

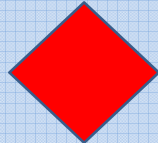


ECE606: Solid State Devices

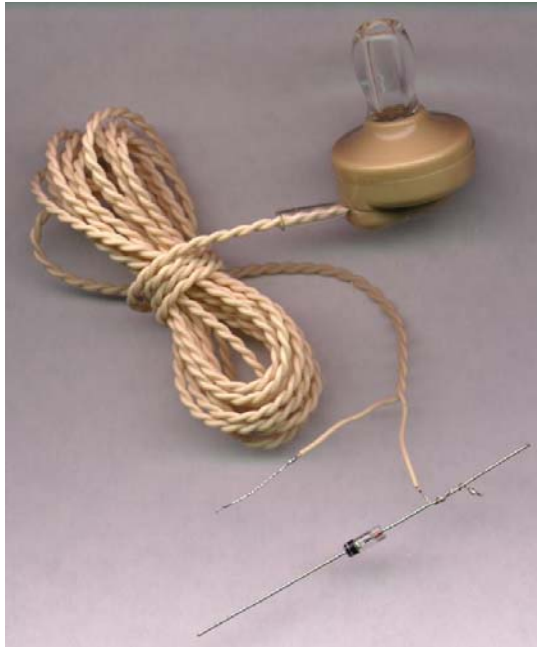
Lecture 23: AC Response

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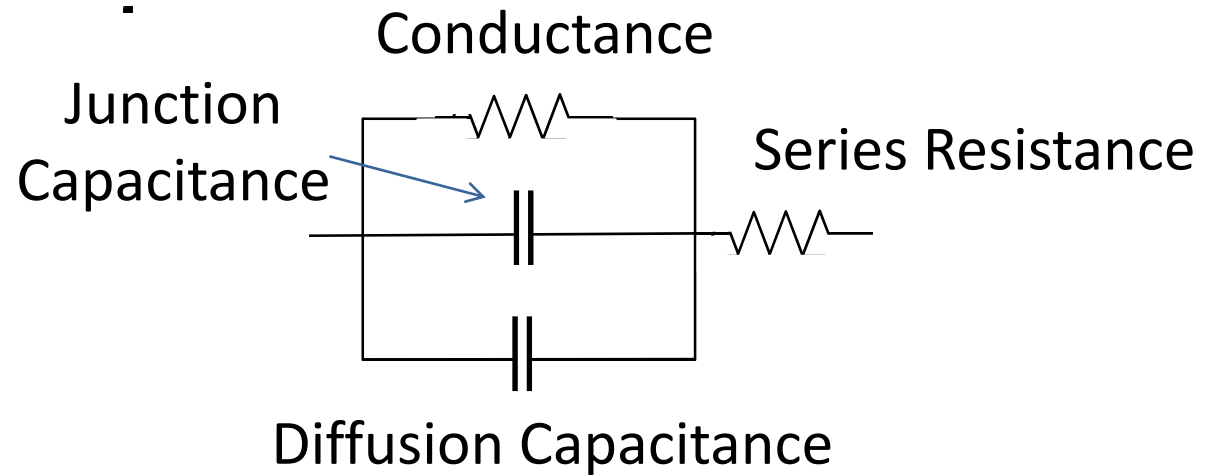
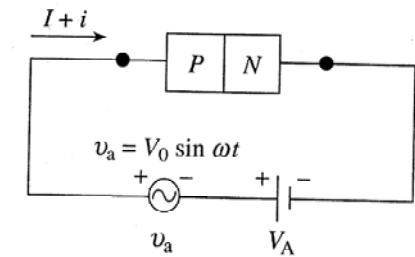
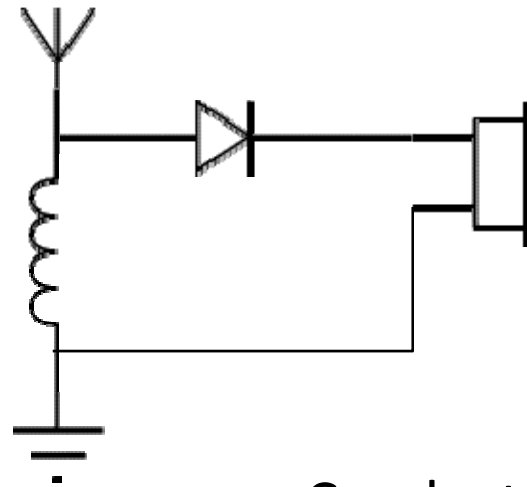
Topic Map

	Equilibrium	DC	Small signal	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT					
MOSFET					

Why should we study AC Response?



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Outline

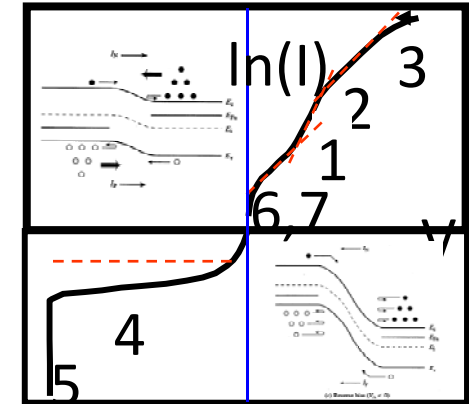
- 1) **Conductance and series resistance**
- 2) Majority carrier junction capacitance
- 3) Minority carrier diffusion capacitance
- 4) Conclusion

Ref. SDF, Chapter 7

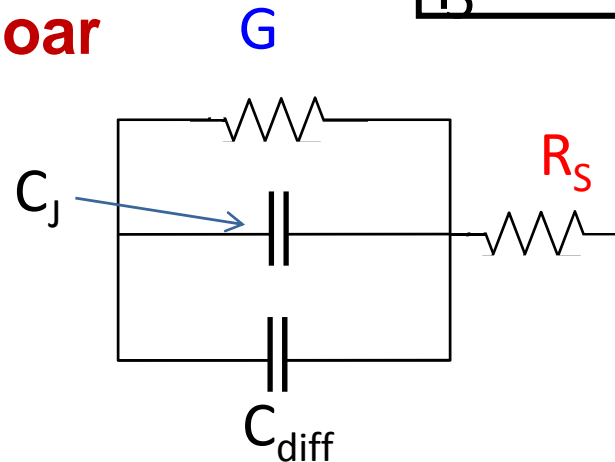
Forward Bias Conductance

$$I = I_o \left(e^{q(V_A - R_S I) \beta / m} - 1 \right)$$

RG, diff, ambiploar



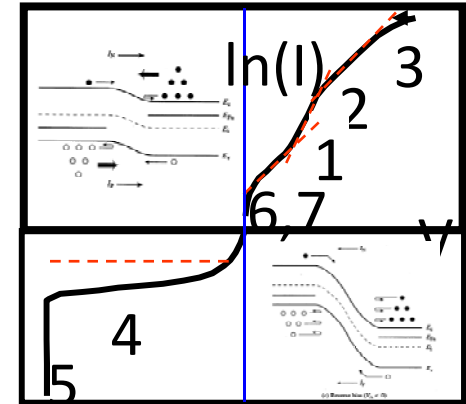
$$\ln \frac{I + I_o}{I_o} = q(V_A - R_S I) \frac{\beta}{m}$$



$$\frac{m}{q\beta(I + I_o)} = \frac{dV_A}{dI} - R_S$$

$$\frac{1}{g_{FB}} = R_S + \frac{m}{q\beta(I + I_o)}$$

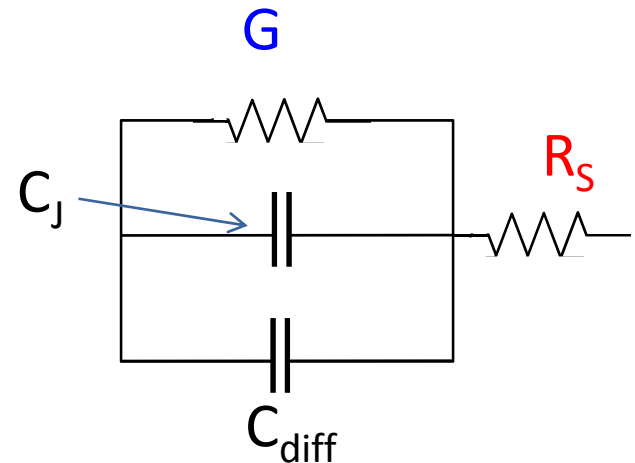
Reverse Bias Conductance



$$I = I_o \left(e^{q(V_A - R_S I)\beta / m} - 1 \right) - \frac{qn_i}{2\tau} B_0 \sqrt{V_{bi} - V_A}$$

$$\approx -I_o - \frac{qn_i}{2\tau} B_0 \sqrt{V_{bi} - V_A}$$

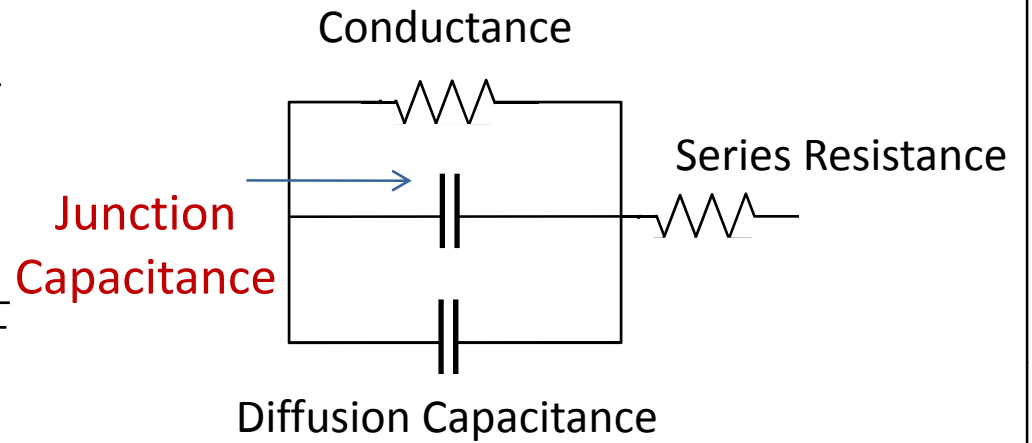
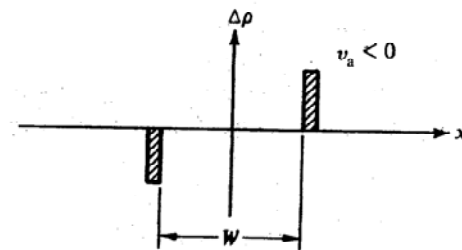
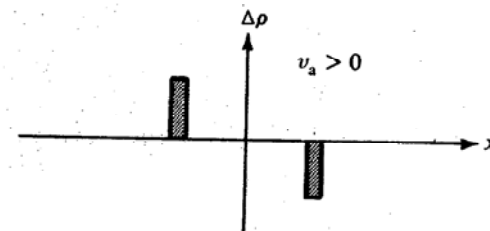
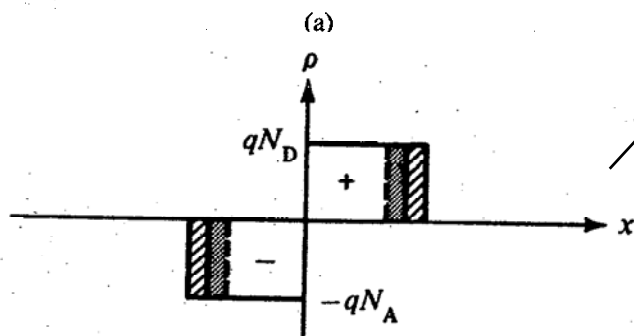
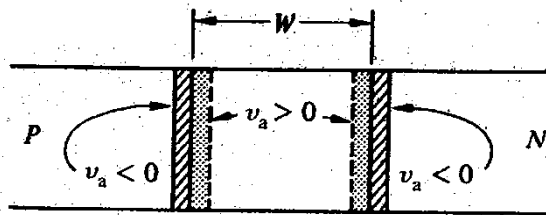
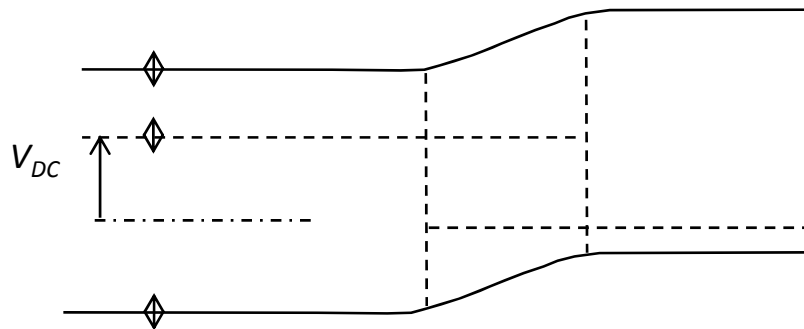
$$\frac{1}{g_{RB}} = \frac{qn_i B_0}{2\tau \sqrt{V_{bi} - V_A}}$$



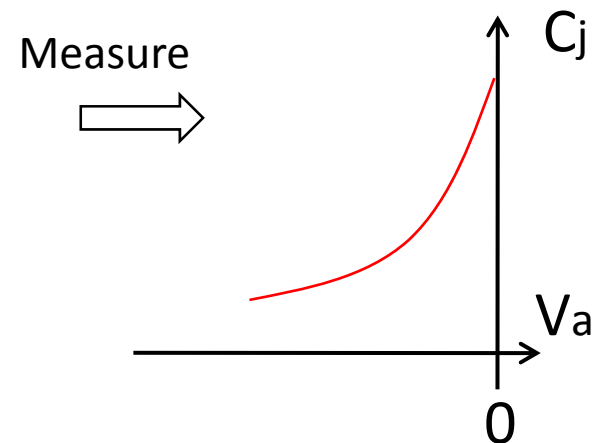
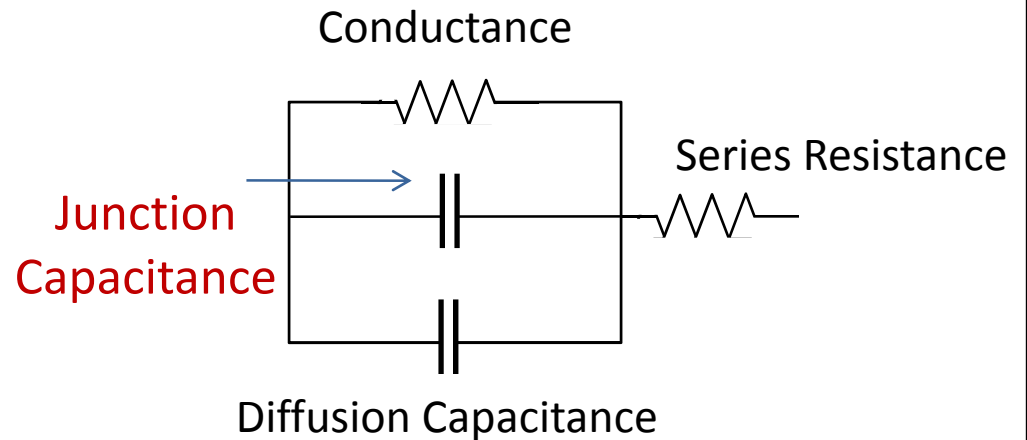
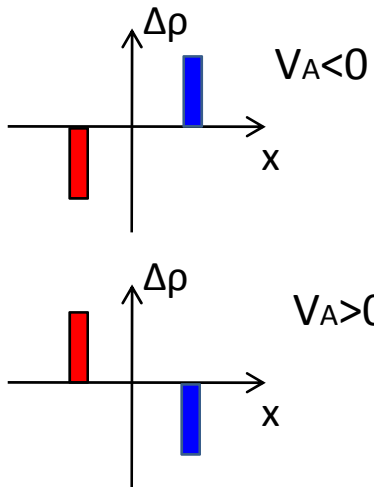
Outline

- 1) Conductance and series resistance
- 2) Majority carrier junction capacitance**
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Junction Capacitance



Majority Carrier Junction Capacitance

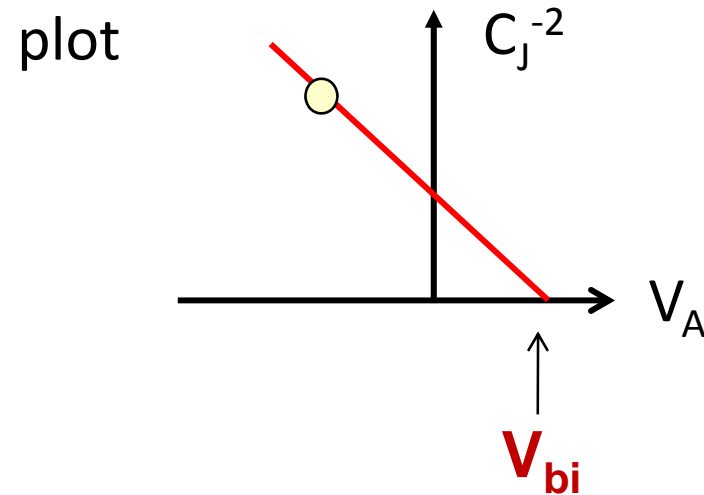
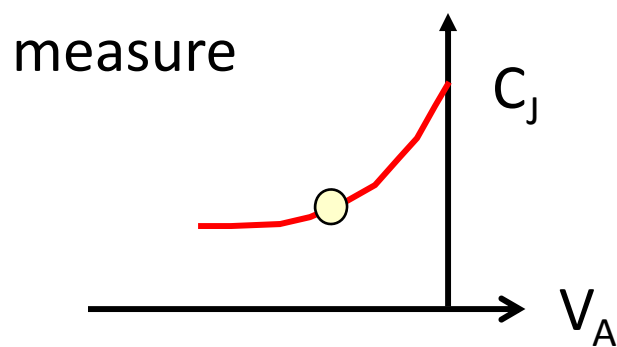


$$C_J = \frac{K_s \epsilon_0 A}{W_n + W_p} = \frac{K_s \epsilon_0 A}{\sqrt{\left(\frac{2K_s \epsilon_0}{qN_D} + \frac{2K_s \epsilon_0}{qN_A} \right) (V_{bi} - V_A)}}$$

Measurement of Built-in Potential

$$\frac{1}{C_J^2} \approx \frac{2}{qN_D(x)K_s\epsilon_0A^2}(V_{bi} - V_A)$$

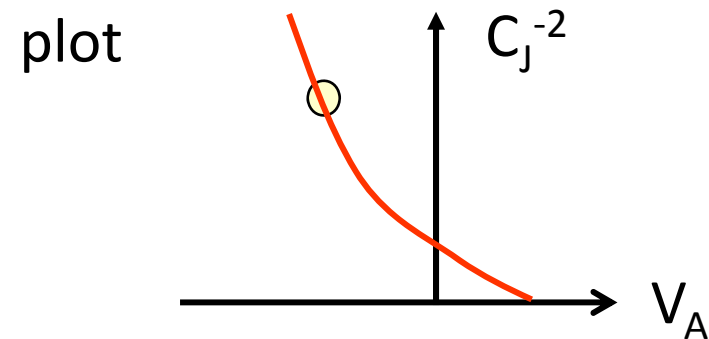
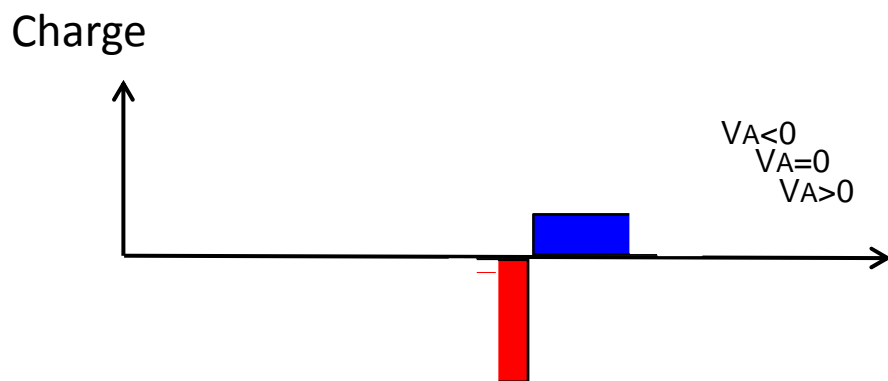
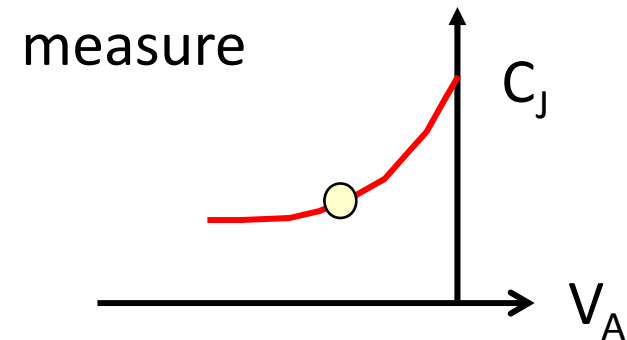
(Assume single sided p⁺-n junction)



.. And Variable Doping

$$\frac{1}{C_J^2} \approx \frac{2}{qN_D(x)K_s\epsilon_0A^2}(V_{bi} - V_A)$$

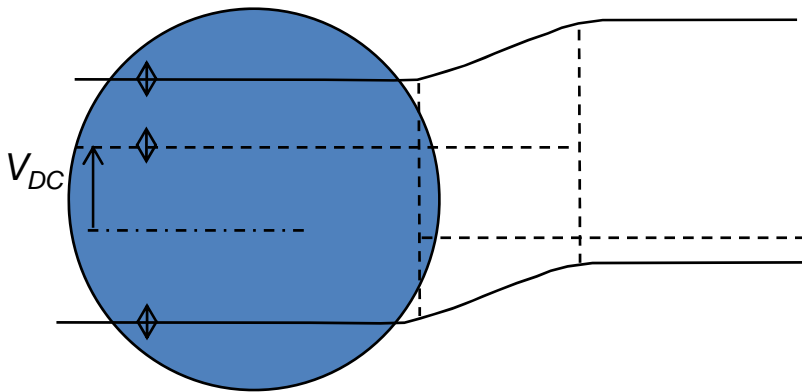
$$N_D(x) = \frac{2}{qK_s\epsilon_0A^2} \frac{1}{d(1/C_J^2)/dV_A}$$



Dielectric Relaxation Time (majority side)

$$J_n = qn\mu_N E + qD_N \nabla n$$

$$\frac{dn}{dt} = \frac{1}{q} \frac{dJ_n}{dx} - \cancel{R_n} + \cancel{G_n}$$



$$\frac{d(\Delta n)}{dt} = \frac{1}{q} \frac{d(qn\mu_N \mathcal{E})}{dx} = N_D \mu_N \frac{d\mathcal{E}}{dx}$$

$$\frac{d\mathcal{E}}{dx} = \frac{q}{k_s \epsilon_0} (\cancel{p} - n_0 - \Delta n + N_D - \cancel{N_A})$$

$$\frac{d(\Delta n)}{dt} = -\frac{qN_D \mu_N}{k_s \epsilon_0} \Delta n = -\frac{\sigma_0 \Delta n}{k_s \epsilon_0}$$

$$\tau_d = \frac{K_s \epsilon_0}{\sigma} \approx 0.1 \text{ ps}$$

$$\Delta n(t) = n_0 e^{-\frac{\sigma_0 t}{k_s \epsilon_0}} = n_0 e^{-\frac{t}{\tau_d}}$$

Outline

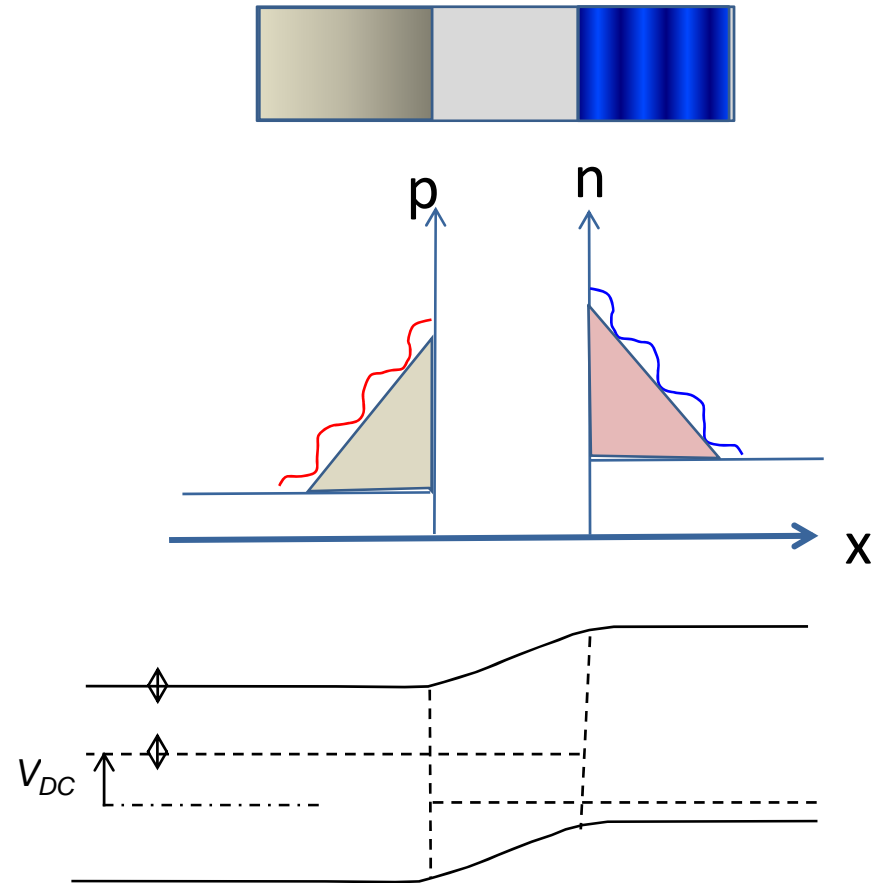
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Diffusion Capacitance for Minority Carriers

$$\mathbf{J}_N = qn\mu_N \mathbf{E} + qD_N \frac{dn}{dx}$$

$$\frac{\partial n}{\partial t} = \frac{1}{q} \frac{dJ_n}{dx} - r_N + g_N$$

$$\frac{\partial (n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t})}{\partial t} = D_N \frac{d^2 (n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t})}{dx^2} - \frac{\Delta n_{dc} + \Delta n_{ac} e^{j\omega t}}{\tau_n}$$



Diffusion Capacitance for Minority Carriers

$$\frac{\partial (n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t})}{\partial t} = D_N \frac{d^2 (n_0 + \Delta n_{dc} + \Delta n_{ac} e^{j\omega t})}{dx^2} - \frac{\Delta n_{dc} + \Delta n_{ac} e^{j\omega t}}{\tau_n}$$

$$j\omega \Delta n_{ac} e^{j\omega t} = D_N \frac{d^2 \Delta n_{dc}}{dx^2} + e^{j\omega t} \frac{d^2 \Delta n_{ac}}{dx^2} - \frac{\Delta n_{dc}}{\tau_n} - e^{j\omega t} \frac{\Delta n_{ac}}{\tau_n}$$

$$\text{DC: } 0 = D_N \frac{d^2 \Delta n_{dc}}{dx^2} - \frac{\Delta n_{dc}}{\tau_n} \Rightarrow \Delta n_{dc} = A e^{-\frac{x}{L_n}} + B e^{+\frac{x}{L_n}}$$

$$\text{AC: } 0 = D_N \frac{d^2 \Delta n_{ac}}{dx^2} - (j\omega \tau_n + 1) \frac{\Delta n_{ac}}{\tau_n} \Rightarrow \Delta n_{ac} = C e^{-\frac{x}{L_n^*}} + D e^{+\frac{x}{L_n^*}} \rightarrow C e^{-\frac{x}{L_n^*}}$$

$$L_n^* = \sqrt{D_n \tau_n / (1 + j\omega \tau_n)} \quad \tau_n^* = \tau_n / (1 + j\omega \tau_n)$$

AC Boundary Conditions

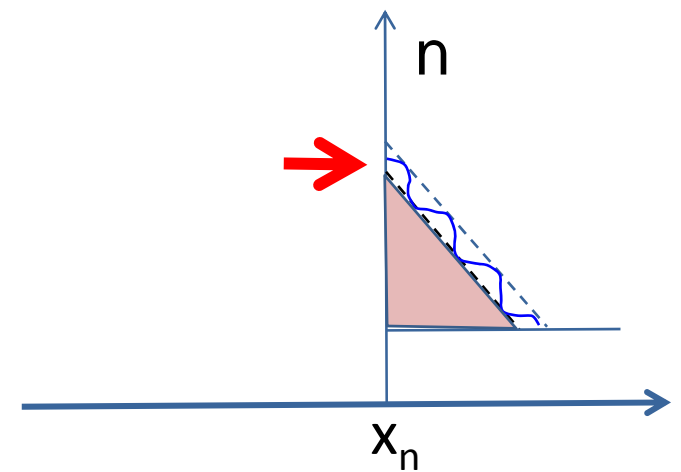
$$\Delta n_{dc}(x=0) = \frac{n_i^2}{N_A} \left(e^{\frac{qV_{dc}}{kT}} - 1 \right)$$

$$\left(\Delta n_{dc} + \Delta n_{ac} e^{j\omega t} \right) = \frac{n_i^2}{N_A + \Delta p_{ac} e^{j\omega t}} \left(e^{q \frac{V_{dc} + V_{ac} e^{j\omega t}}{kT}} - 1 \right)$$

$$\left(\Delta n_{dc} + \Delta n_{ac} e^{j\omega t} \right) \approx \frac{n_i^2}{N_A} \left(e^{\frac{qV_{dc}}{kT}} e^{\frac{qV_{ac} e^{j\omega t}}{kT}} - 1 \right)$$

$$\approx \frac{n_i^2}{N_A} \left\{ e^{\frac{qV_{dc}}{kT}} \left(1 + \frac{qV_{ac} e^{j\omega t}}{kT} \right) - 1 \right\}$$

$$\Delta n_{ac}(x=0) = \frac{qV_{ac}}{kT} \frac{n_i^2}{N_A} e^{\frac{qV_{dc}}{kT}} = C$$



AC Current and Impedance

$$\Delta n_{ac}(x=0) = \frac{qV_{ac}}{kT} \frac{n_i^2}{N_A} e^{\frac{qV_{dc}}{kT}} = C$$

$$\Delta n_{ac}(x) = Ce^{-\frac{x}{L_n^*}} + De^{+\frac{x}{L_n^*}} \rightarrow Ce^{-\frac{x}{L_n^*}}$$

$$J_{ac} = -qD_n \left. \frac{d\Delta n_{ac}}{dx} \right|_{x=0} = \frac{qD_n}{L_n^*} \frac{qV_{ac}}{kT} \frac{n_i^2}{N_A} e^{\frac{qV_{dc}}{kT}}$$

$$Y_{ac} = \frac{J_{ac}}{V_{ac}} = \frac{q^2 D_n}{L_n^* kT} \frac{n_i^2}{N_A} e^{\frac{qV_{dc}}{kT}} \equiv G_0 \sqrt{1 + j\omega\tau_n}$$

Diffusion Conductance and Capacitance

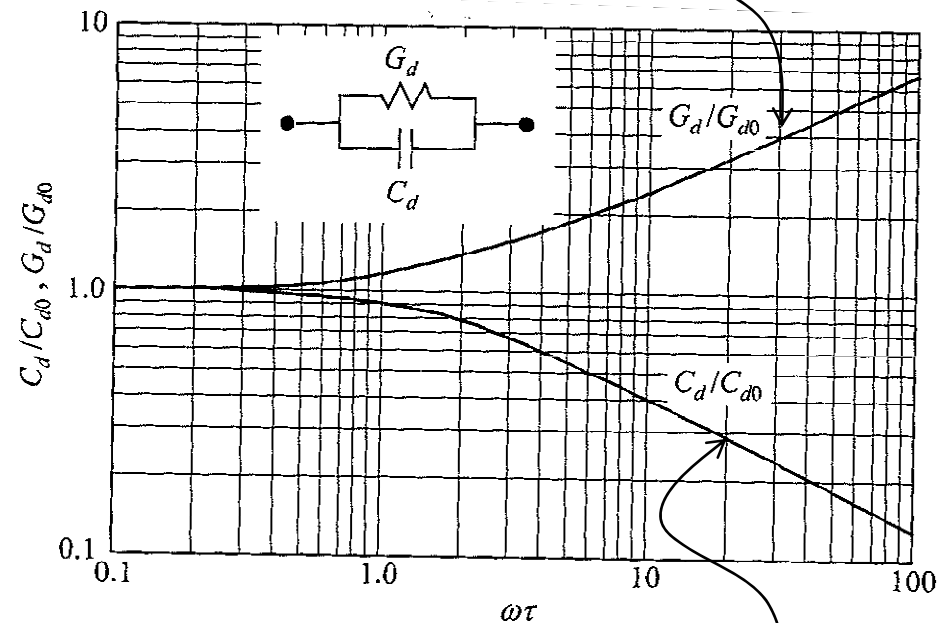
$$G_D \propto \sqrt{\omega}$$

$$Y_{ac} = G_D + j\omega C_D \equiv G_0 \sqrt{1 + j\omega\tau_n}$$

Separate in real & imaginary parts ...

$$G_D = \frac{G_0}{\sqrt{2}} \left[\sqrt{1 + \omega^2 \tau_n^2} + 1 \right]^{1/2}$$

$$\omega C_D = \frac{G_0}{\sqrt{2}} \left[\sqrt{1 + \omega^2 \tau_n^2} - 1 \right]^{1/2}$$



$$C_D \propto 1/\sqrt{\omega}$$

Conclusion

- 1) Small signal response relevant for many analog applications.
- 2) Small signal parameters always refer to the DC operating conditions, as such the parameter changes with bias condition.
- 3) Important to distinguish between majority and minority carrier capacitance. Their relative importance depends on specific applications.