

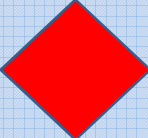


# **ECE606: Solid State Devices**

## **Lecture 24: Large Signal Response**

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# Topic Map

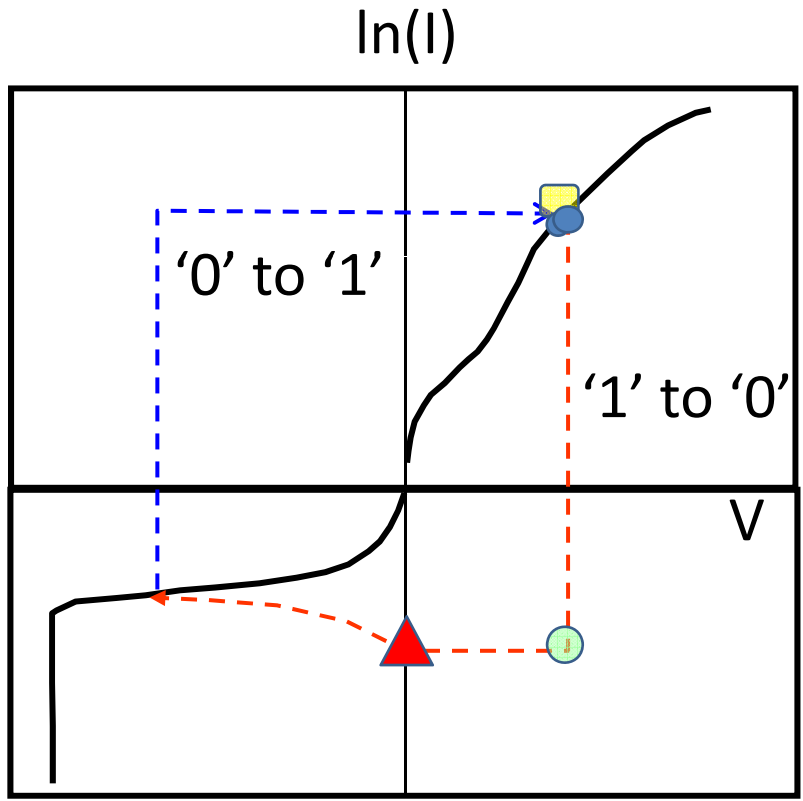
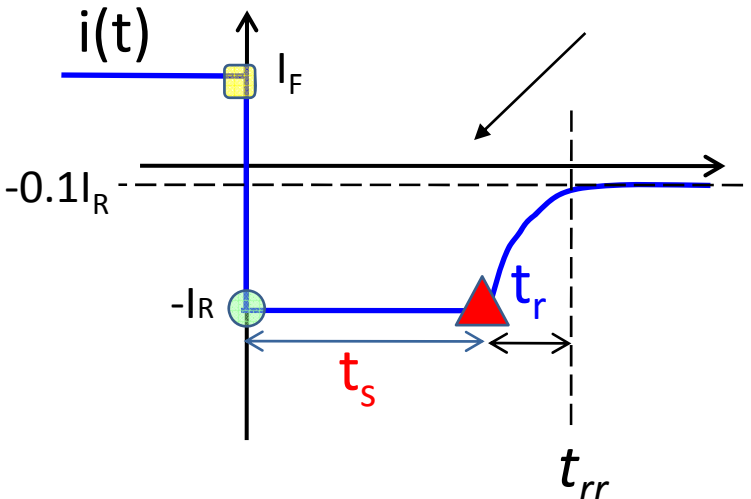
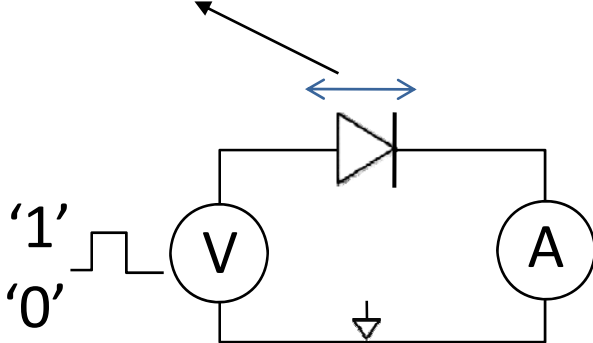
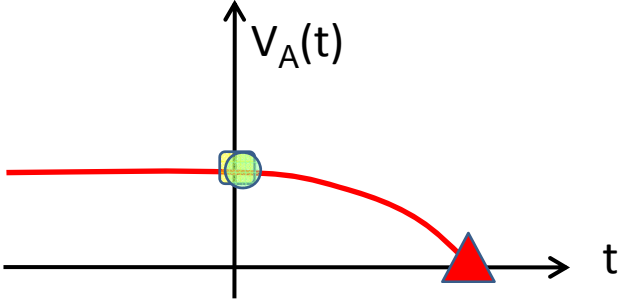
	Equilibrium	DC	Small signal	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT					
MOSFET					

# Outline

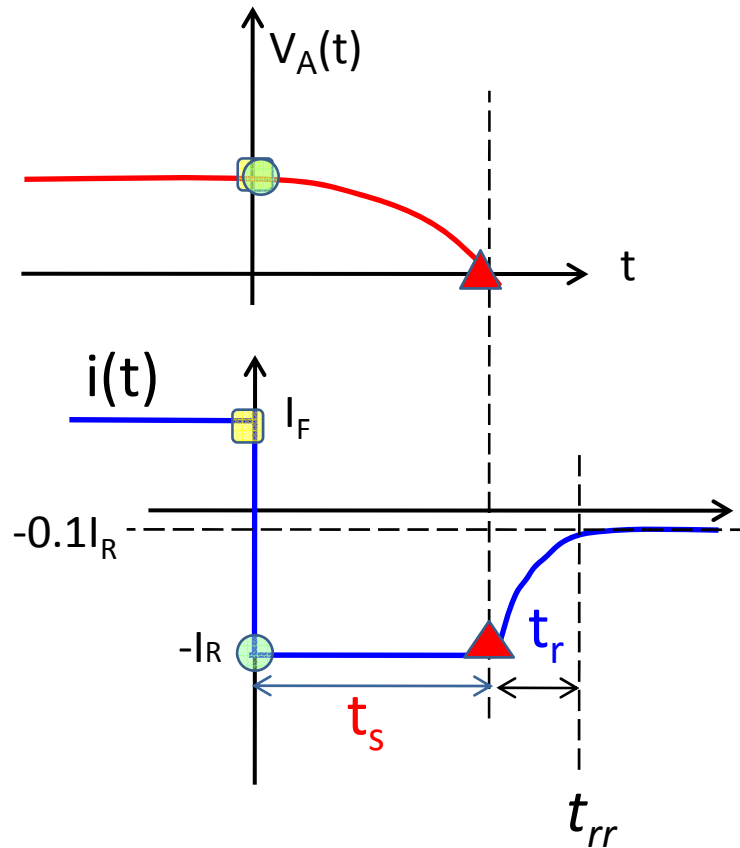
- 1) Large signal response and charge control model**
- 2) Turn-off characteristics
- 3) Turn-on characteristics
- 4) Other applications
- 5) Conclusion

Ref. SDF, Chapter 8

# Digital, Large Signal Applications



# Definitions



$t_s$  ... Charge storage time

$t_r$  ... Recovery time

$t_{rr}$  .. Reverse recovery time

# Continuity Equations

Full analytical solution impossible for large signal....

$$\nabla \cdot \mathbf{D} = q(p - n + N_D^+ - N_A^-)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N\mathbf{E} + qD_N\nabla n$$

$$\frac{\partial p}{\partial t} = \frac{1}{q} \nabla \cdot \mathbf{J}_P - r_P + g_P$$

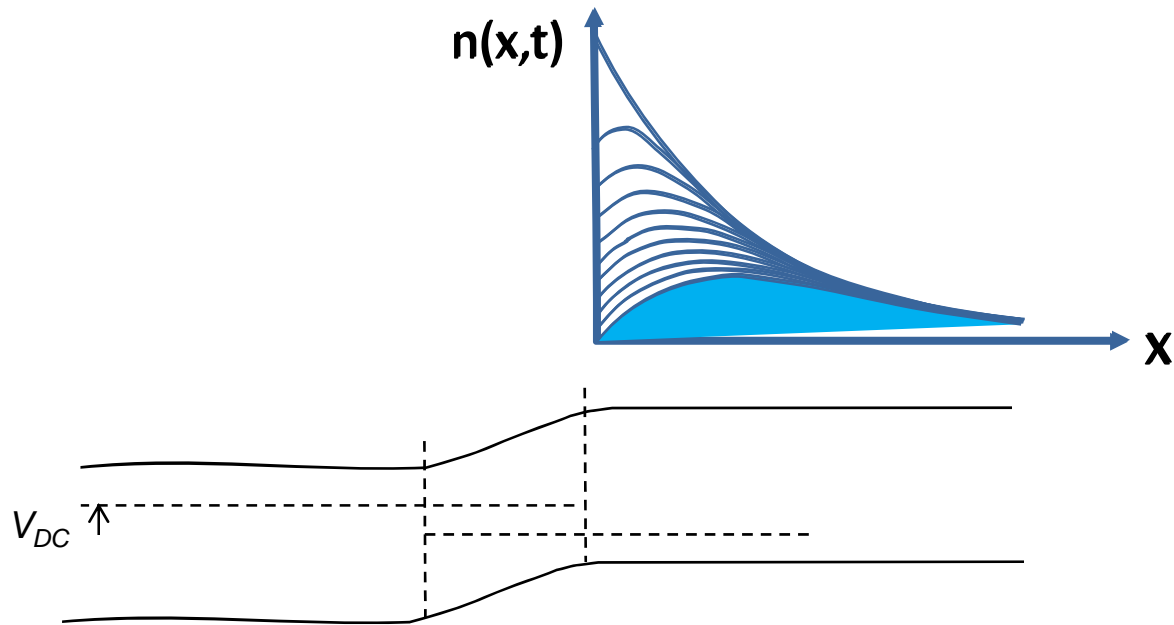
$$\mathbf{J}_P = qp\mu_P\mathbf{E} - qD_P\nabla p$$

$$\frac{\partial Q_n}{\partial t} = i_{n,diff} - \frac{Q_n}{\tau_n}$$

$$\frac{\partial Q_p}{\partial t} = i_{p,diff} - \frac{Q_p}{\tau_n}$$

**Charge control equations ...**

# Large Signal Charge Control Model



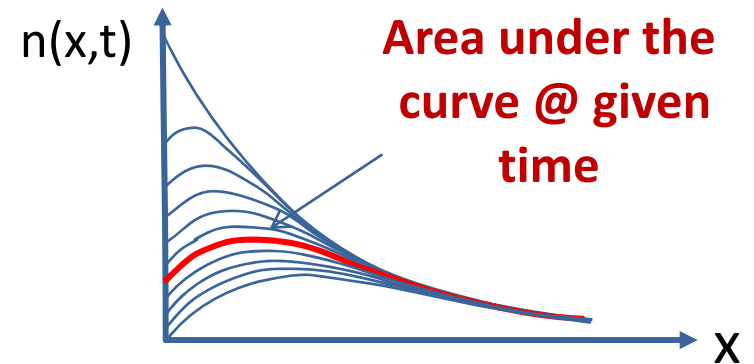
$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{dJ_n}{dx} - r_N + g_N$$

$$\frac{\partial(\Delta n)}{\partial t} = D_N \frac{d^2(\Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}$$

$$\mathbf{J}_N = qn/\mu_N \mathbf{E} + qD_N \frac{dn}{dx}$$

# Large Signal Charge Control Model

$$\frac{\partial(\Delta n)}{\partial t} = D_N \frac{d^2(\Delta n)}{dx^2} - \frac{\Delta n}{\tau_n}$$



$$\int_0^{W_p} \frac{\partial(qA\Delta n)}{\partial t} dx = \int_0^{W_n} D_N \frac{d}{dx} \frac{d(qA\Delta n)}{dx} dx - \int_0^{W_n} \frac{qA\Delta n}{\tau_n} dx$$

$$\frac{\partial Q}{\partial t} = D_N \left. \frac{d(qA\Delta n)}{dx} \right|_{x=W_p} - D_N \left. \frac{d(qA\Delta n)}{dx} \right|_{x=0} - \frac{Q}{\tau_n}$$

$$Q \equiv \int_0^{W_p} (qA\Delta n) dx$$

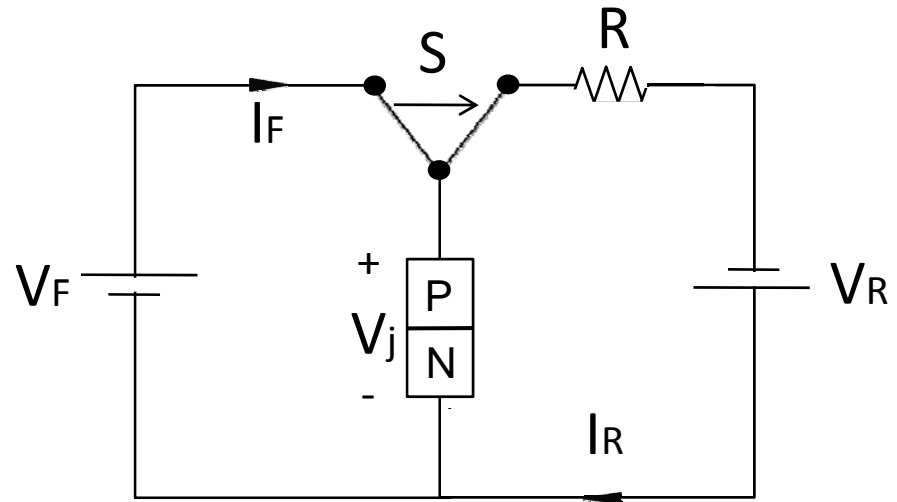
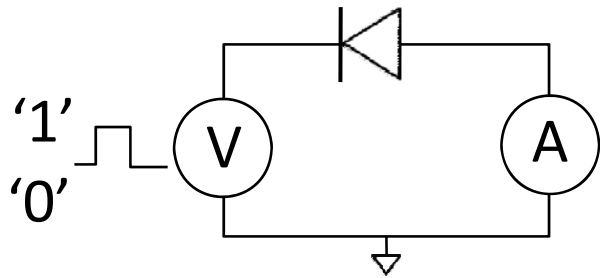
$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$



# Outline

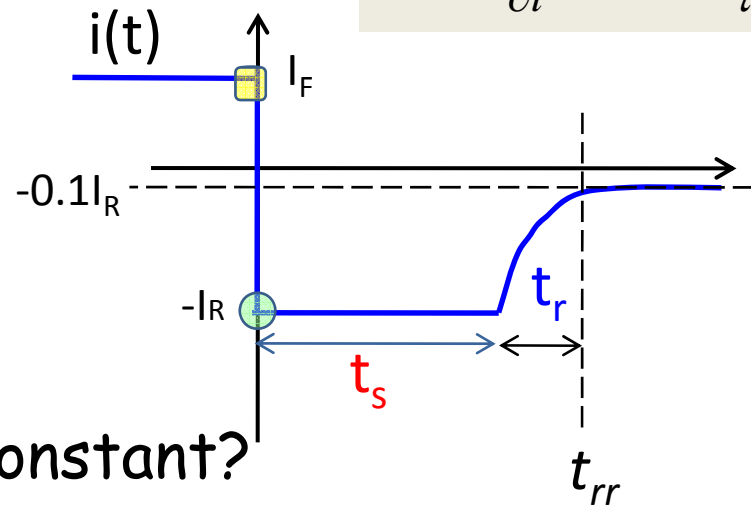
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# Turn-off Characteristics: Determine ( $t_s$ )



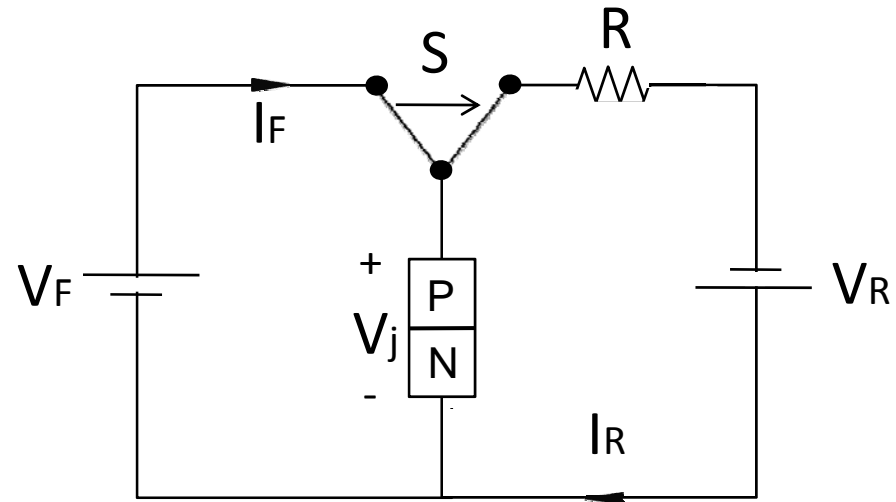
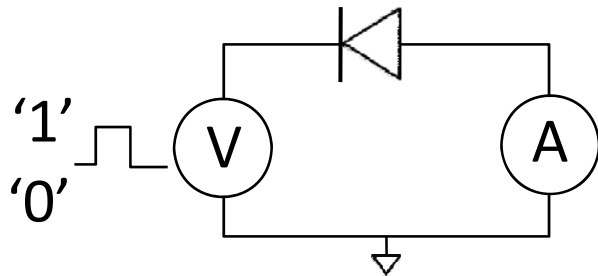
$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$t < 0 \quad \frac{\partial Q}{\partial t} = I_F - \frac{Q(0^-)}{\tau_n}$$



Why does the current remain constant?

# Boundary Condition



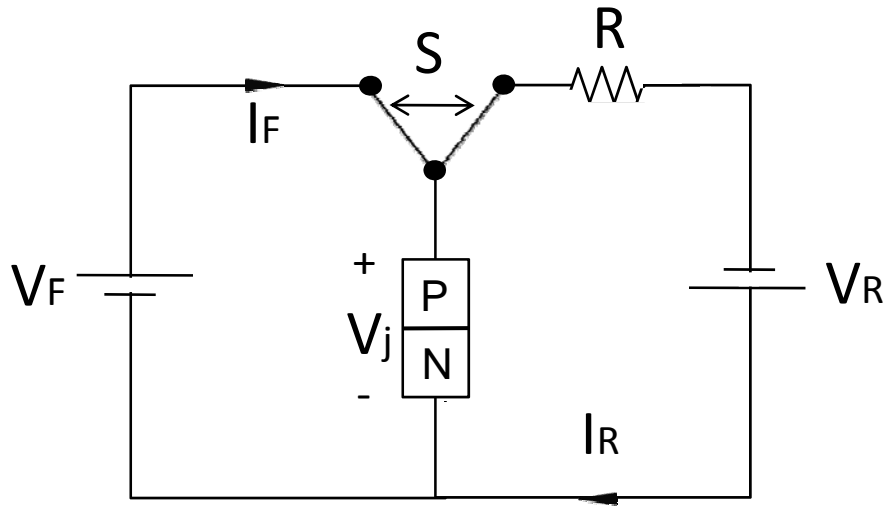
$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$t < 0 \quad \frac{\partial Q}{\partial t} = I_F - \frac{Q(0^-)}{\tau_n}$$

$$Q(0^-) = I_F \tau_n = Q(0^+)$$

Note. For a capacitor, charge can not change instantly ...

# Turn-off *Current* Transient



constant!

$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

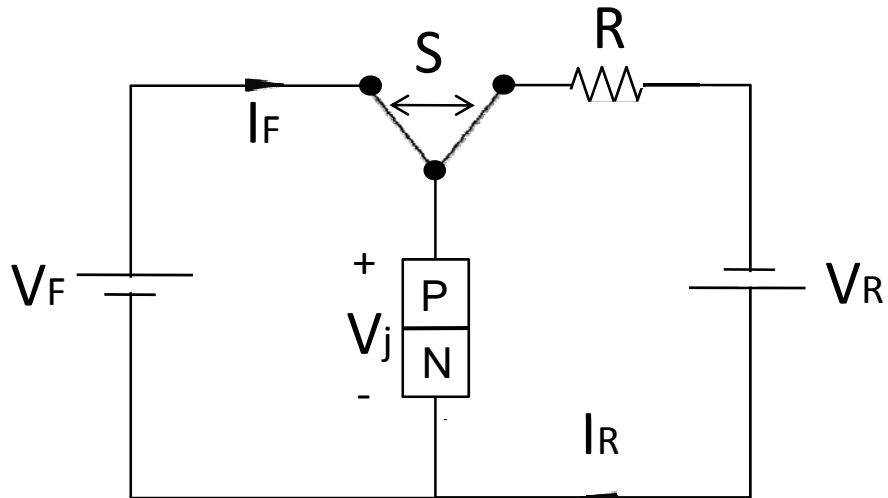
$$t > 0 \quad \frac{\partial Q}{\partial t} = -I_R - \frac{Q}{\tau_n}$$

↓

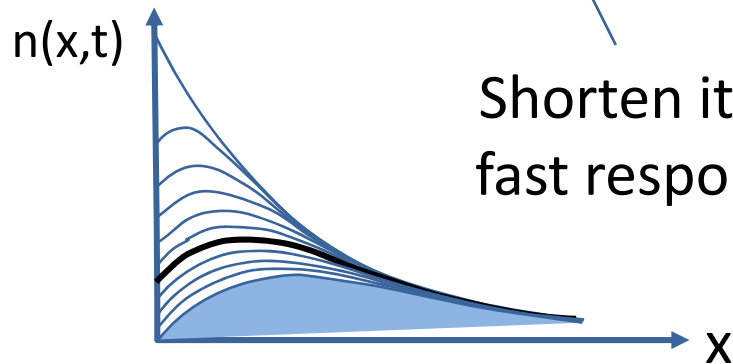
$$\int_{Q(0^+)}^{Q(t_s)} \frac{dQ}{- \left( I_R + \frac{Q}{\tau_n} \right)} = \int_0^{t_s} dt$$

$$t_s = \tau_n \ln \frac{I_R + Q(0^+)/\tau_n}{I_R + Q(t_s)/\tau_n}$$

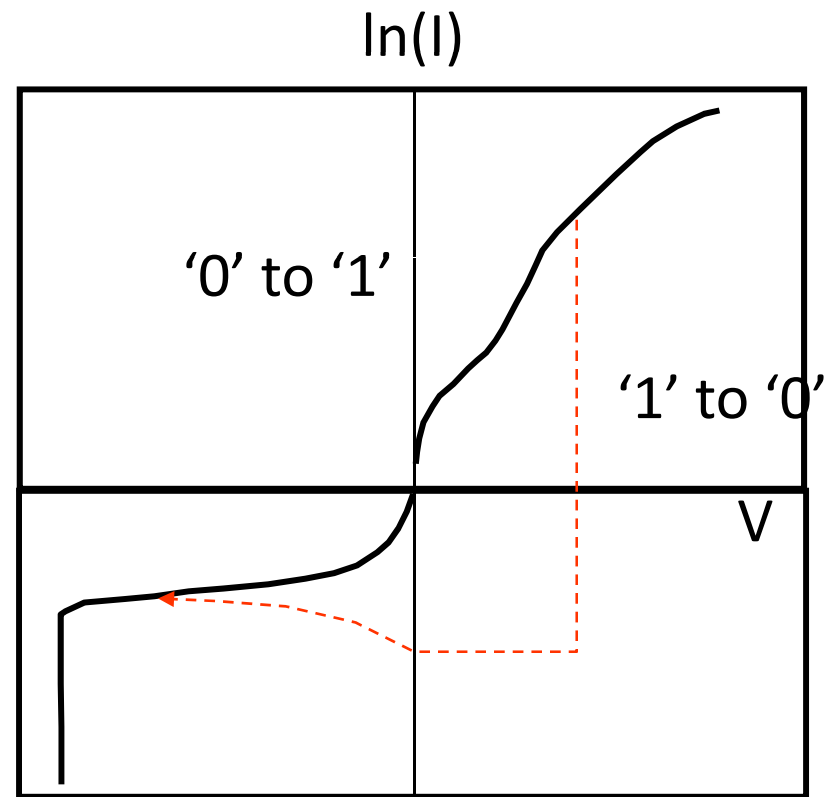
# Storage Time



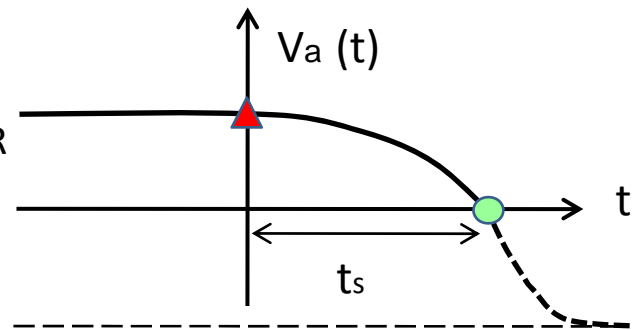
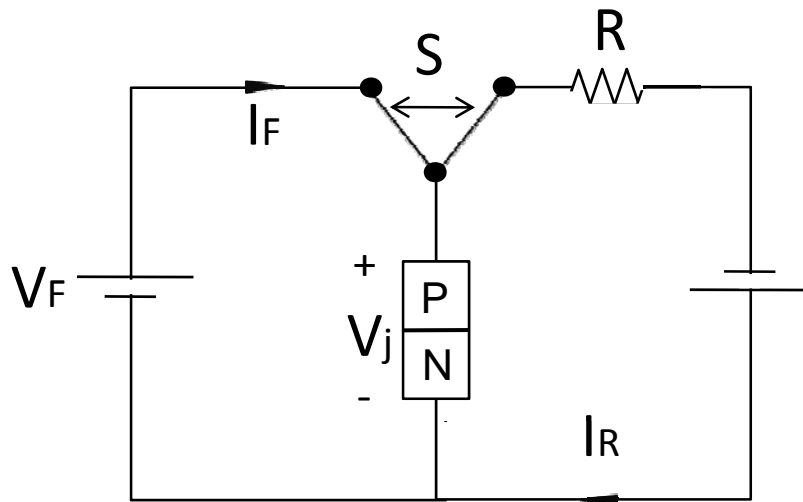
$$t_s = \tau_n \ln \frac{I_R + Q(0^+)/\tau_n}{I_R + Q(t_s)/\tau_n} = \tau_n \ln \frac{I_R + I_F}{I_R}$$



Shorten it for fast response!

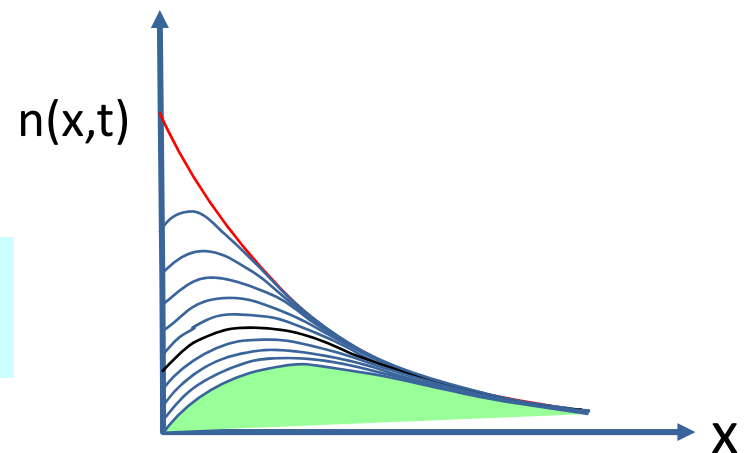


# Turn-off *Voltage* Transient



$$v_A(t) = \frac{kT}{q} \ln \frac{n_p(0,t)}{n_{po}}$$

$$Q_n(t) = \tau_p \ln \left( -I_R + (I_R + I_F) e^{-t/\tau_p} \right)$$



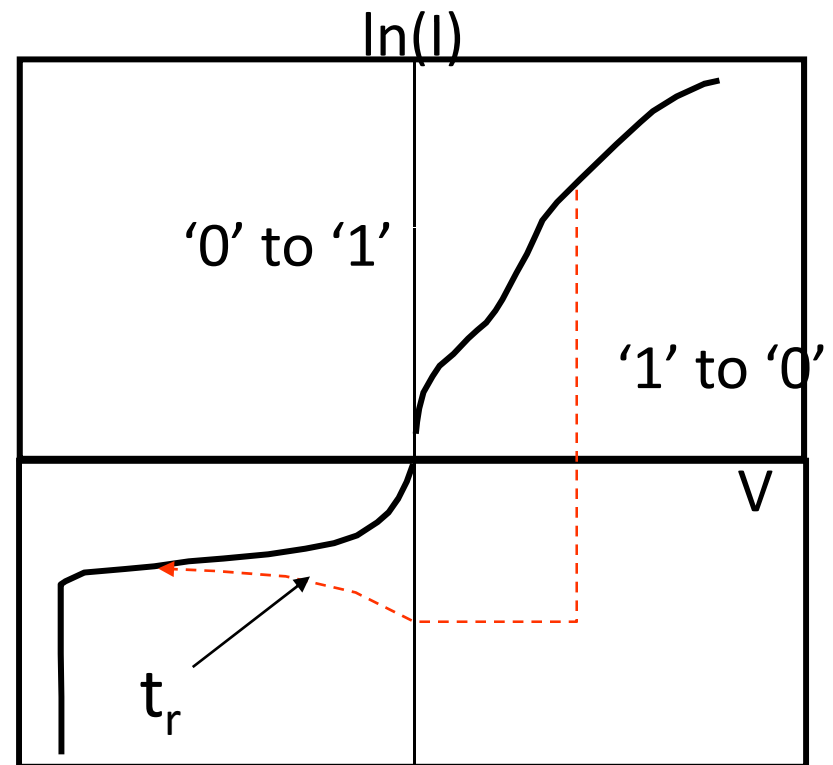
Allows easy calculation of  $n_p(0,t)$ .

# Recovery Time

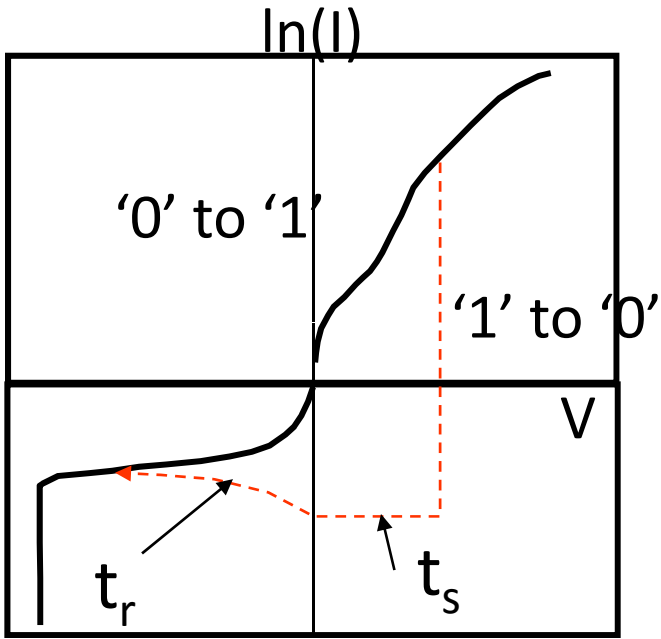
$$\operatorname{erf} \sqrt{\frac{t_r}{\tau_p}} + \frac{e^{-\frac{t_r}{\tau_p}}}{\sqrt{\pi \frac{t_r}{\tau_p}}} = 1 + 0.1 \frac{I_F}{I_R}$$

Useful formula ...

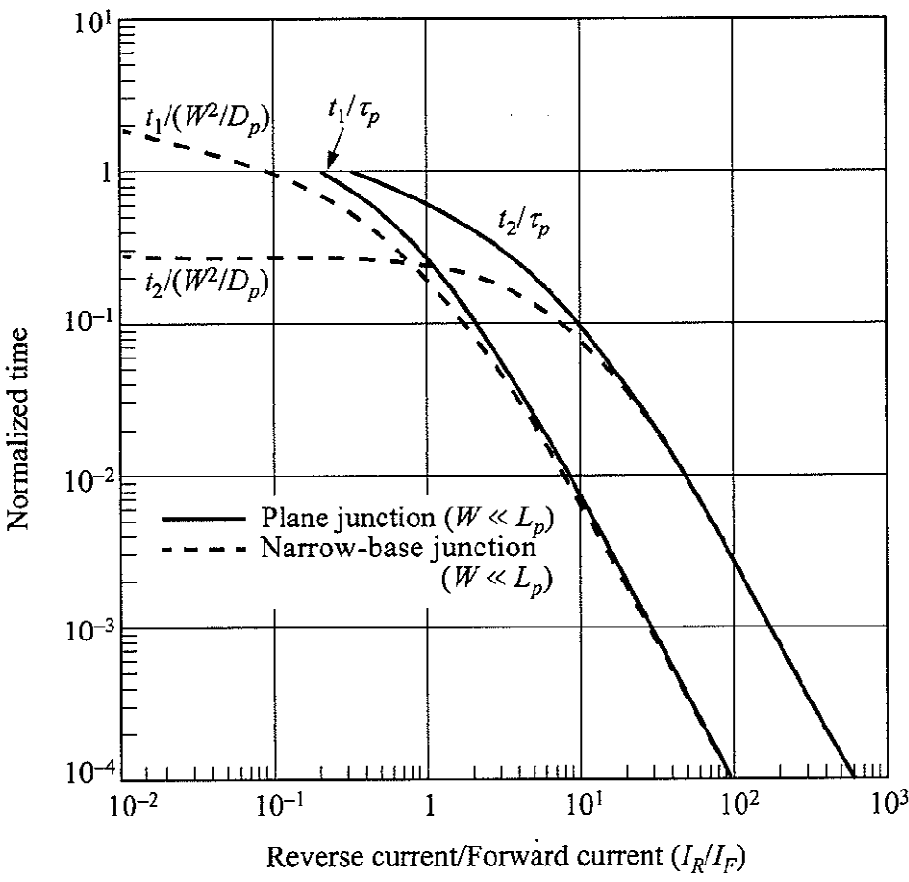
$$\operatorname{erf}(\sqrt{x}) = \left[ 1 - e^{-x \frac{1.27 + 0.15x}{1 + 0.15x}} \right]^{0.5}$$



# Recovery Time



Ref. Sze/Ng, p. 117



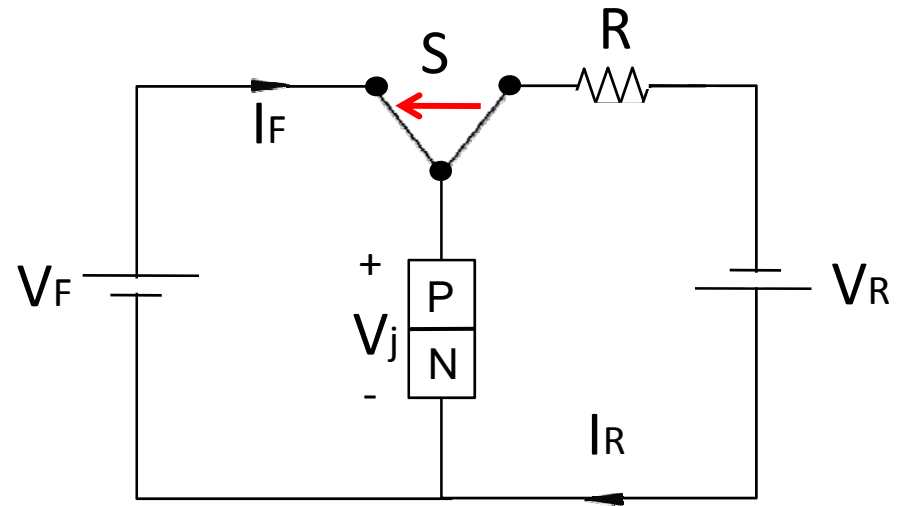
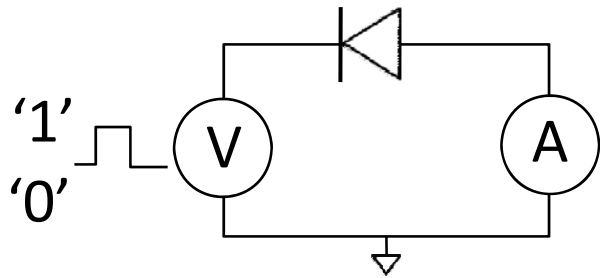
$$t_{rr} = t_r + t_f \approx \begin{cases} \frac{W_p^2}{2D_n} \left( \frac{I_R}{I_F} \right)^{-2} & (W_p \ll L_n) \\ \frac{\tau_p}{2} \left( \frac{I_R}{I_F} \right)^{-2} & (W_p \gg L_n) \end{cases}$$



# Outline

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# Turn-on Characteristics: Boundary Condition

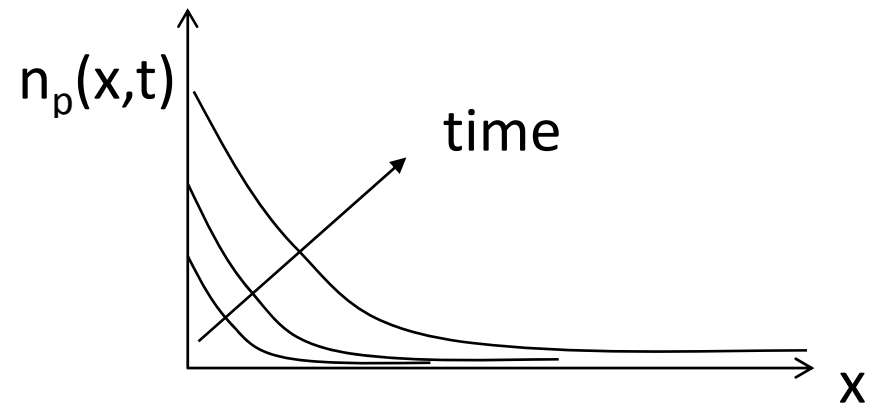
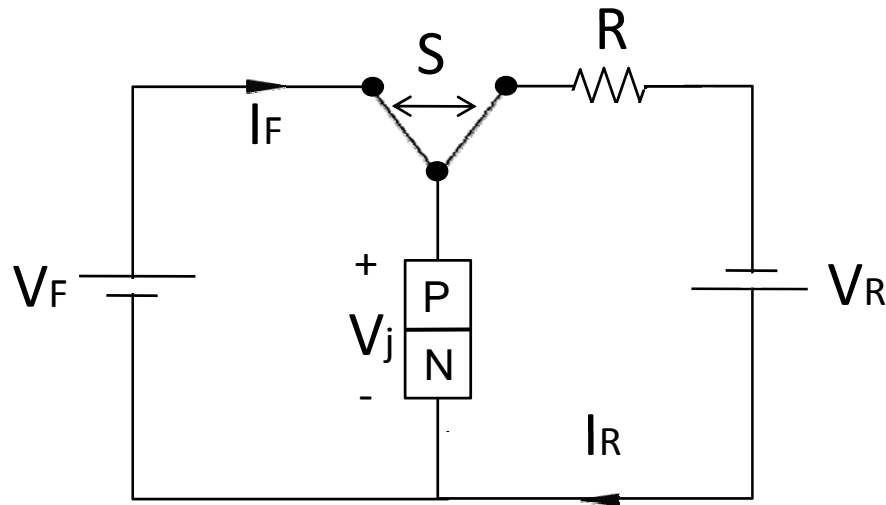


$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$t \rightarrow \infty \quad \frac{\partial Q}{\partial t} = I_F - \frac{Q(t = \infty)}{\tau_n}$$

$$Q(t = \infty) = I_F \tau_n$$

# Turn-on Characteristics



$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$t > 0 \quad \frac{\partial Q}{\partial t} = I_F - \frac{Q}{\tau_n}$$

$$Q(t) = Q(t \rightarrow \infty) \left( 1 - e^{-\frac{t}{\tau_n}} \right) = I_F \tau_n \left( 1 - e^{-\frac{t}{\tau_n}} \right)$$

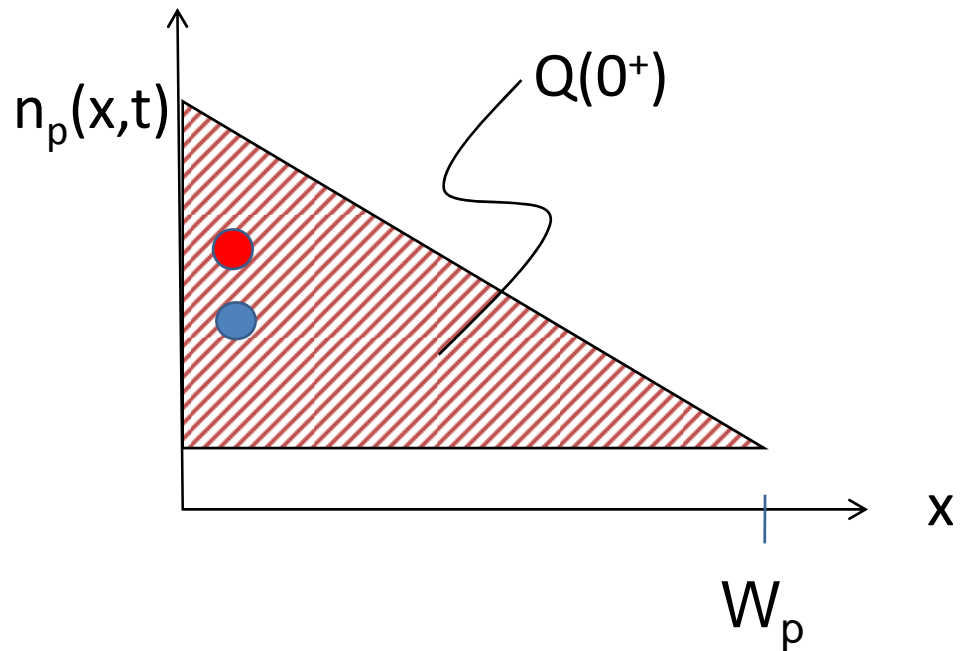
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# Calculating Diffusion Time by Charge Control

$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$\frac{Q(0^+) - Q(t = \infty)}{\tau_{diff}} = i_{diff}$$



$$\tau_{diff} = \frac{Q(0^+)}{i_{diff}} = \frac{q \left[ \frac{\Delta n_p(0)}{2} \right] W_p}{q D_n \frac{\Delta n_p(0)}{W_p}} = \frac{W_p^2}{2 D_n} \sim \frac{1}{2} \times \frac{W_p}{(D_n / W_p)}$$

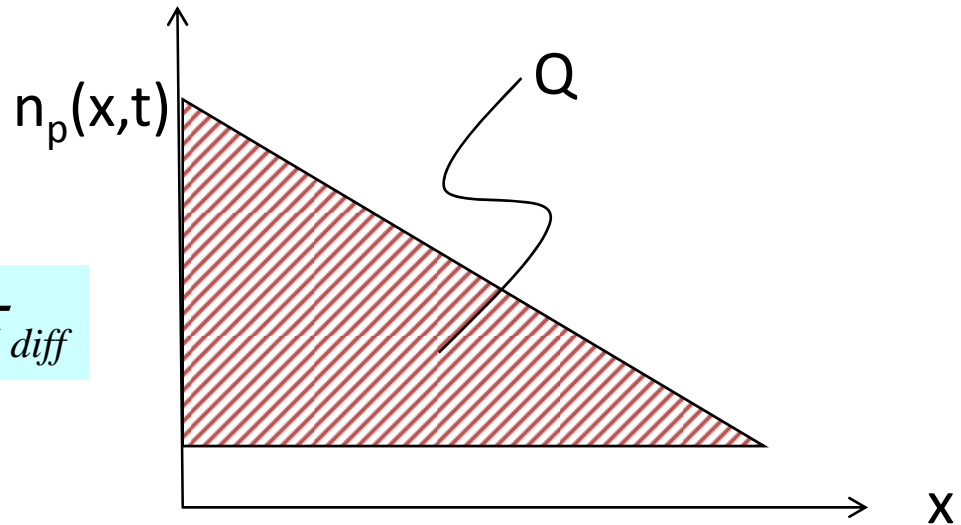
Diffusion velocity

# Two line derivation of **Steady State** Diode Current

$$\frac{\partial Q}{\partial t} = i_{diff} - \frac{Q}{\tau_n}$$

$$\tau_n \rightarrow \tau_{diff}$$

$$i_{diff} = \frac{Q}{\tau_{diff}}$$



$$i_{diff} = \frac{Q}{\tau_{diff}} = \frac{q \times \frac{1}{2} \frac{n_i^2}{N_A} (e^{qV_{A\beta}} - 1) \times W_p}{\frac{W_p^2}{2D_n}} = q \frac{D_n}{W_p} \frac{n_i^2}{N_A} (e^{qV_{A\beta}} - 1)$$

Exact  
Expression!

## Conclusion

Large signal response of devices of great importance for digital applications.

Analytical solution of partial differential equation often difficult (if not impossible), therefore approximate methods like Charge-control approximation often help simplify the solution and still provide a great deal of insight into the dynamics of switching operation.

Be careful in using the boundary condition which is often dictated by external circuit conditions.