO

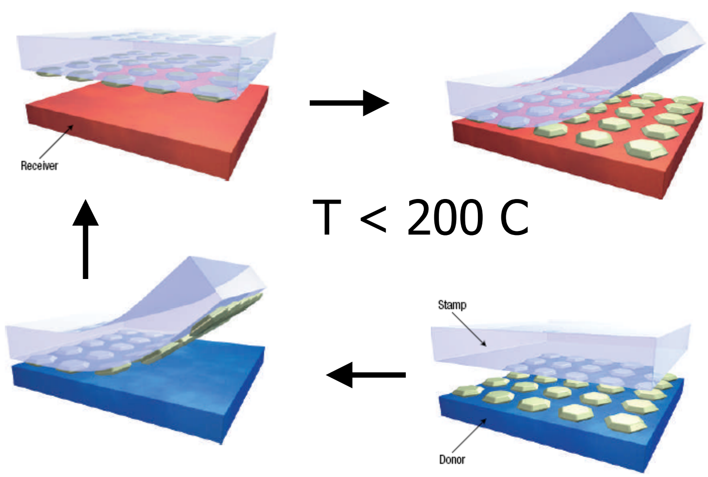
Solution Process



T > 500 C T < 200 C



Nanotransfer Printing



Rogers el al. Nature Mat. 2006

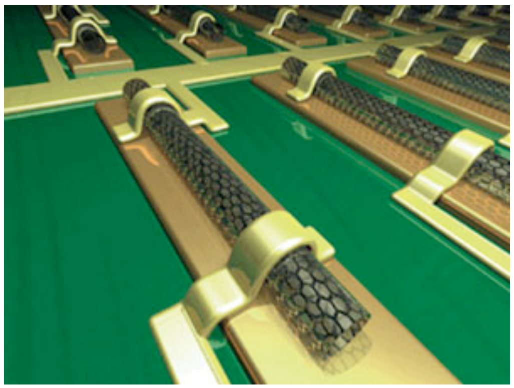
Advantages

- Highly crystalline CNT / SiNW by high temperature process
- Plastic, glass or organic substrate :

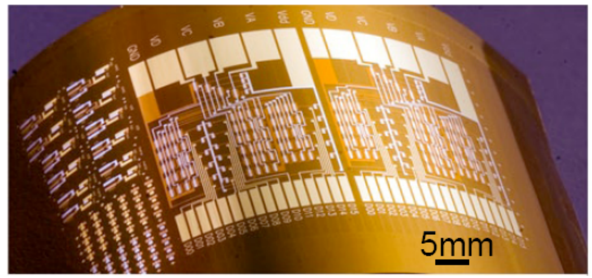
  Low temperature final step
- Transparent and conducting

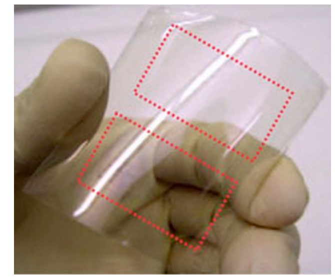
why do we make nanonet transistors ?

Ideally …

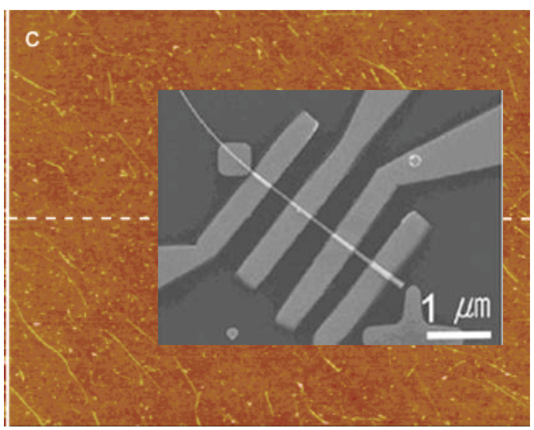


flexible electronics ?

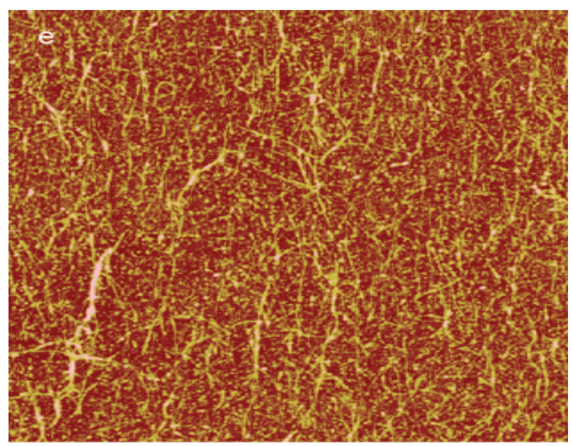


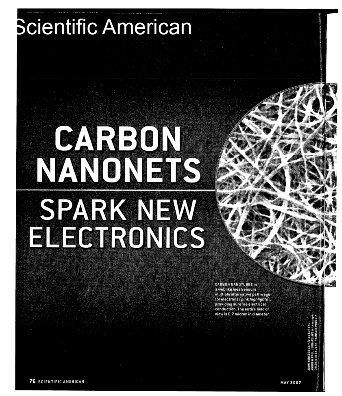


In practice …



…better if …





… lot of interest in this topic.

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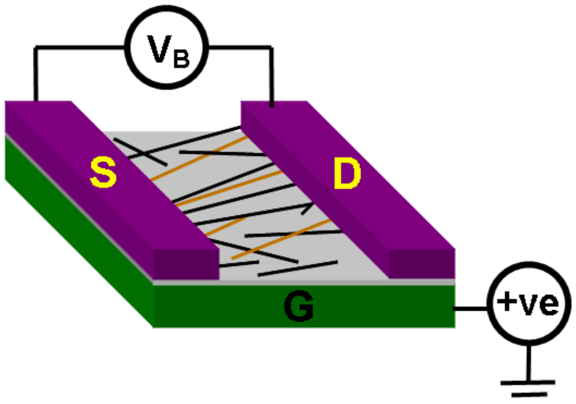
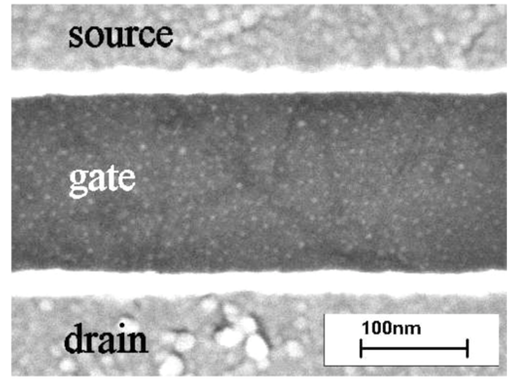
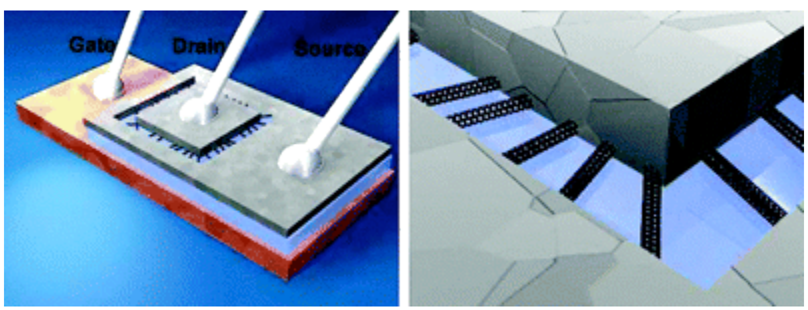
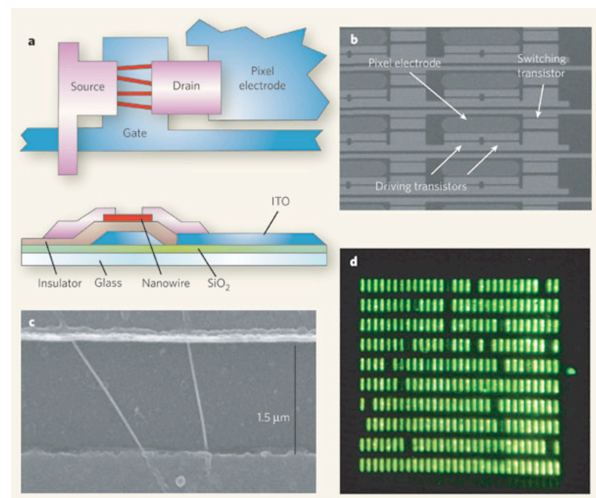
short channel nanonet transistors





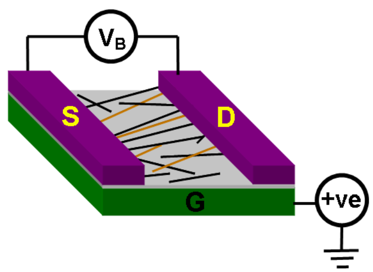


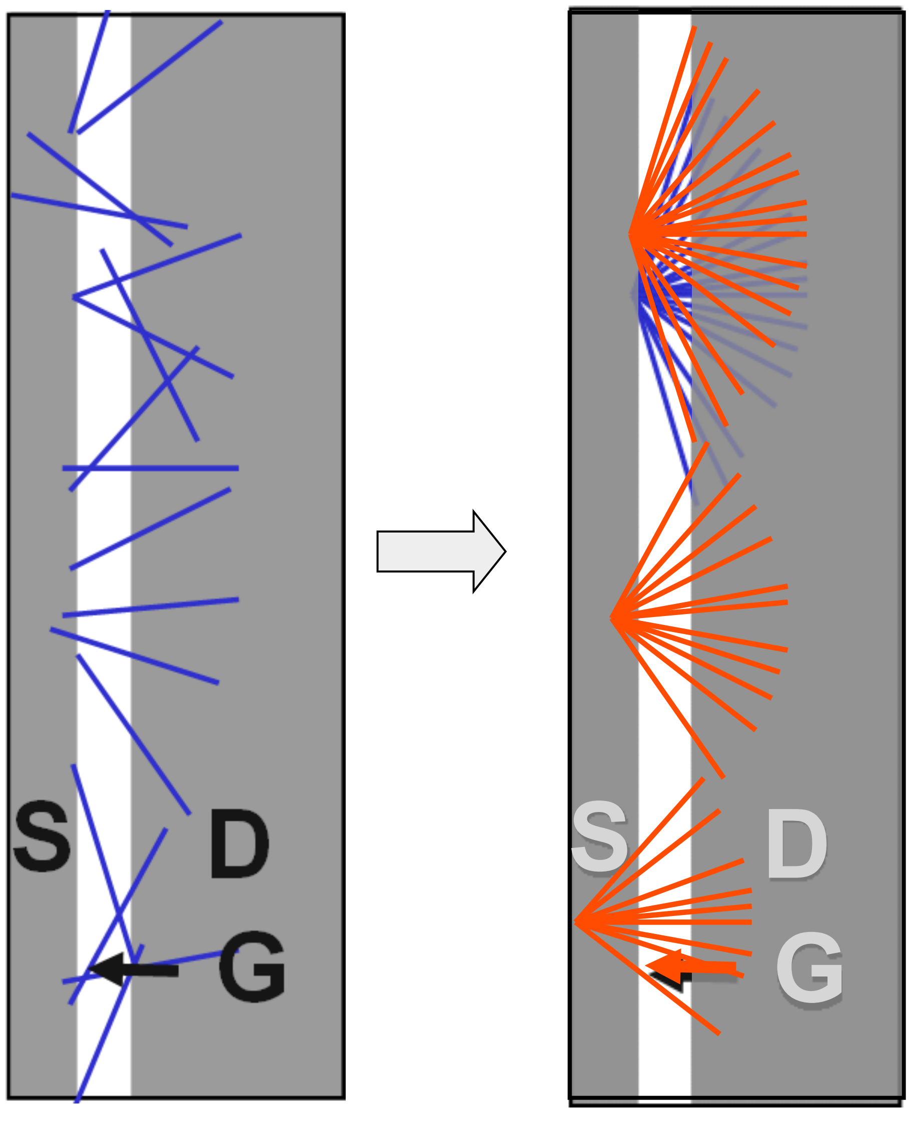


Seidel, NanoLetters, 2004.

Janes, Nature Nanotech, 2008.

short channel stick percolation

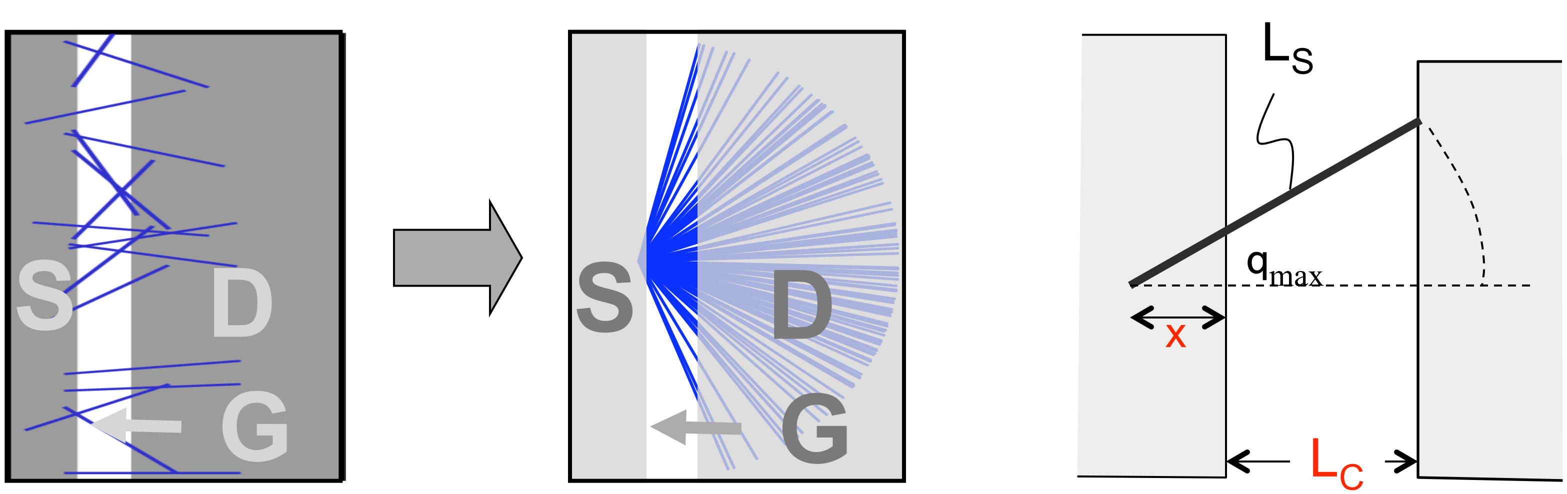




Fan Diagram:

Collect all the sticks to one point

number of bridging sticks



$$\cos\theta_{max} = \frac{L_C + x}{L_S}$$

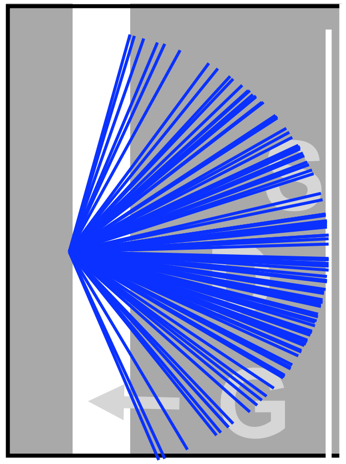
$$N_s = \int_0^{L_S - L_C} D_x dx \frac{\theta_{max}(x)}{\pi/2} = \int_0^{L_S - L_C} \frac{2}{\pi} D_x dx \cos^{-1}\frac{x + L_C}{L_S}$$

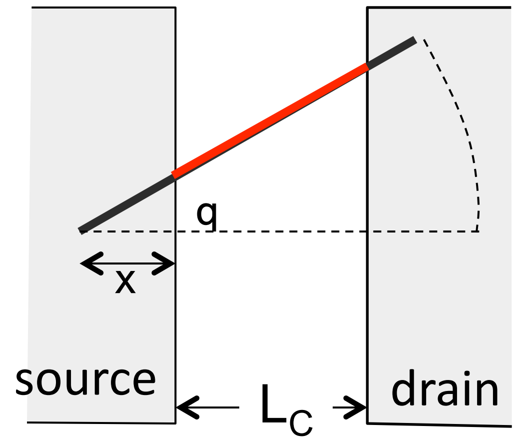
$$N_S = \frac{2 D_C L_S}{\pi}\left[\sqrt{1 - R^2} - R\cos^{-1} R\right]$$

$$R = \frac{L_C}{L_S}$$

- Generalized Buffon Needle Problem
- If needle is curved, use chord length

I-V characteristics: ballistic transport





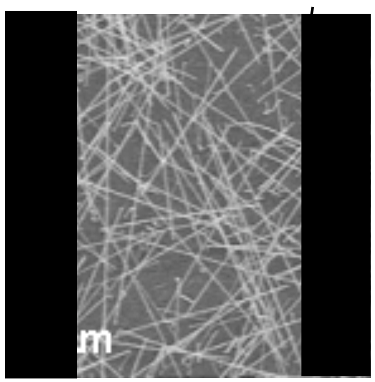
$$I_D^{(i)} = \frac{q}{\pi\hbar}\int_{E_b(V_G)}^{\infty} dE\left[f_0\left(E_{FS}\right) - f_0\left(E_{FD}\right)\right] \equiv f(V_G, V_D)$$

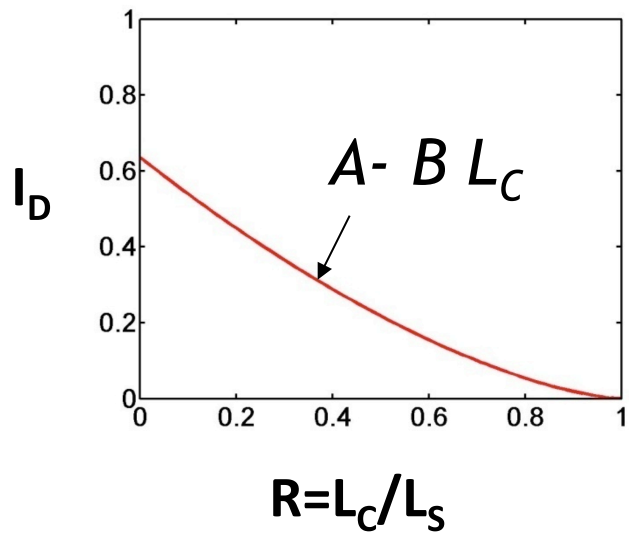
$$I_D = \sum_1^N f = \int_0^{L_S - L_C} D_x dx \frac{2}{\pi}\theta_{\max} f = f\,\frac{2D_C L_S}{\pi}\left[\sqrt{1-\left(\frac{L_C}{L_S}\right)^2} - \frac{L_C}{L_S}\cos^{-1}\frac{L_C}{L_S}\right]$$

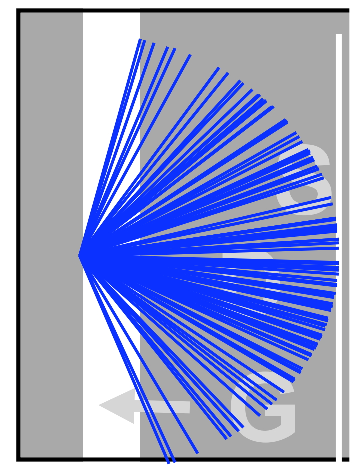
Electrical Part Geometrical Part

ballistic transport and length-scaling ….

$$I_D = f(V_G, V_D) \times \frac{2 D_C L_S}{\pi} \left[\sqrt{1 - \left(\frac{L_C}{L_S} \right)^2} - \frac{L_C}{L_S} \cos^{-1} \frac{L_C}{L_S} \right]$$

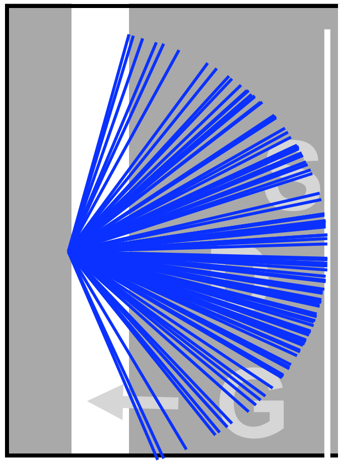


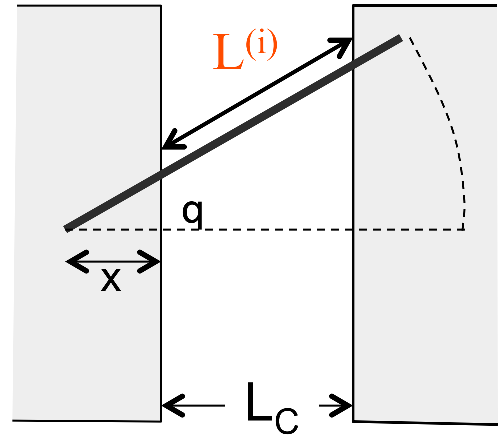




length dependence even for ballistic transport,
nothing to do it with mobility !

I-V characteristics: with scattering





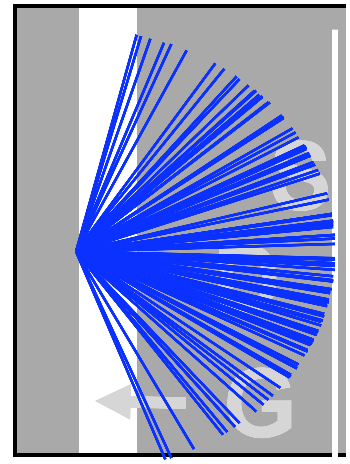
$$I_D^{(i)} = \frac{q}{\pi\hbar} \int_{E_b(V_G)}^{\infty} dE \left[\frac{\lambda}{\lambda + L^{(i)}} \right] \left[f_0(E_{FS}) - f_0(E_{FD}) \right]$$

$$\approx \frac{\langle\lambda\rangle}{\langle\lambda\rangle + L^{(i)}} \frac{q}{\pi\hbar} \int_0^{\infty} dE \left[f_0(E_{FS}) - f_0(E_{FD}) \right]$$

$$I_D = \sum_1^N \frac{\langle\lambda\rangle}{\langle\lambda\rangle + L^{(i)}} \times f = f \times \sum_1^N \frac{\langle\lambda\rangle}{\langle\lambda\rangle + L_C/\cos(\theta)}$$

I-V characteristics: with scattering

$$I_D = f \times \sum_{1}^{N} \frac{\langle \lambda \rangle}{\langle \lambda \rangle + L_C / \cos(\theta)}$$



$$\frac{I_D}{f} = \frac{2D_C}{\pi b^2}\left[b g_B\left(\frac{L_C}{L_S}\right) - \cos^{-1}\frac{L_C}{L_S} + \frac{2(bL_C/L_S+1)}{\sqrt{b^2-1}} \tanh^{-1}\frac{(b-1)\tan(\theta_S/2)}{\sqrt{b^2-1}} \right]$$

$$I_D \;=\; f\left(V_G, V_D\right) \;\; \xi\left(\frac{L_C}{L_S}, D_C L_S\right) \sim \sigma_0 \frac{W}{L^{\alpha}}$$

Electrical Part   Geometrical Part

Experimental verification of fan-diagram in the appendix …

…. a remarkable formula

Scaling variables …

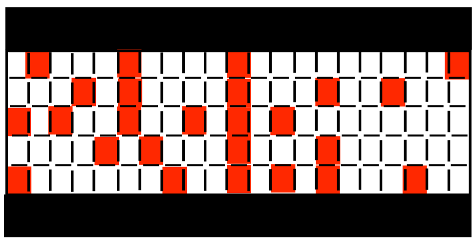
$$I_T = f\left(V_D, V_G\right) \times \xi\left(\frac{L_C}{L_S}, D_C L_S{}^2\right)$$

Classical Transistor Theory (1950s --)

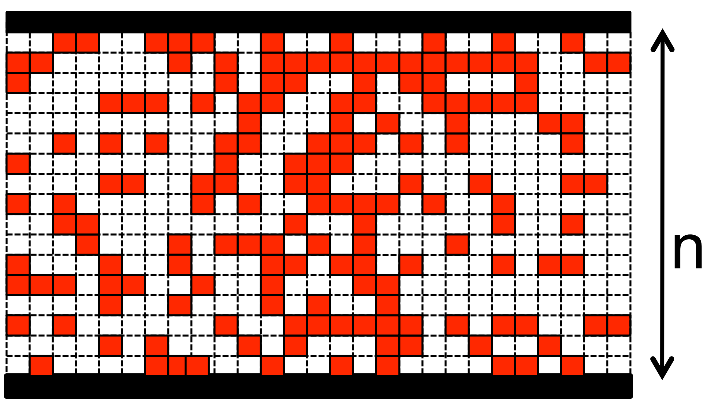
Classical percolation Theory (1970s --)

lecture 3 vs. lecture 4

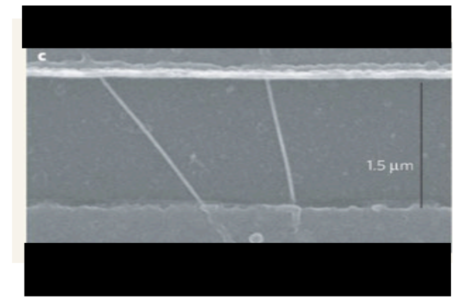
$$G \sim \sigma_{row} p^L \frac{W}{L}$$



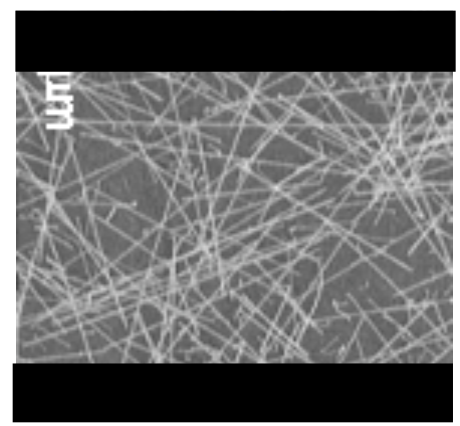
$$G \sim \sigma_{row} \frac{W}{L^{\alpha}}$$



$$G = \frac{2q^2}{\pi^2 \hbar} D_C L_S \left[\sqrt{1 - \left(\frac{L_C}{L_S} \right)^2} - \frac{L_C}{L_S} \cos^{-1} \frac{L_C}{L_S} \right]$$



G ?



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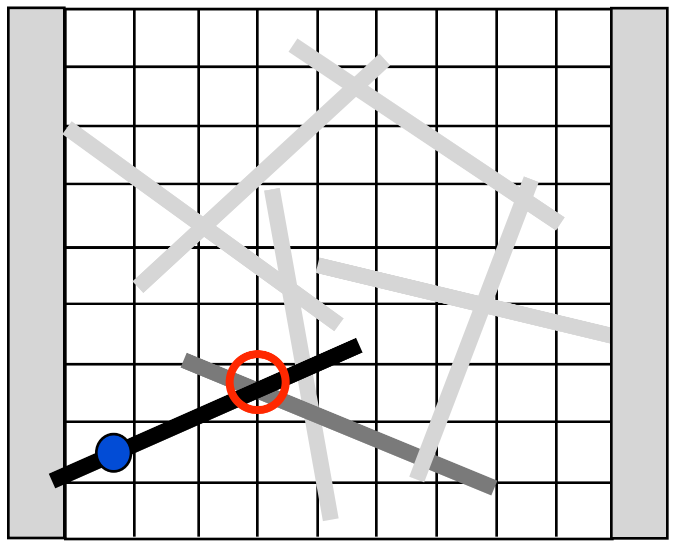
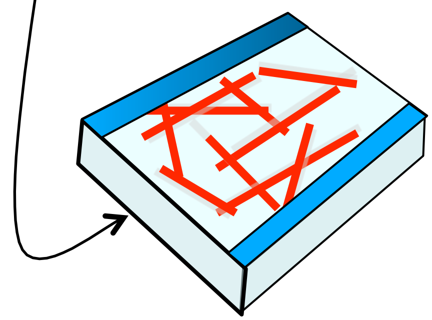
long channel nanonets: numerical model

$$\sum_i \nabla^2 \Phi_i + \frac{\rho_i}{\varepsilon} = \frac{d^2\Phi_i}{ds^2} + \frac{\rho_i}{\varepsilon} + \sum_{j \neq i} \frac{\Phi_j - \Phi_i}{\lambda_{ij}^2} - \frac{\Phi_i - V_G}{\lambda^2} = 0$$



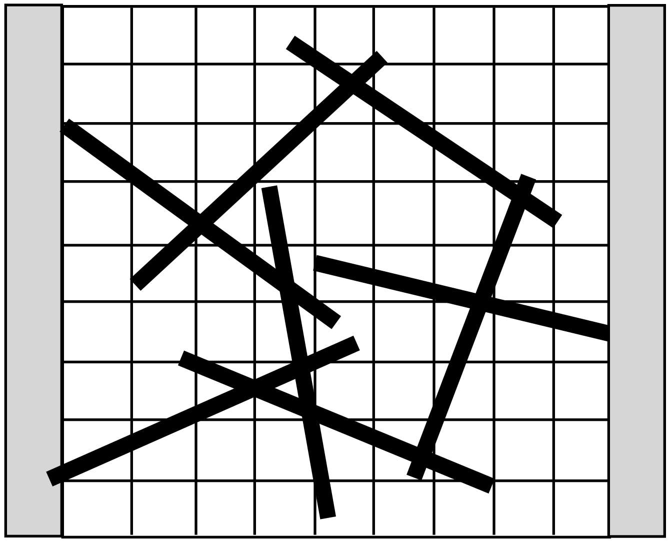


$$J_{n,i} = qn\mu E - qD\frac{dn}{ds}$$

$$\sum_i \frac{dJ_{n,i}}{ds} - \sum_{i \neq j} c_{ij}^n (n_i - n_j) = 0$$

don't try this at home: use nanohub

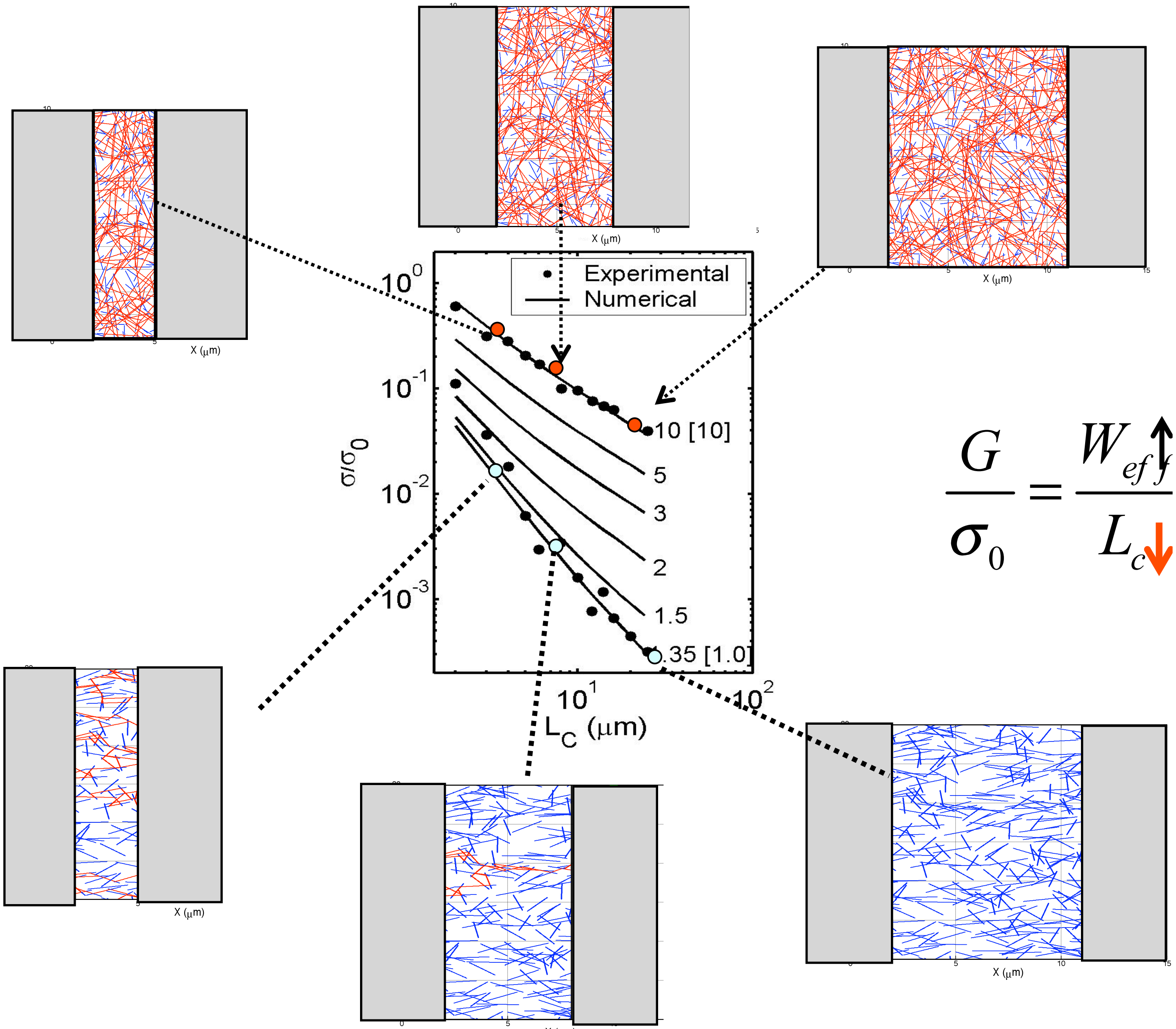
- Analytical solution not possible
- Self consistent numerical DD-Poisson solver
- Solve for hundreds of configuration
- Solve for various biases



Simulator at www.nanohub.org as 'NanoNET'

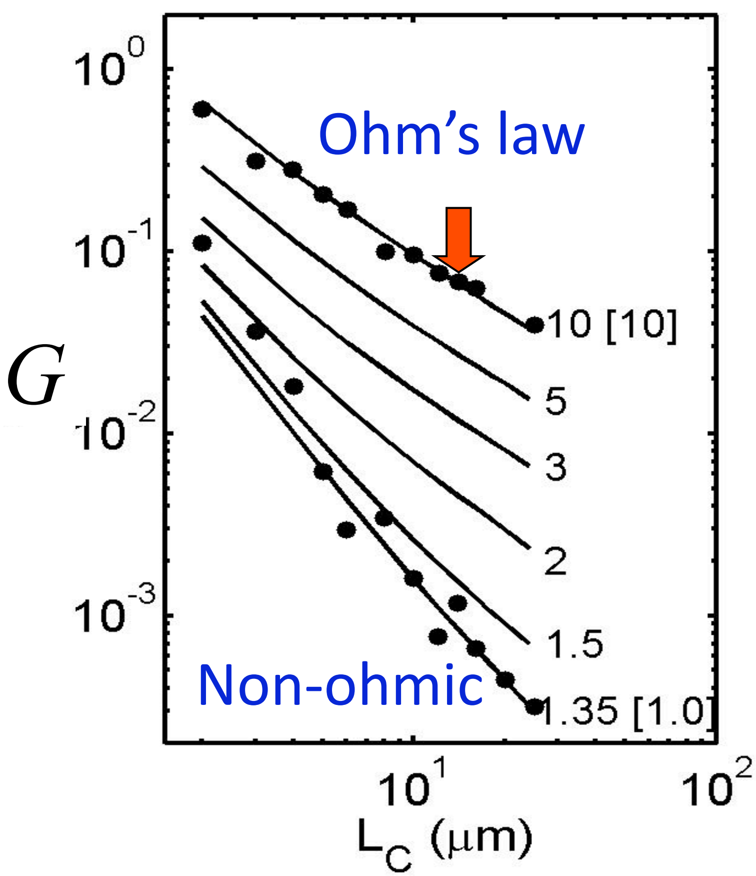
the end of Ohm's law …

$$I_D = f(V_D, V_G) \times \xi\left(\frac{L_S}{L_C}, DL_S^{\,2}\right)$$



$$\frac{G}{\sigma_0} = \frac{W_{eff}\uparrow}{L_c\downarrow}$$

geometrical scaling function



$$I_D = f(V_D, V_G) \times \xi\left(\frac{L_C}{L_S}, D_C L_S\right)$$

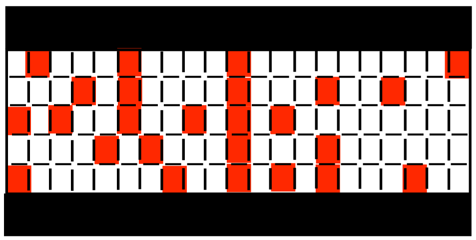
$$= A(V_G - V_{th})V_D \times \xi\left(\frac{L_C}{L_S}, D_C L_S\right)$$

$$G = \frac{I_D}{V_D} \propto \xi\left(\frac{L_S}{L_C}, DL_S^2\right) \simeq \frac{1}{L_S}\left(\frac{L_S}{L_C}\right)^{m(DL_S^2)}$$

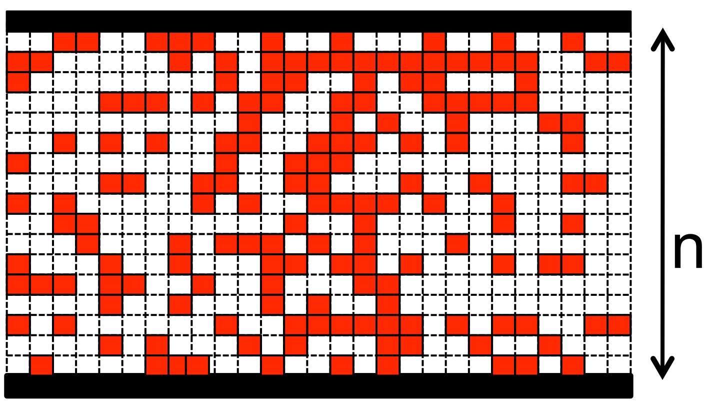
Do you see ohm's law at high D?

lecture 3 vs. lecture 4

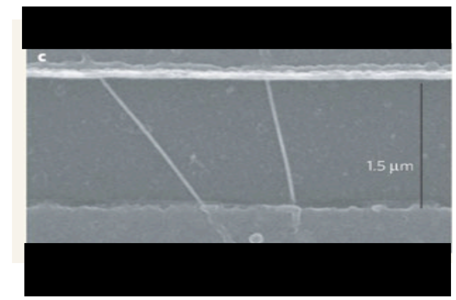
$$G \sim \sigma_{row} p^L \frac{W}{L}$$



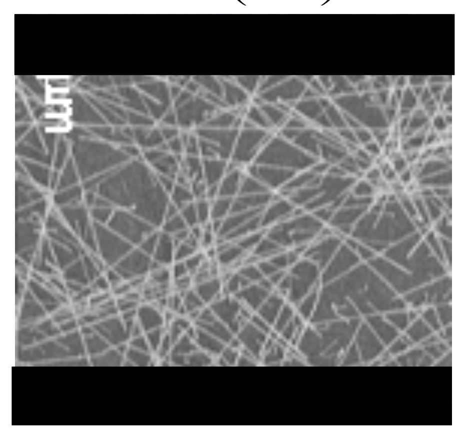
$$G \sim \sigma_{row} \frac{W}{L^{\alpha}}$$



$$G = \frac{2q^2}{\pi^2 \hbar} D_C L_S \left[\sqrt{1 - \left(\frac{L_C}{L_S} \right)^2} - \frac{L_C}{L_S} \cos^{-1} \frac{L_C}{L_S} \right]$$



$$G \sim \frac{1}{L_S} \left(\frac{L_S}{L_C} \right)^{m(DL_S^2)}$$



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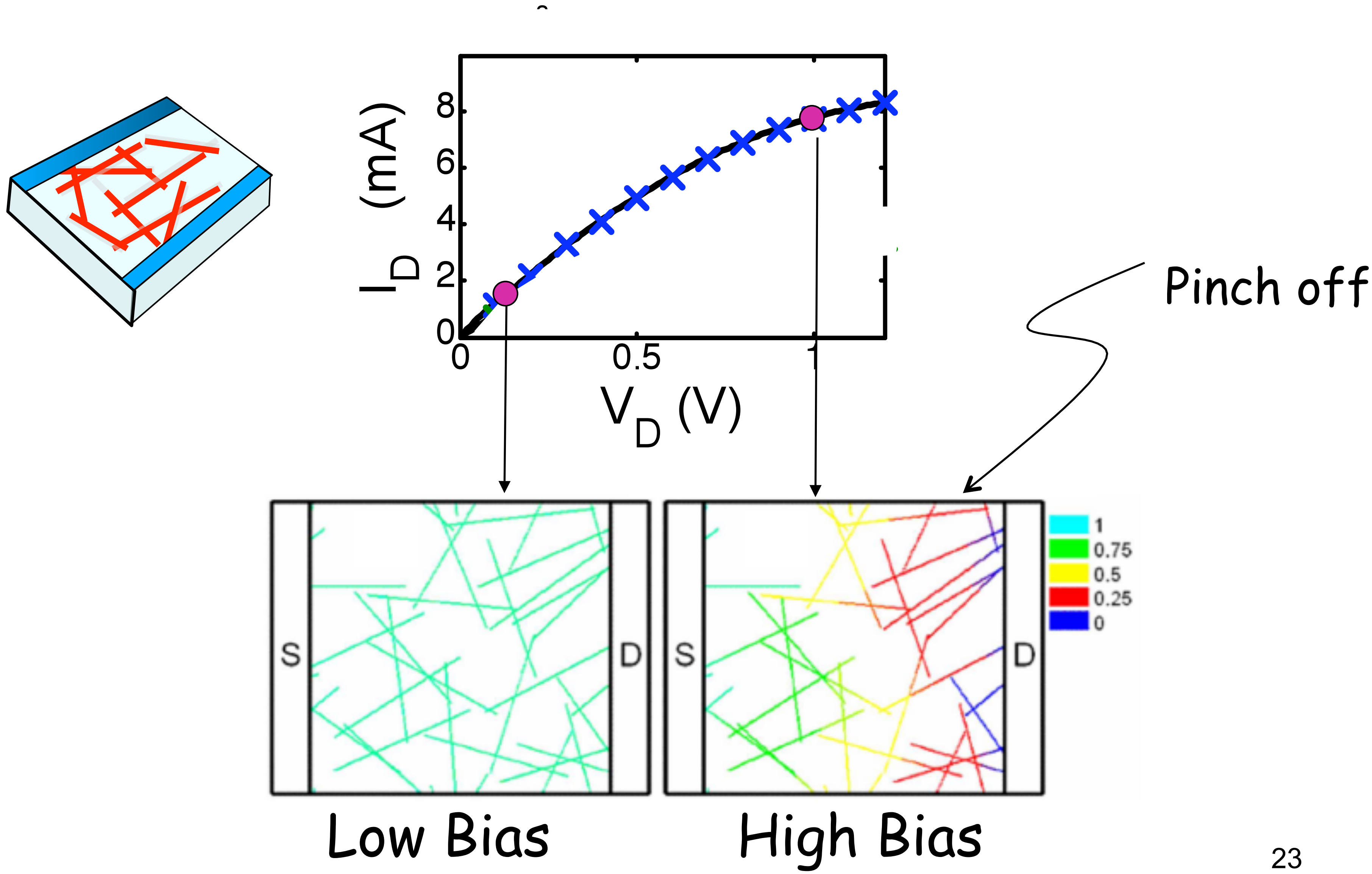
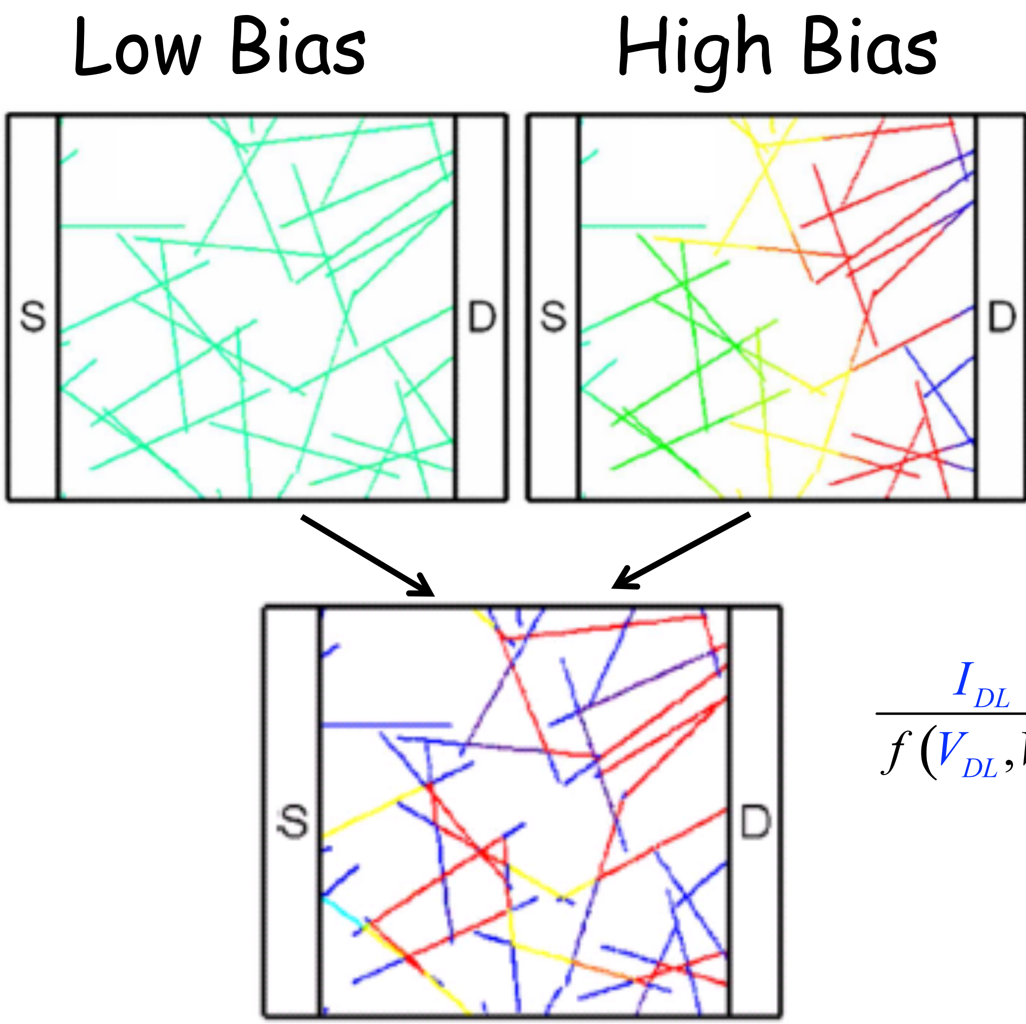
nonlinear I-V of nanonet transistors



voltage scaling in theory …



$$I_D = f(V_D, V_G) \times \xi\left(\frac{L_C}{L_S}, DL_S^2\right)$$

$$\frac{I_{DL}}{f(V_{DL}, V_G)} = \xi\left(\frac{L_C}{L_S}, DL_S^{\ 2}\right) = \frac{I_{DH}}{f(V_{DH}, V_G)}$$

… scales exactly, as anticipated from short channel formula

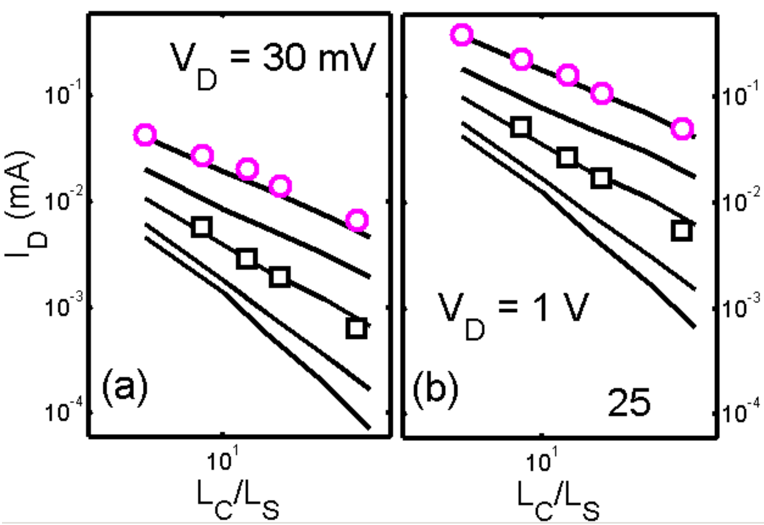
… and in practice!

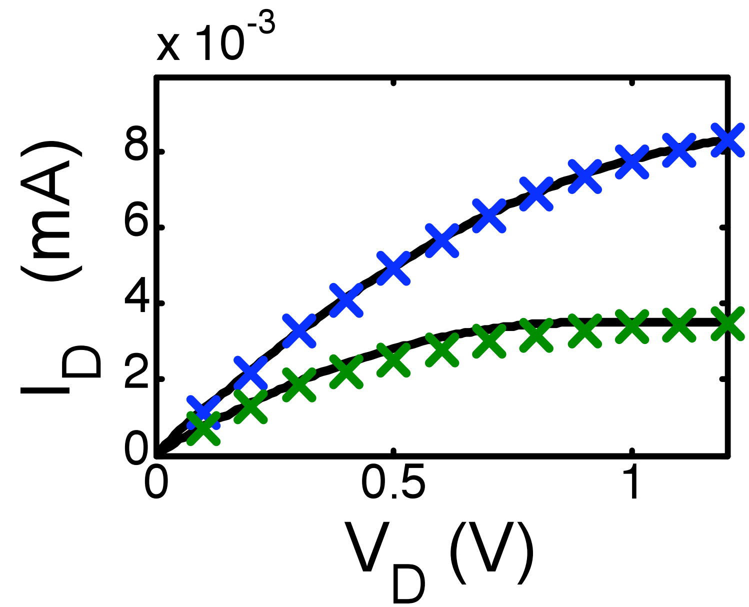
$$I_D = \xi\left(\frac{L_S}{L_C}, DL_S^{\,2}\right) f(V_D, V_G)$$

Hur et al. JACS, 2005
Pimparkar et al. EDL, 2007

$$\xi\left(\frac{L_S}{L_C}, DL_S^2\right) \simeq \left(\frac{L_S}{L_C}\right)^{m\left(DL_S^2\right)}$$

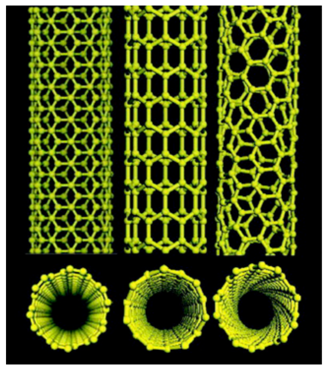
$$f(V_G, V_D) = \left[(V_G - V_{TH})V_D - \beta V_D^2\right]$$

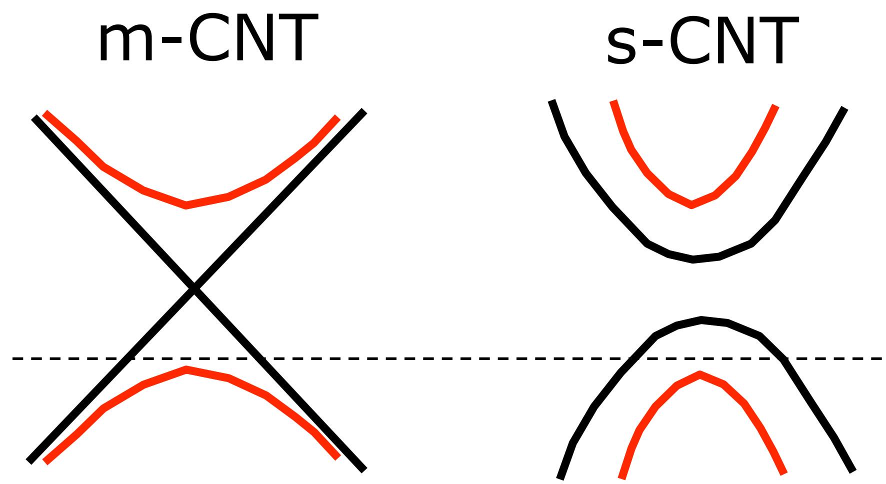


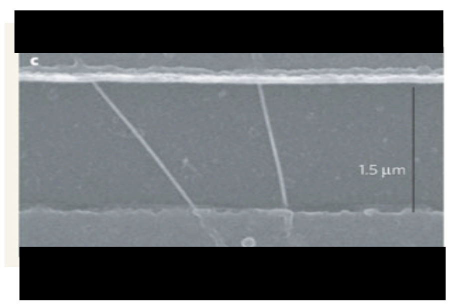


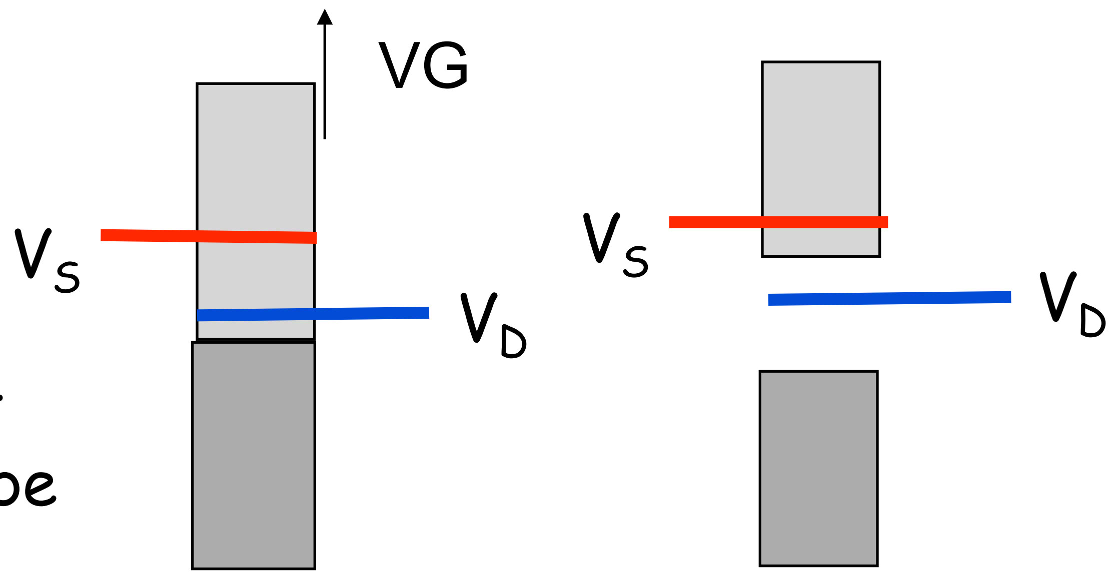
characterizing one transistor is sufficient …

metallic and semiconducting CNTs



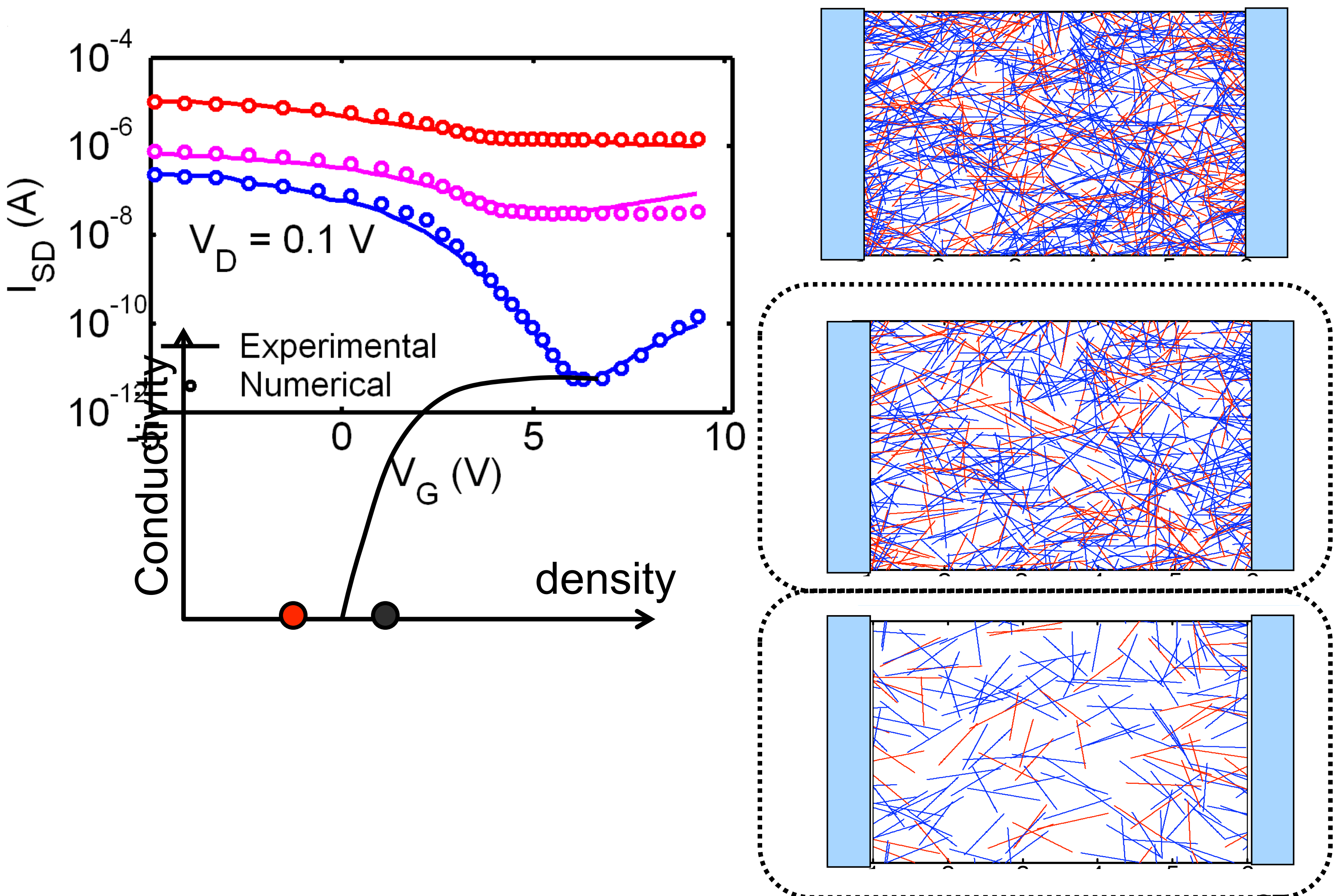




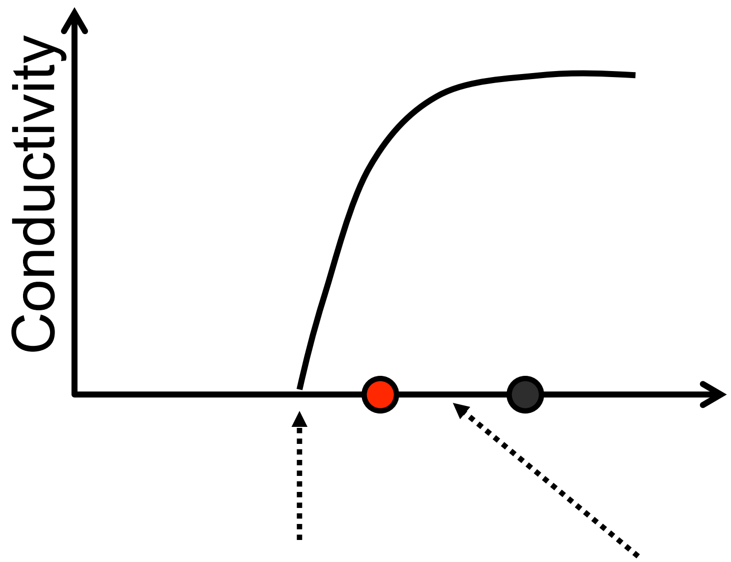


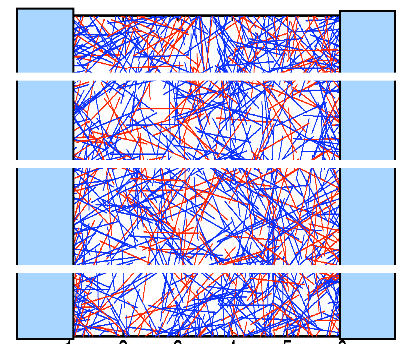
* Metallic CNTs short transistors and must be eliminated.

M-CNT Content: heterogeneous percolation



‘striping’ control of percolation threshold



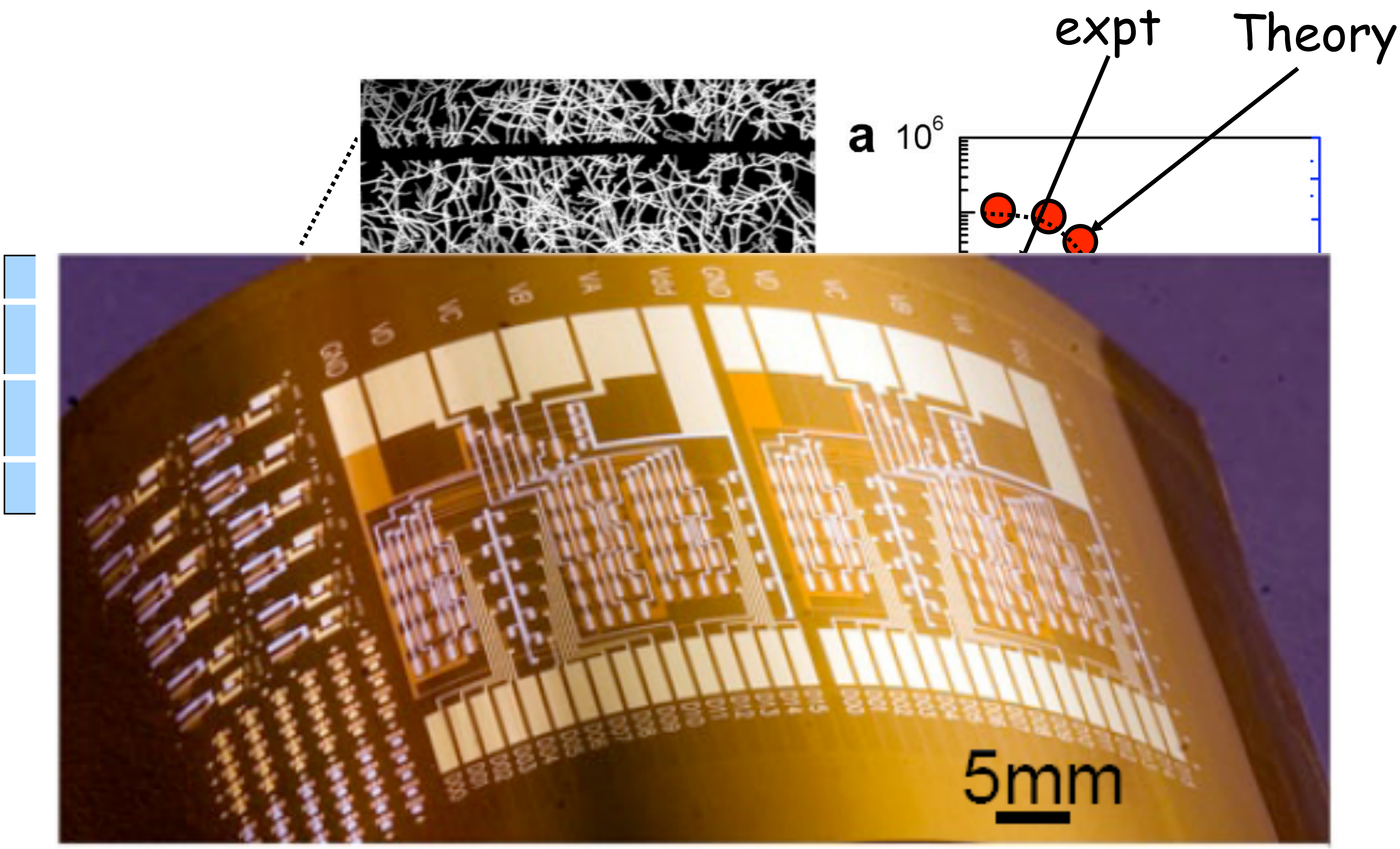


$$\rho^* = \frac{4.26^2}{\pi L_S^{\ 2}}$$

$$\rho^* = \frac{4.26^2}{\pi \left(\frac{L_S}{k} \right)^2}$$

Depending on the ckt, shorten the tubes by a factor k!
(*see lecture 3 for the theory of p_c shift)

Striping for improved on-off ratio



conclusions

- Illustrated the power of stick percolation model by analyzing results of short and long-channel nanonet transistors.

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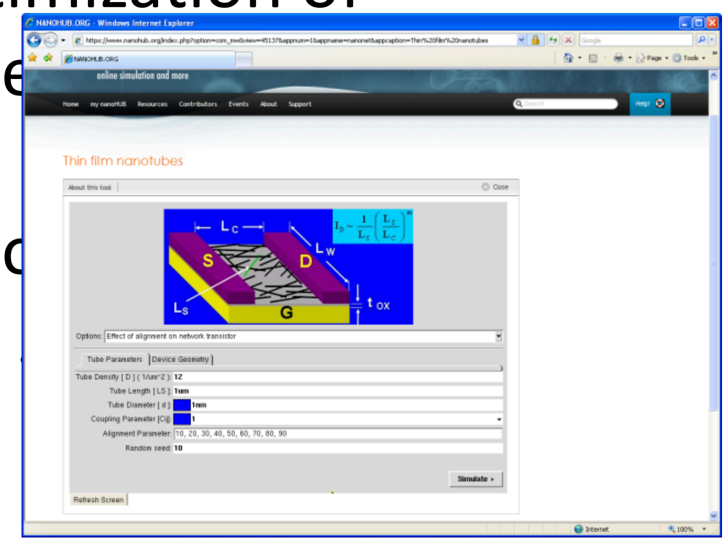
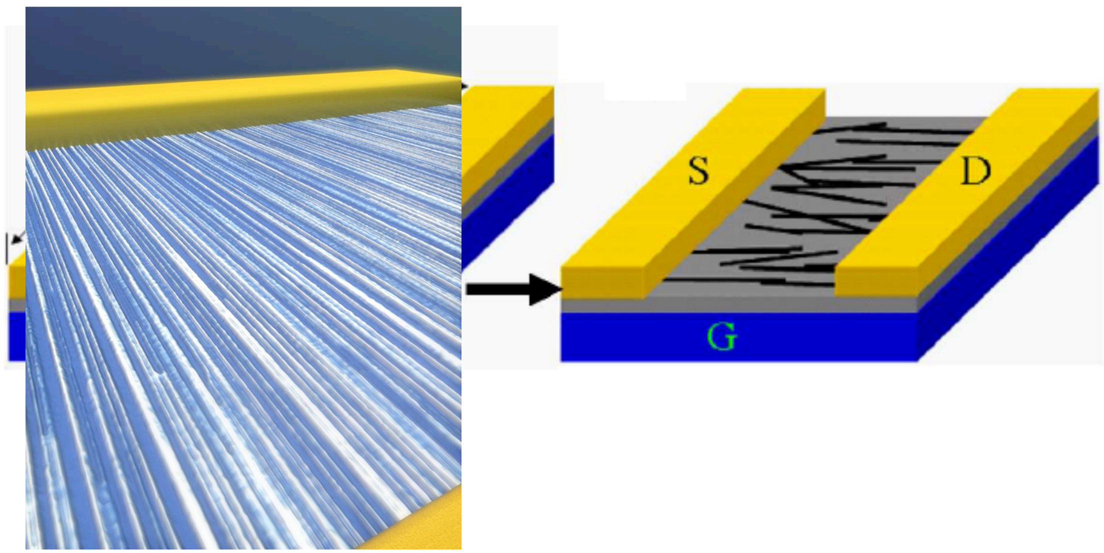
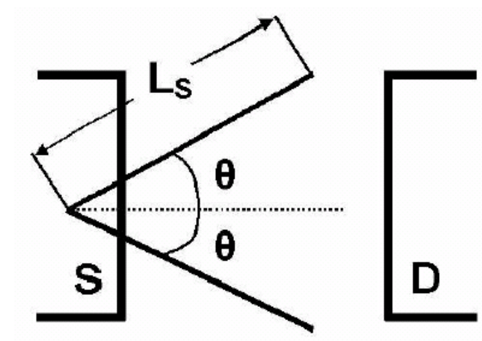
- You will get additional insight int you do the HW using the Nanonet nanohub.org



alignment and asymmetric percolation



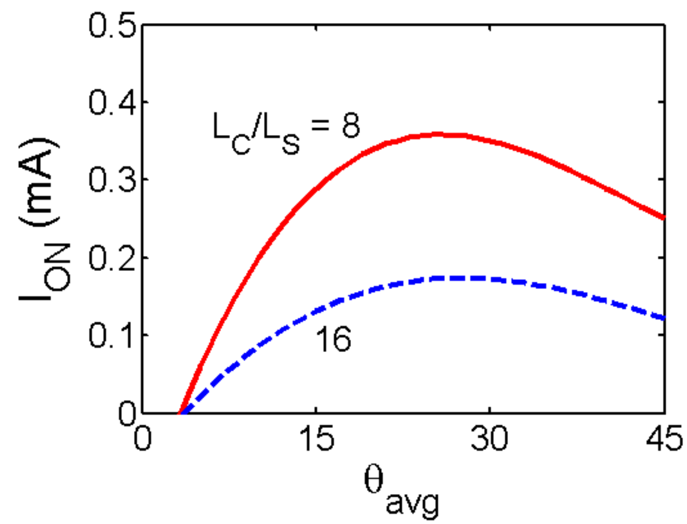


With more alignment

Av. Path length ↓ → I_D ↑

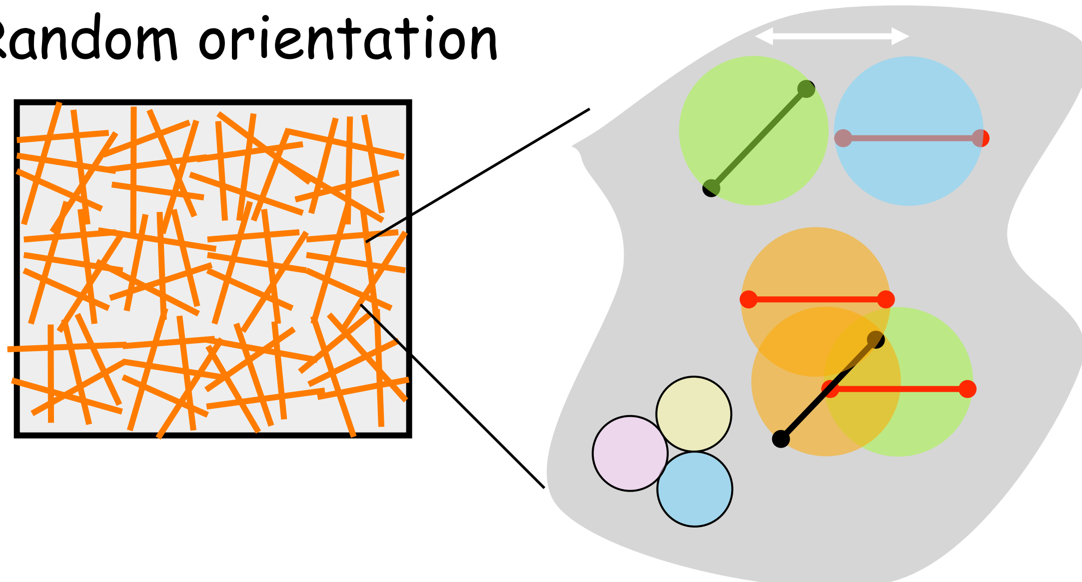
No. of paths ↓ → I_D ↓

Trade-off for optimal alignment



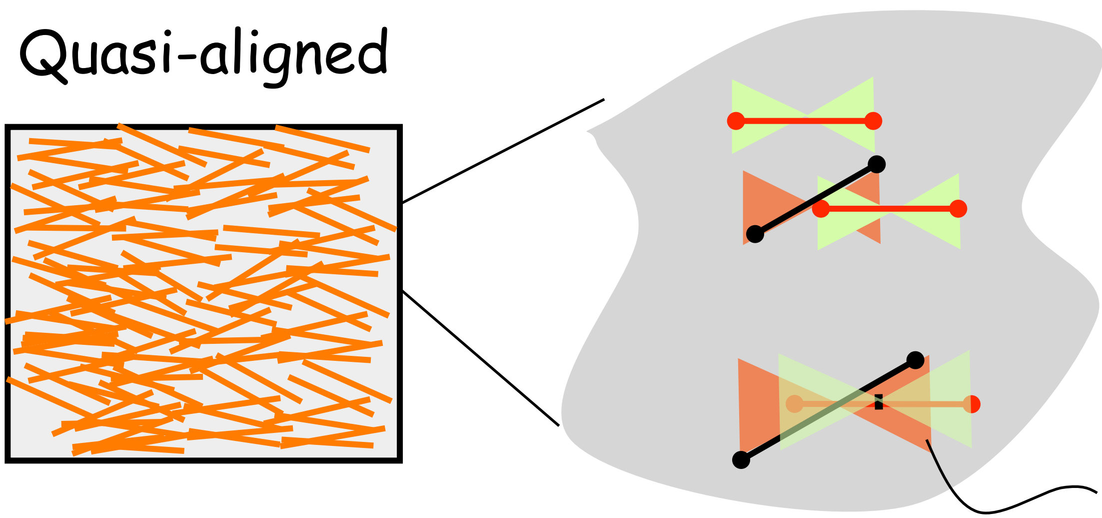
percolation of quasi-aligned sticks

Random orientation



$$N_{C,R} \approx \frac{4}{\pi \left(L_S/2\right)^2}$$

Quasi-aligned



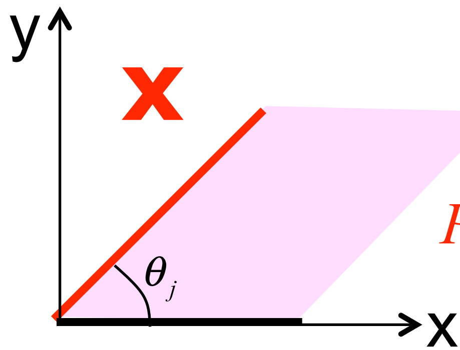
Fine for small q

$$N_{C,\theta} \approx \frac{4}{\pi \left(L_S \sin(\theta)/2\right)^2}$$

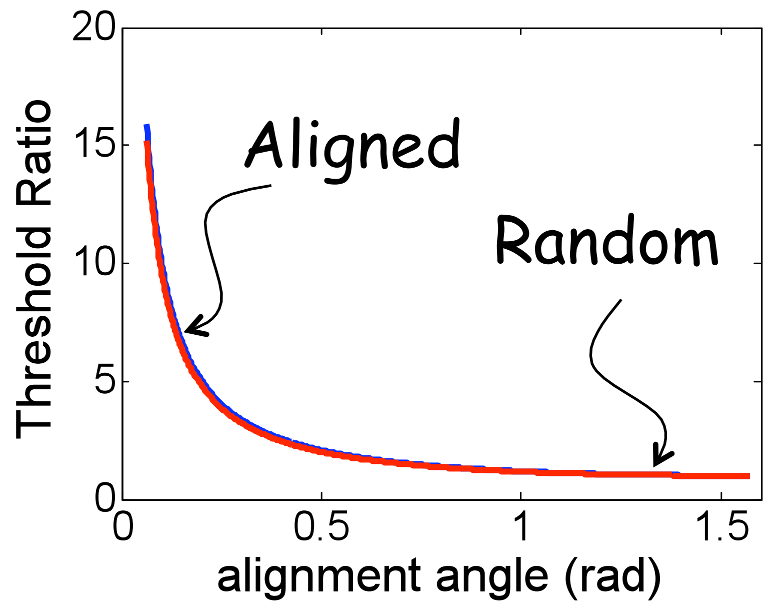
$L_S \sin(\theta)/2$

excluded volume for aligned stick ....

$$A_{\theta_i,\theta_j} = L_S\ L_S \sin(\theta_i - \theta_j)$$



$$P(\theta_j) = \frac{1}{2\theta}$$



$$\langle A_{ex} \rangle_\theta = \int_{-\theta}^{\theta} d\theta_i P(\theta_i) \int_{-\theta}^{\theta} d\theta_j P(\theta_j) \times A_{\theta_i,\theta_j}$$

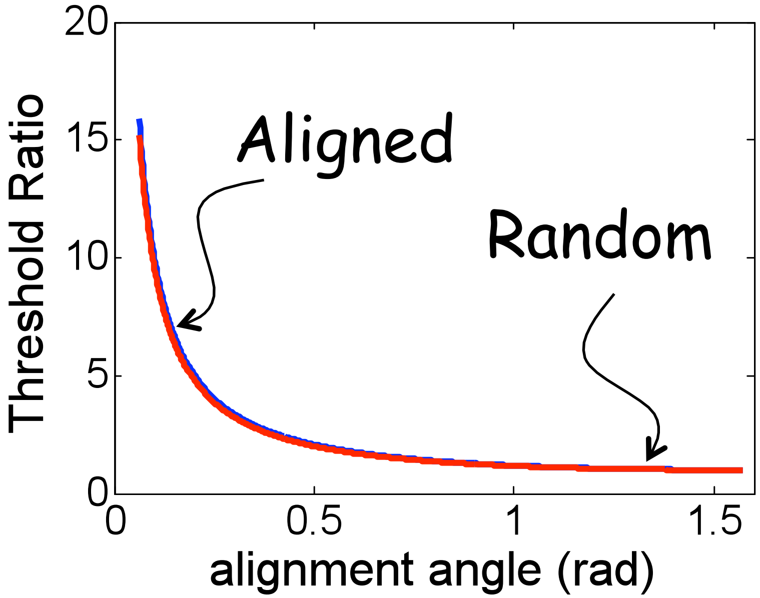
$$= \frac{L_S^2}{4\theta^2}\left[4\theta - 2\sin(2\theta)\right] \sim \frac{2L_S^2}{\pi}\sin\theta$$

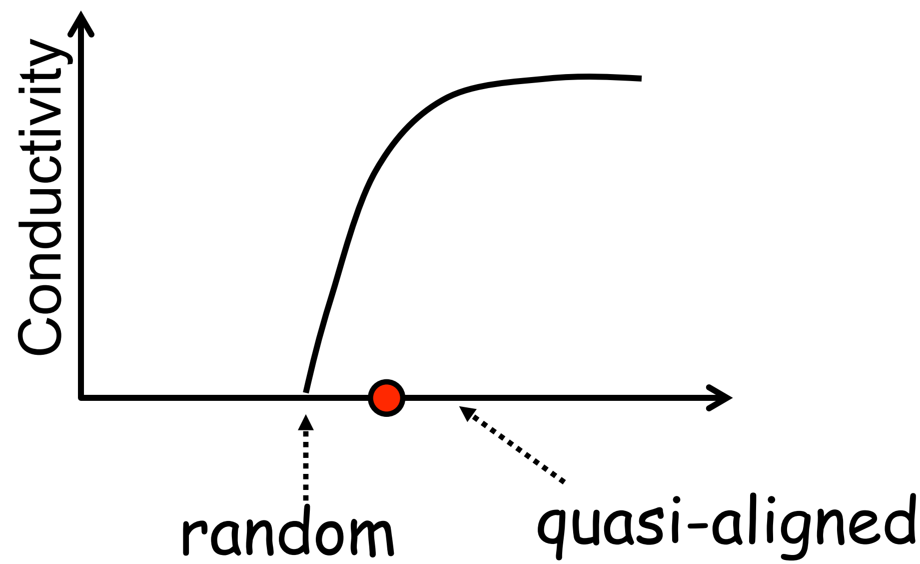
$$\langle A_{ex} \rangle_\theta N_{c,\theta} \sim 1.8 \sim \langle A_{ex} \rangle_R N_{c,R}$$

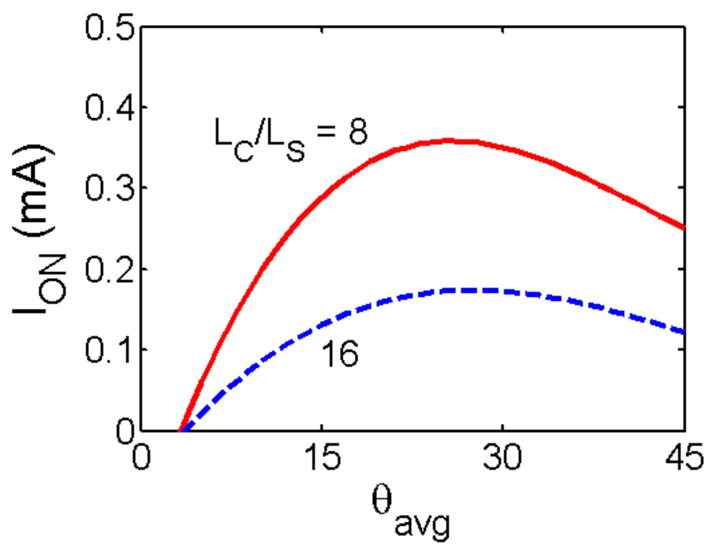
$$\frac{N_{c,\theta}}{N_{c,R}} \sim \frac{1}{\sin(\theta)}$$

Balberg, PRB, 1984

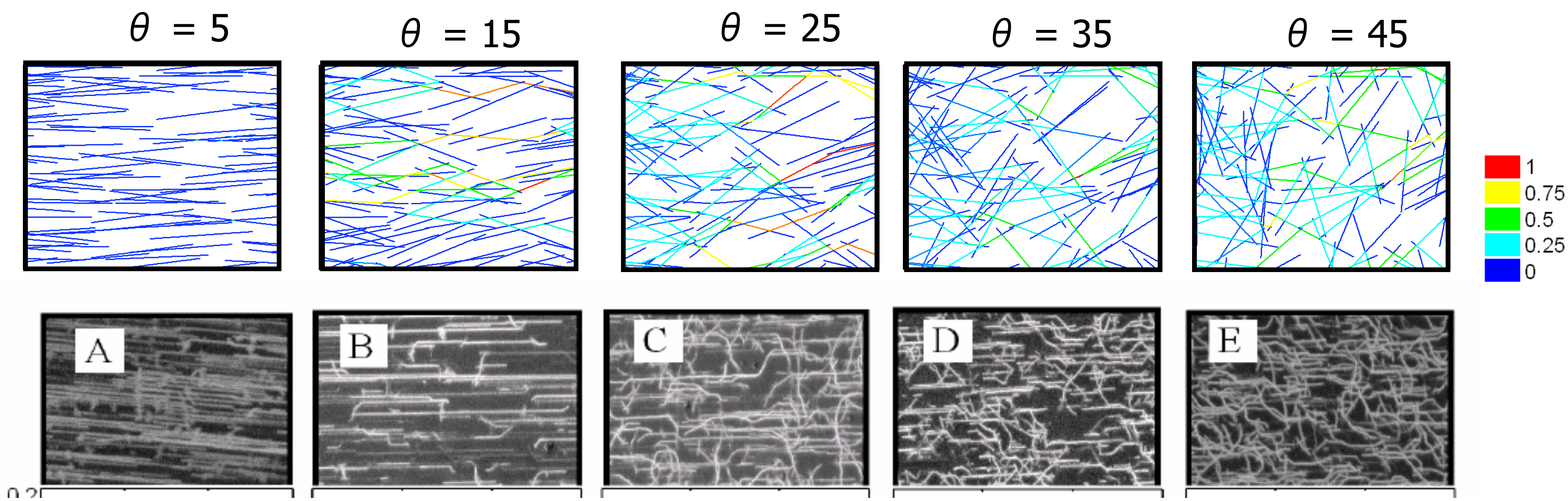
alignment shifts of percolation threshold



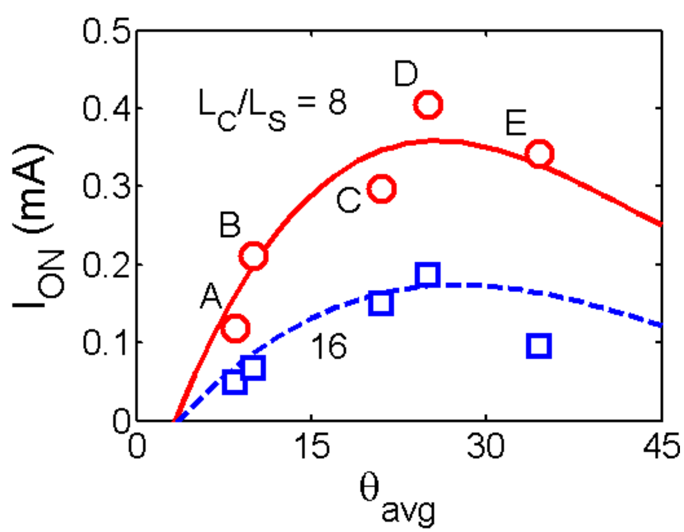




experimental verification



Pimparkar et al. MRS Spring meeting 2007



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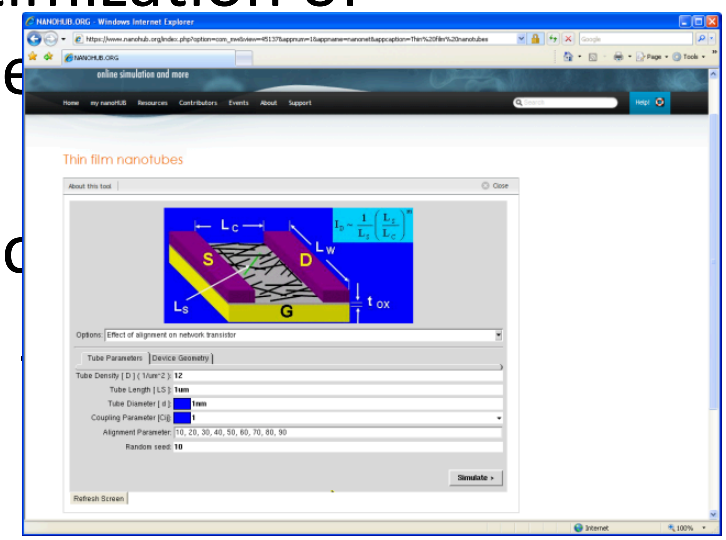
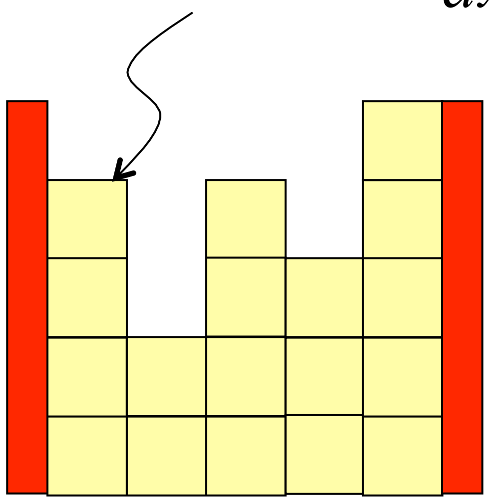


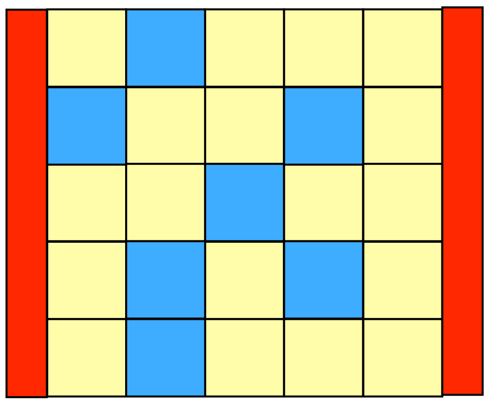
Appendix

scaling relationship: simple argument

$$I_D(x) = W(x) q n \mu \frac{dV}{dx}$$

$$I_D = \frac{A}{L_S}\xi\left(\frac{L_S}{L_C}, DL_S^2\right) \times f(V_G, V_D)$$





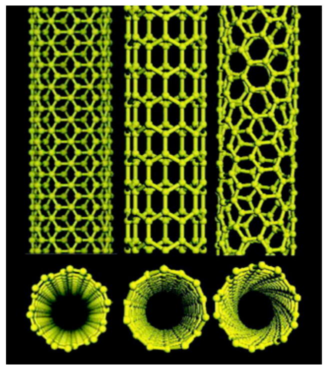
$$\int_0^{L_C} \frac{I\,dx}{W(x)} =$$

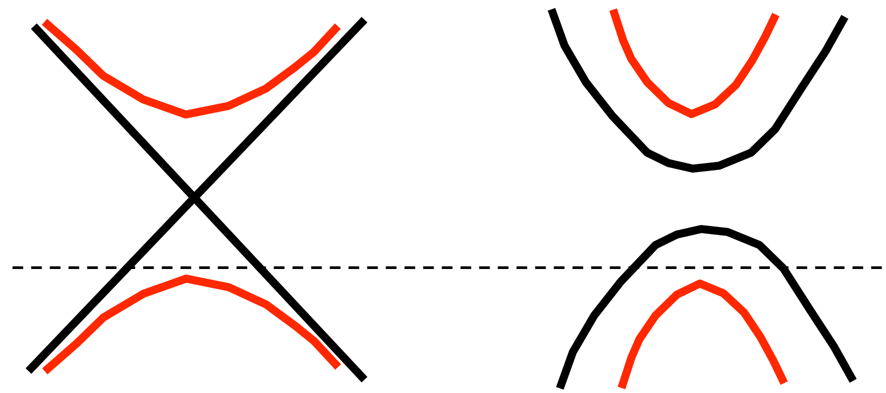
$$\int_0^{V_D} C_{ox}(V_G - V_T)\mu dV$$

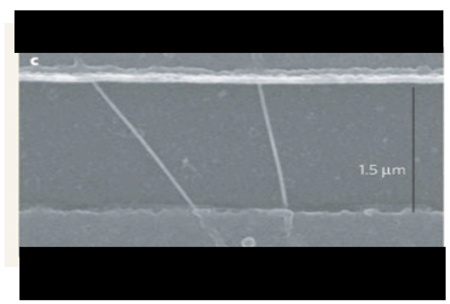
$$I = \frac{\mu C_{ox}}{\int_0^{L_C} W(x)^{-1} dx}\left[(V_G - V_T)V_D - V_D^2/2\right]$$

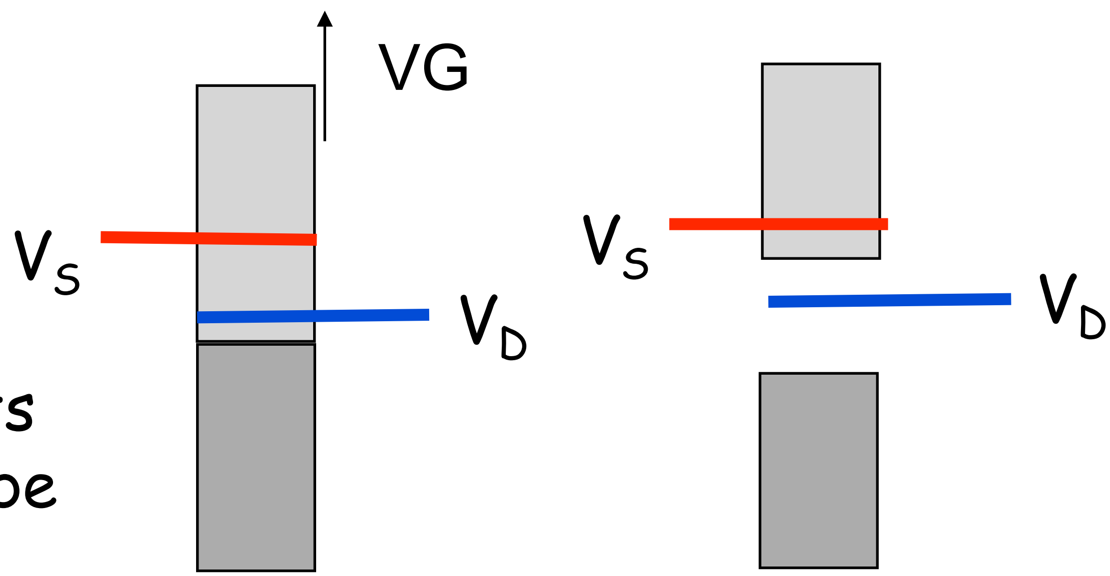
$$= \frac{A}{L_C}\xi(p_i, L_C) \times f(V_G, V_D)$$

metallic and semiconducting CNTs



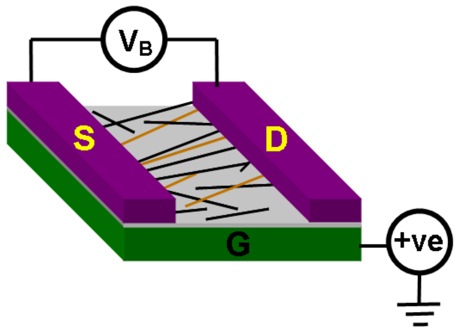


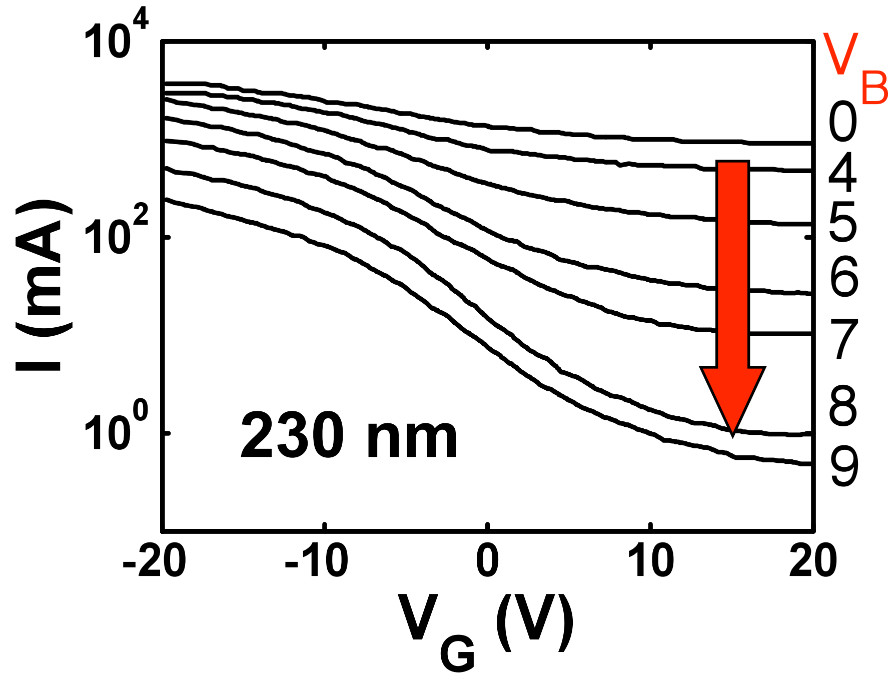


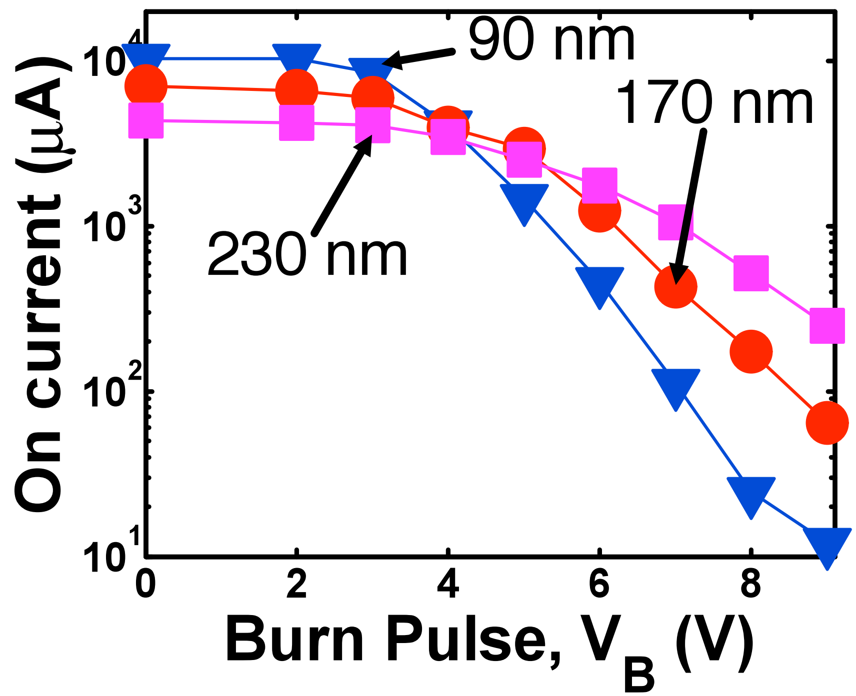


* Metallic CNTs shorts transistors and must be eliminated.

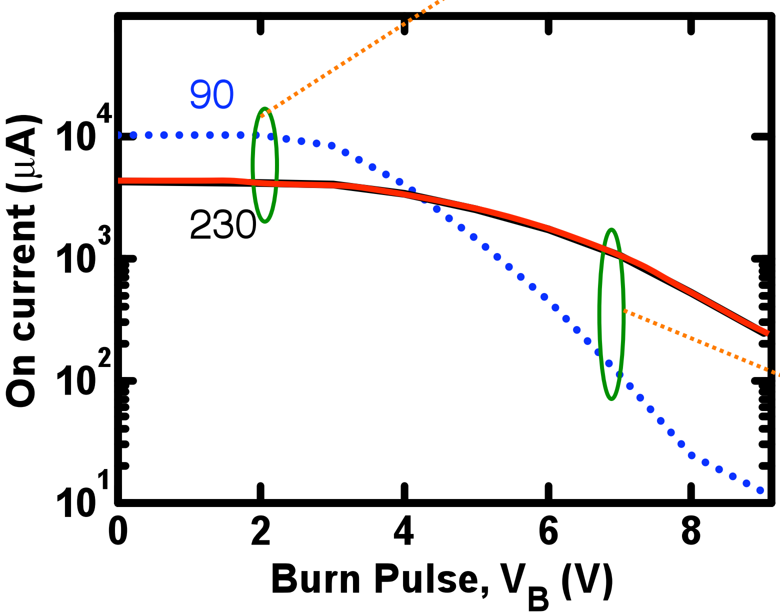
electrical burning of metallic nanotubes

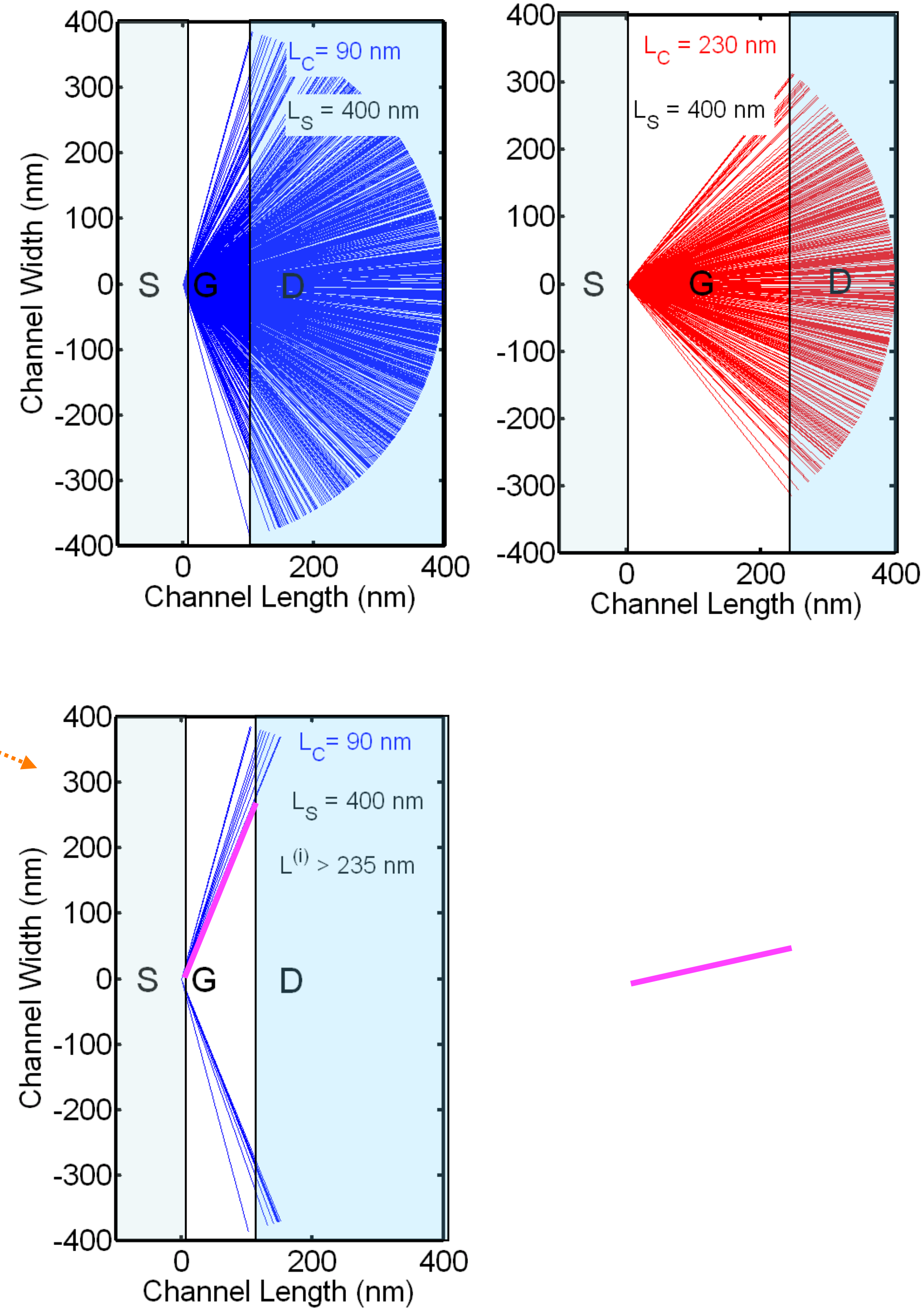






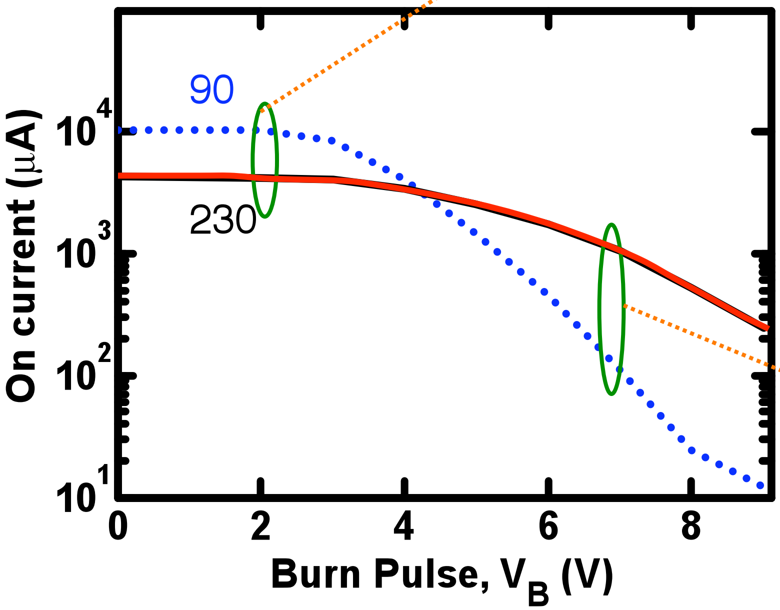
Interpretation ..

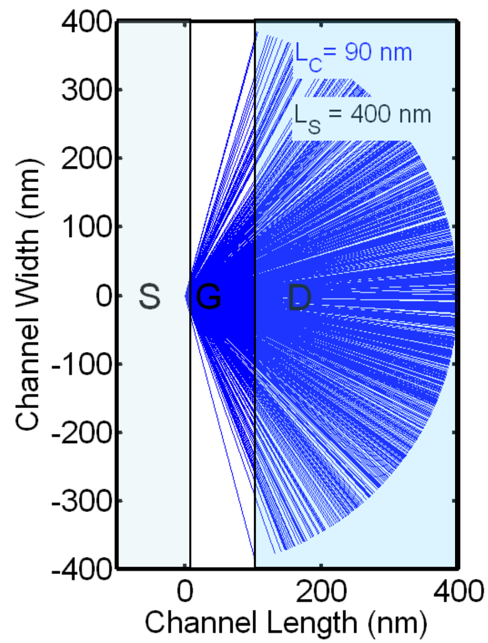


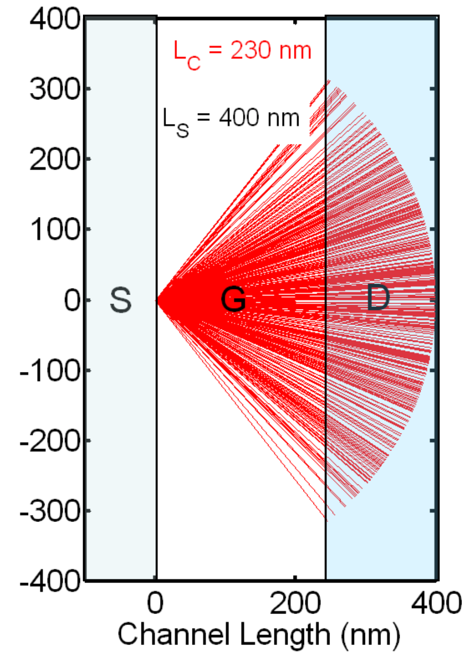


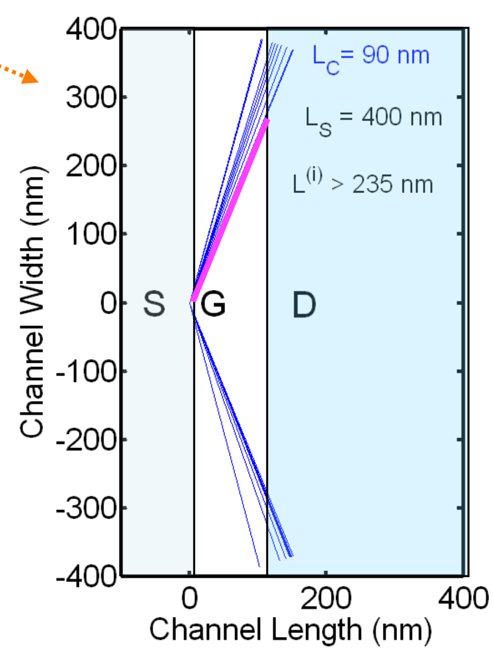
$$L_B = \frac{V_B}{I_{CRIT}}$$

interpretation ...



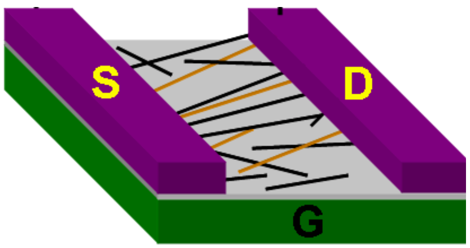


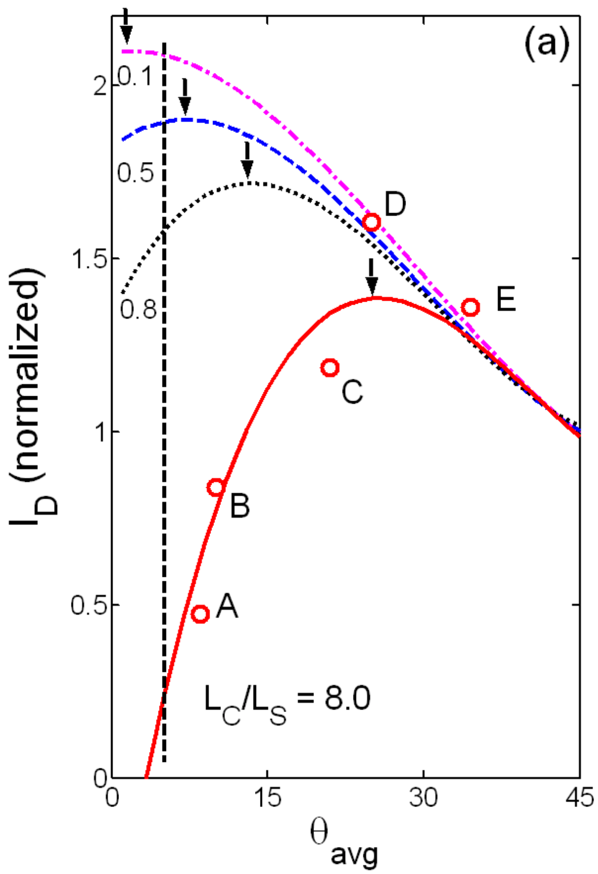




$$L_B = \frac{V_B}{I_{CRIT}}$$

alignment of short nanonet transistors…





Random network is close to optimal !