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ECE606: Solid State Devices
Lecture 20: Electrostatics of p-n junction diodes

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Outline

1) **Introduction to p-n junctions**
2) Drawing band-diagrams
3) Accurate solution in equilibrium
4) Band-diagram with applied bias

Ref. Semiconductor Device Fundamentals, Chapter 5

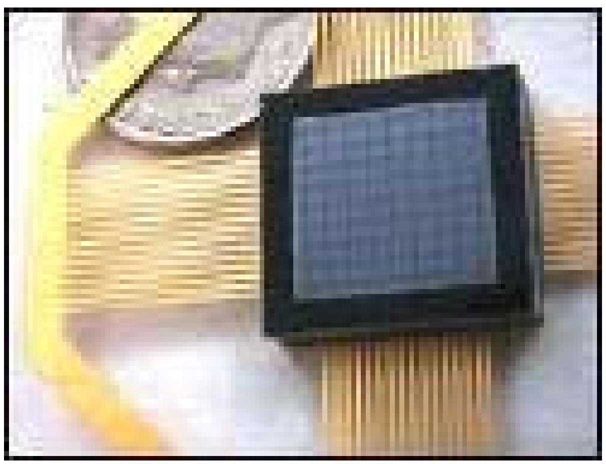
What is a Diode good for ….

solar cells

GaAs lasers

Organic LED

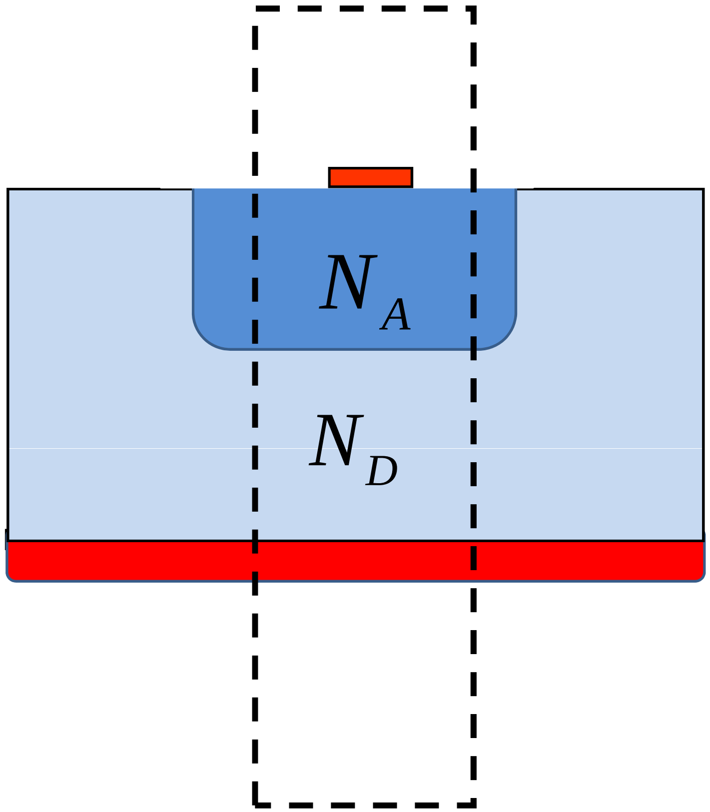
Avalanche Photodiode



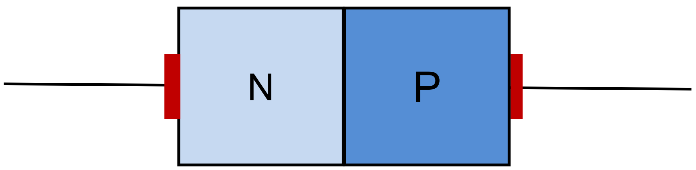
GaN lasers

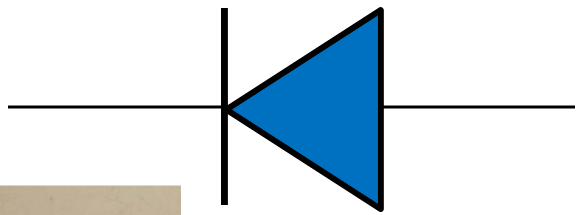
Image.google.com

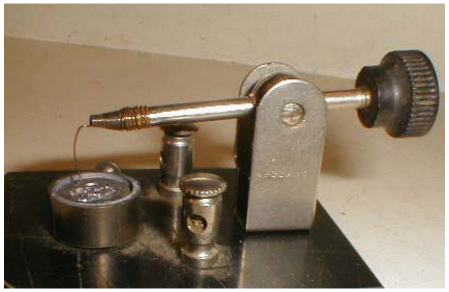
p-n Junction Devices …



Symbols







Topic Map

	Equilibrium	DC	Small signal	Large Signal	Circuits
Diode					
Schottky					
BJT/HBT					
MOS					

Drawing Band Diagram in Equilibrium…

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

← **equilibrium**

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = \frac{-1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

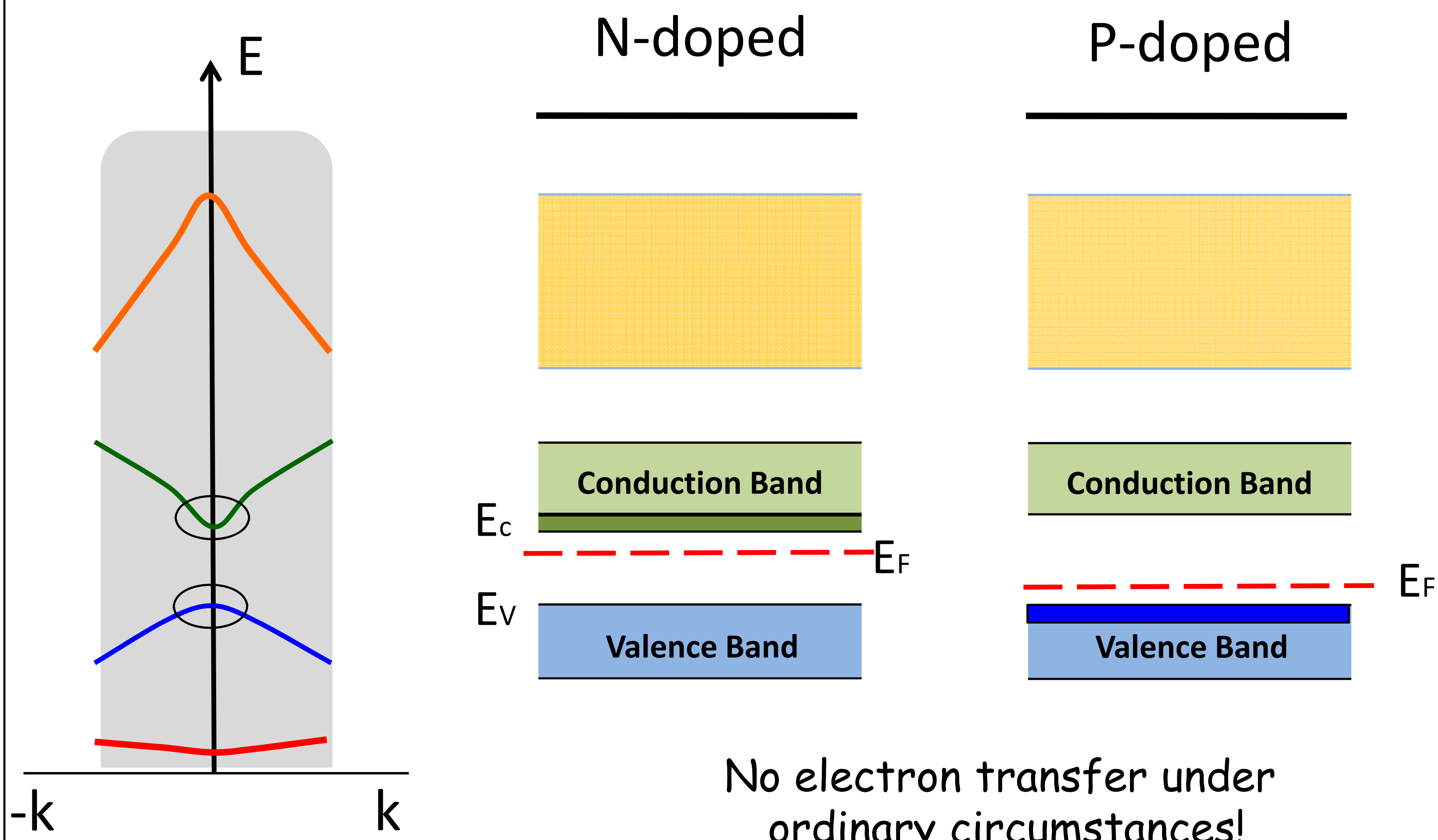
$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

DC dn/dt=0

Small signal dn/dt ~ $j\omega t$ x n

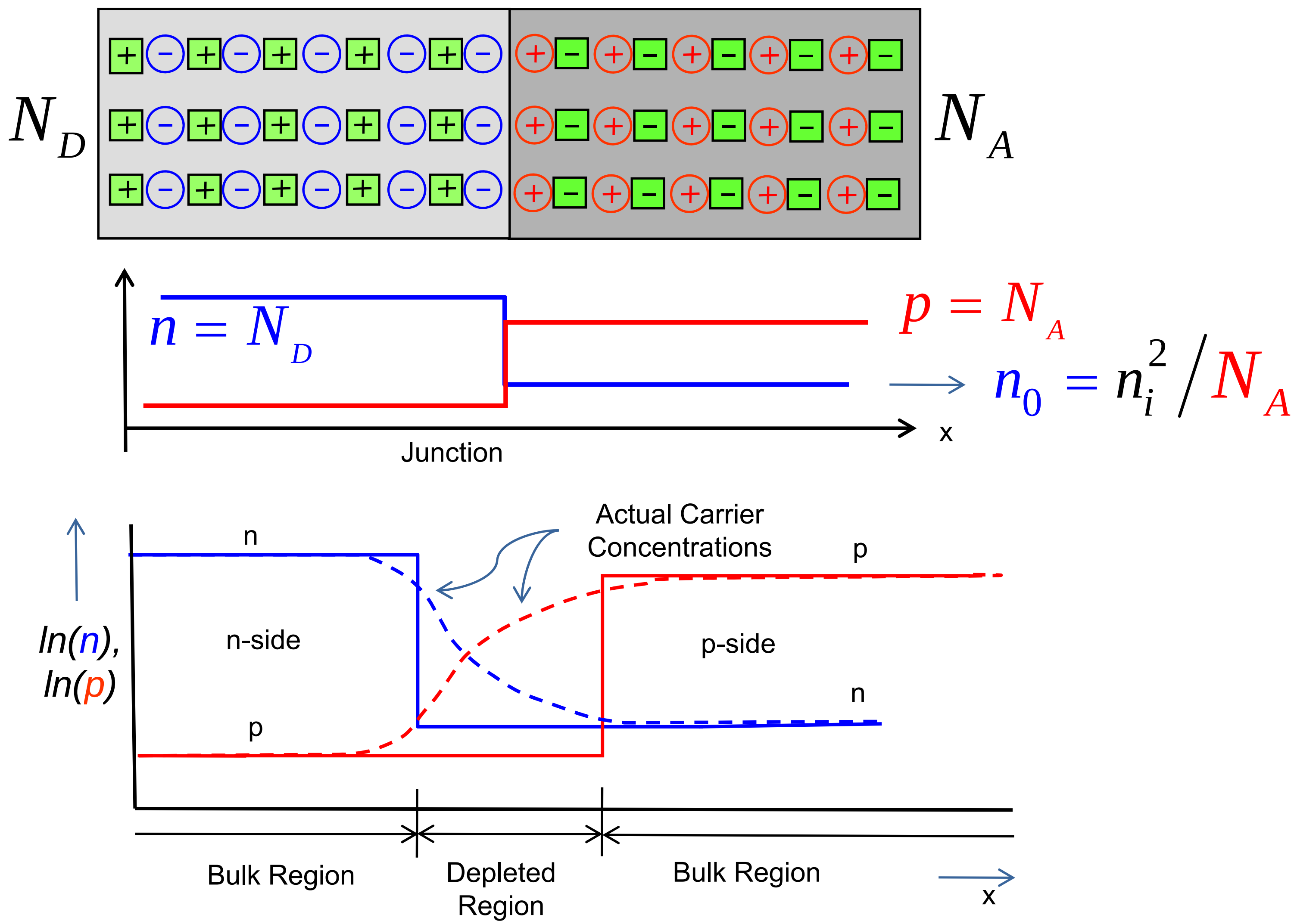
Transient --- full solution

P and N doped Material Side by Side …

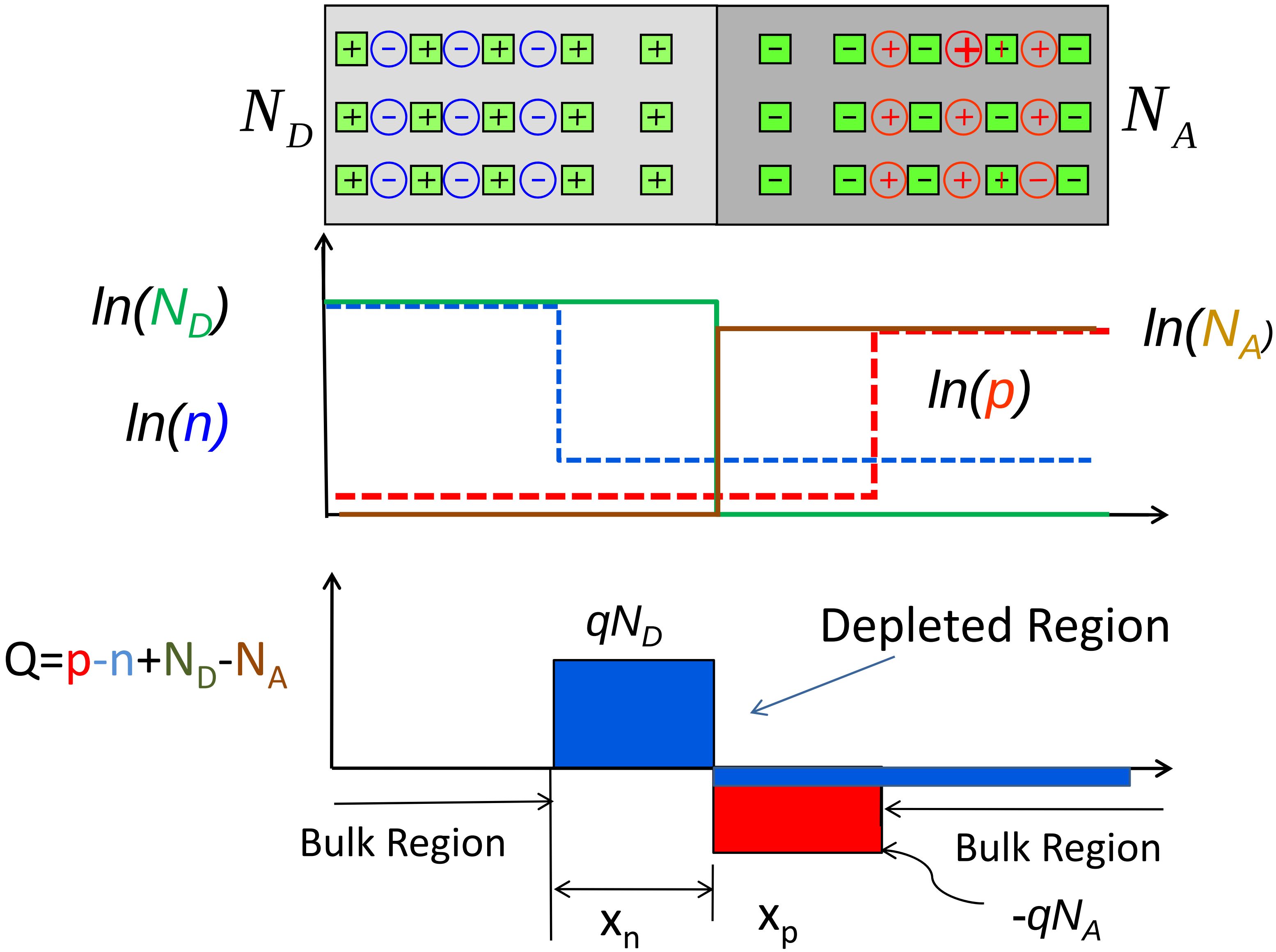


No electron transfer under ordinary circumstances!

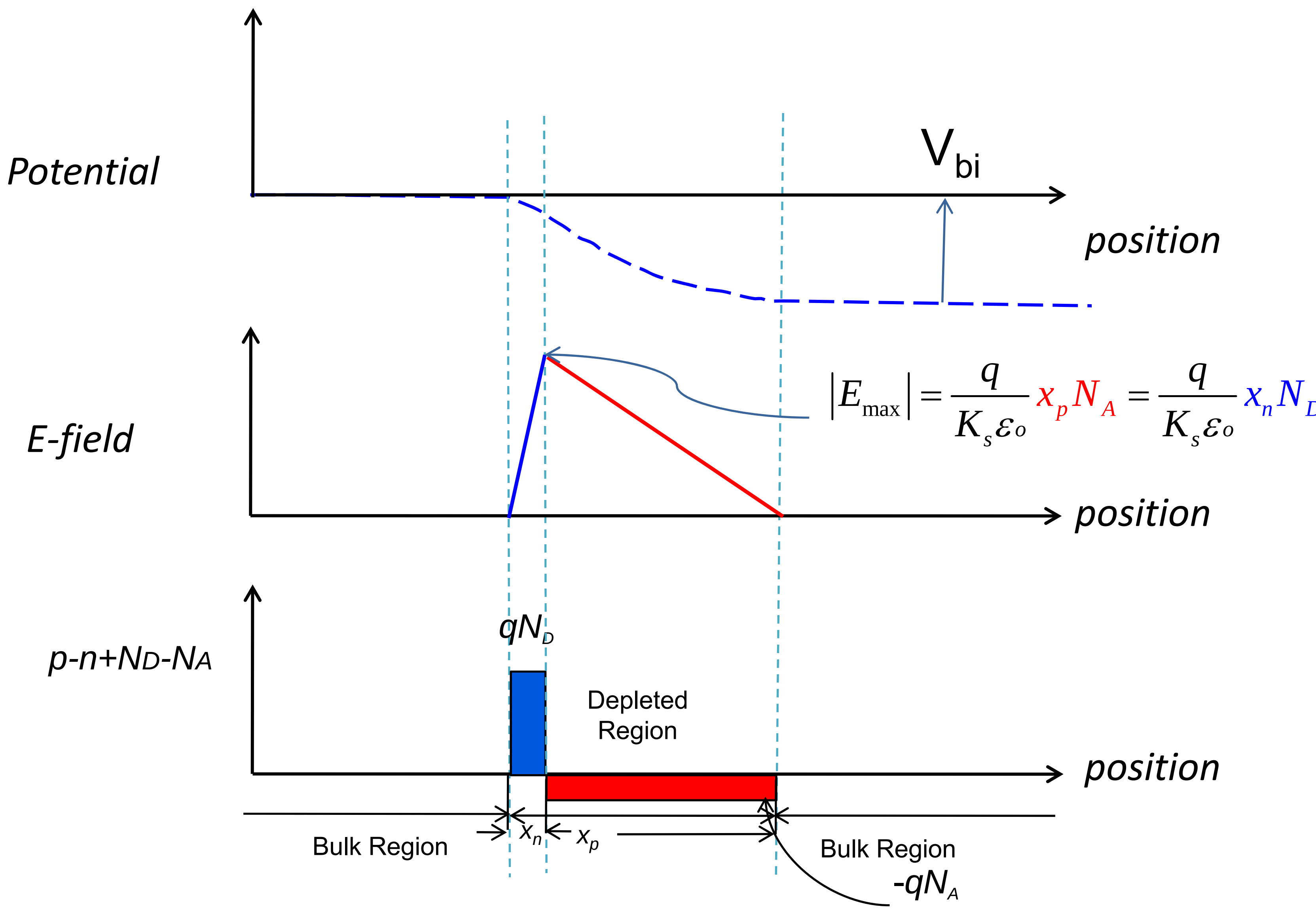
Forming a Junction



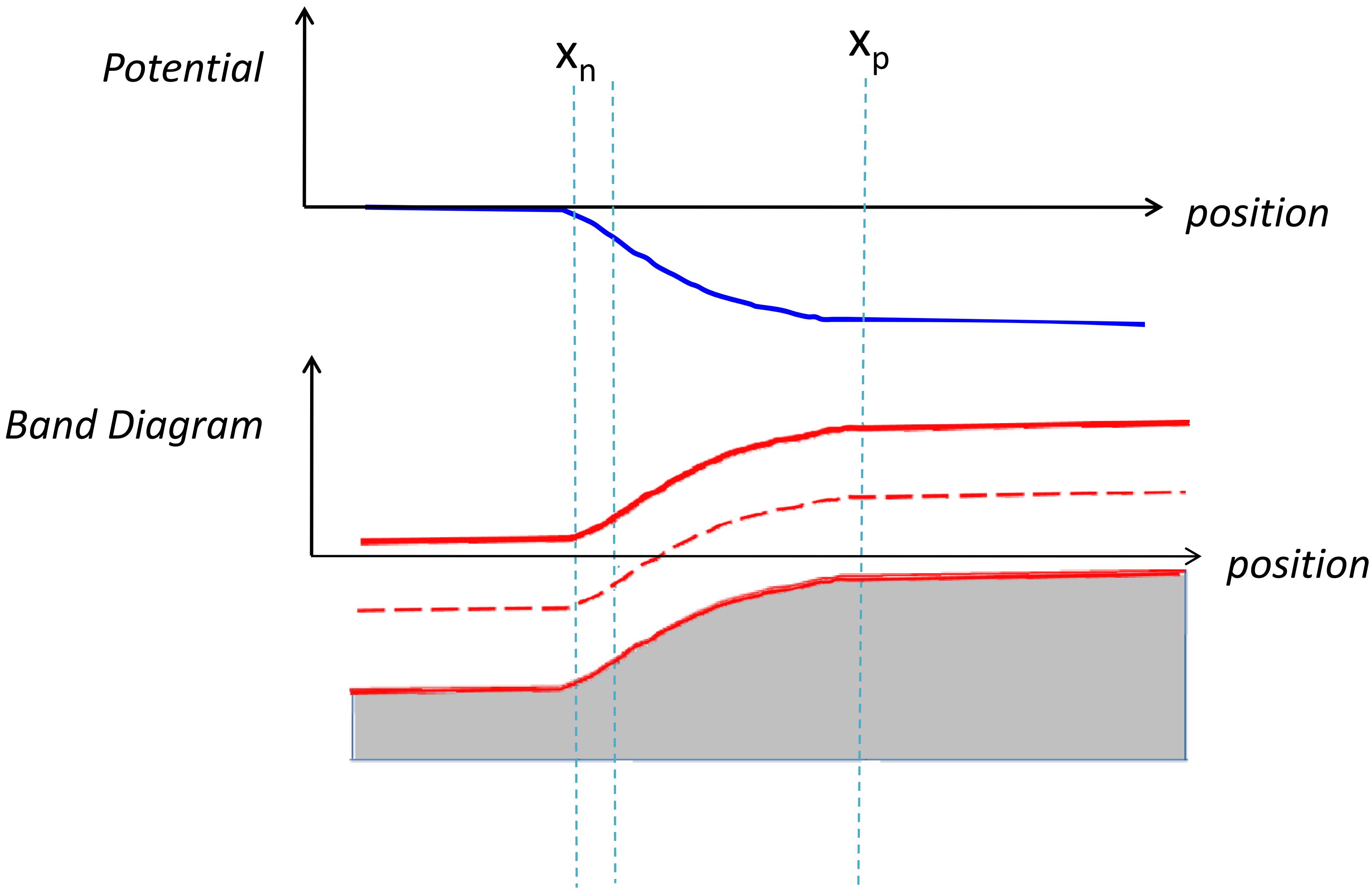
Formation of a Junction



Sketch of Electrostatics



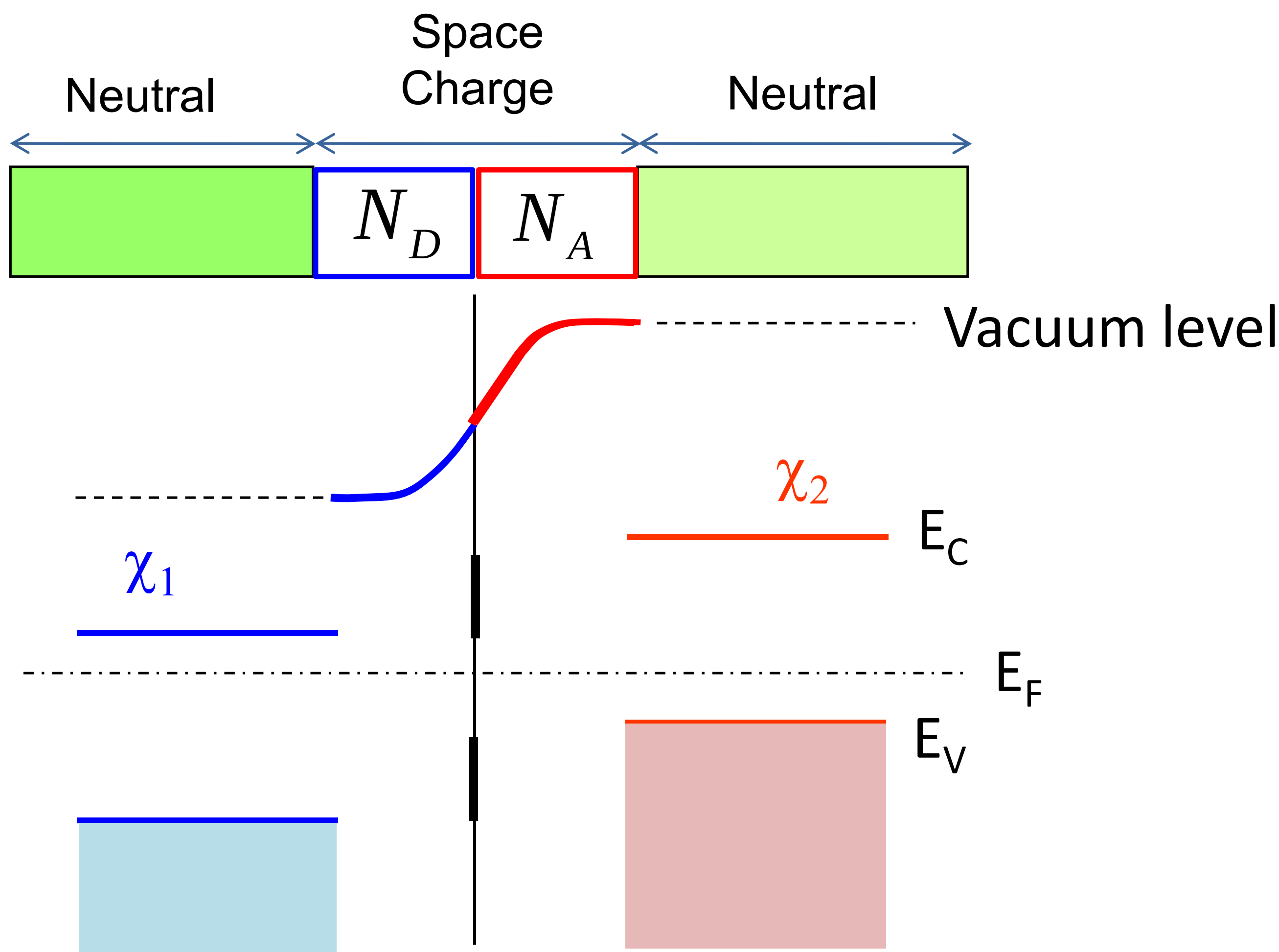
Sketch of Electrostatics



Outline

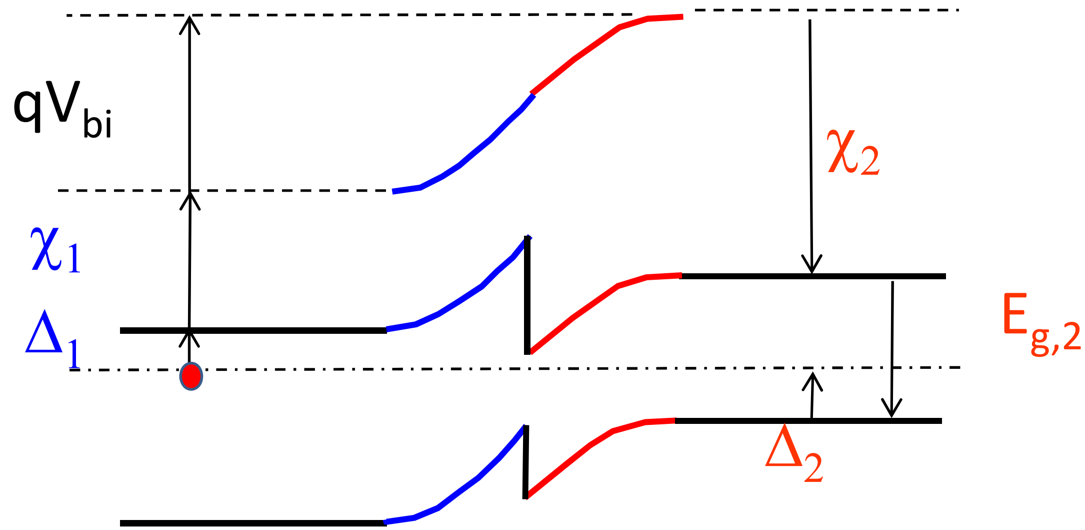
1) Introduction to p-n junctions
2) **Drawing band-diagrams**
3) Analytical solution in equilibrium
4) Band-diagram with applied bias

Short-cut to Band-diagram



… is equivalent to solving the Poisson equation

Built-in Potential: boundary conditions @infinity



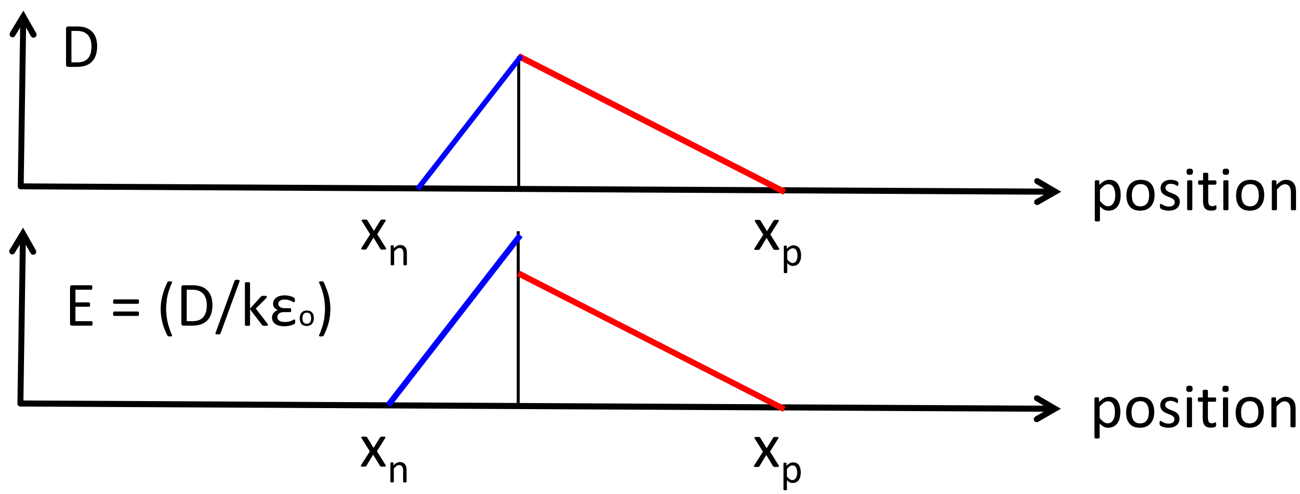
$$\Delta_1 + \chi_1 + qV_{bi} = \chi_2 + E_{g,2} - \Delta_2$$

$$qV_{bi} = E_{g,2} - \Delta_2 - \Delta_1 + \chi_2 - \chi_1$$

$$= \left(E_{g,2} + k_B T \ln \frac{N_A}{N_{V,2}} \right) + k_B T \ln \frac{N_D}{N_{C,1}} + \left(\chi_2 - \chi_1 \right)$$

$$= k_B T \ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + \left(\chi_2 - \chi_1 \right)$$

Interface Boundary Conditions

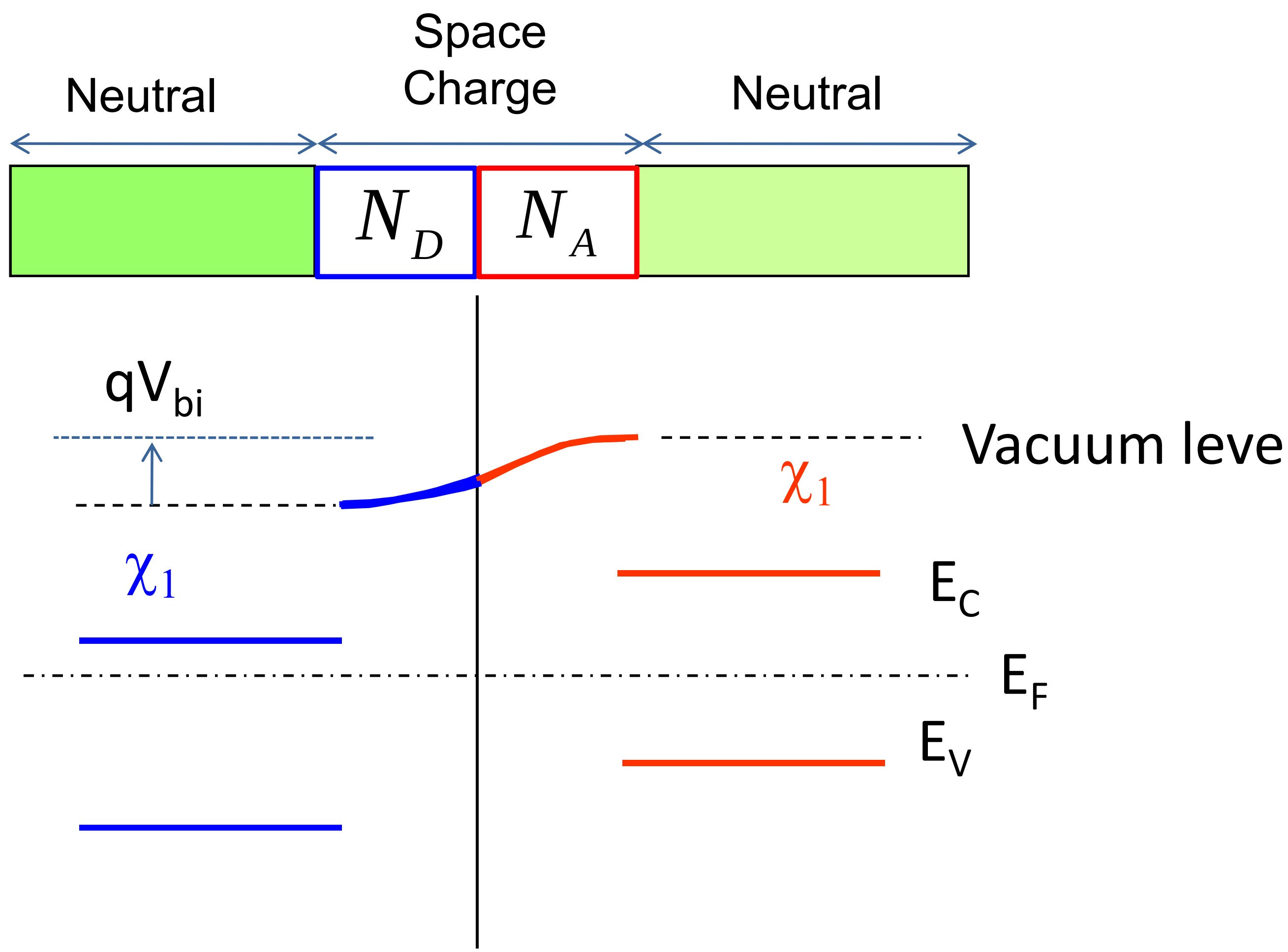


$$D_1 = K_1 \varepsilon_0 E(0^-) = K_2 \varepsilon_0 E(0^-) = D_2$$

$$E(0^-) = \frac{K_2}{K_1} E(0^+)$$

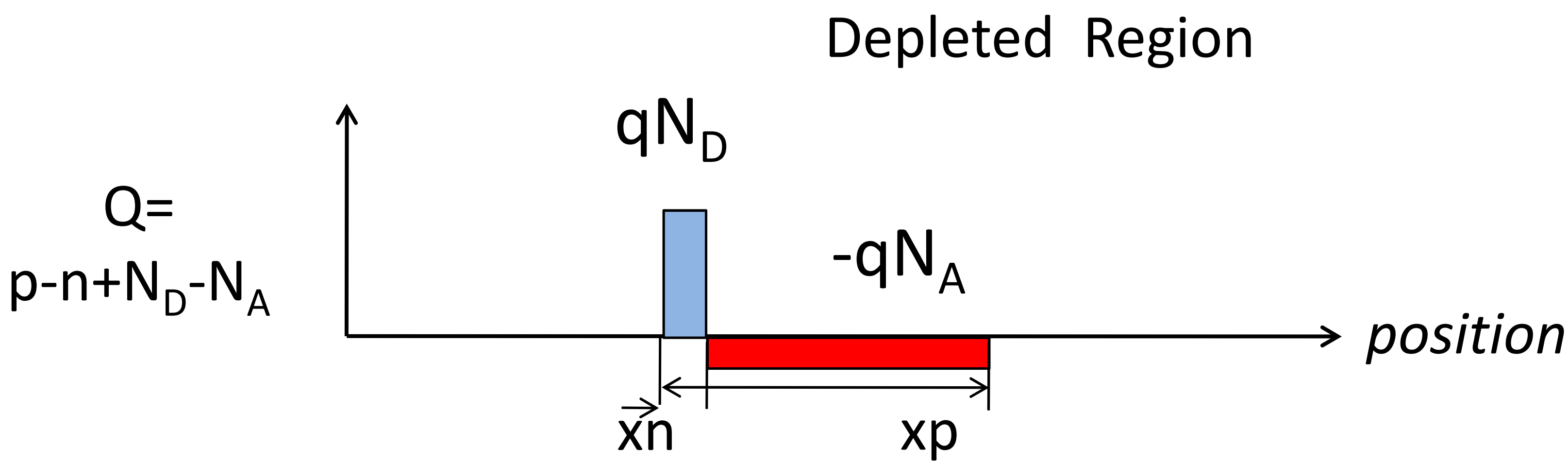
Displacement is continuous across the interface, field need not be ..

Built-in voltage for **Homo-**junctions



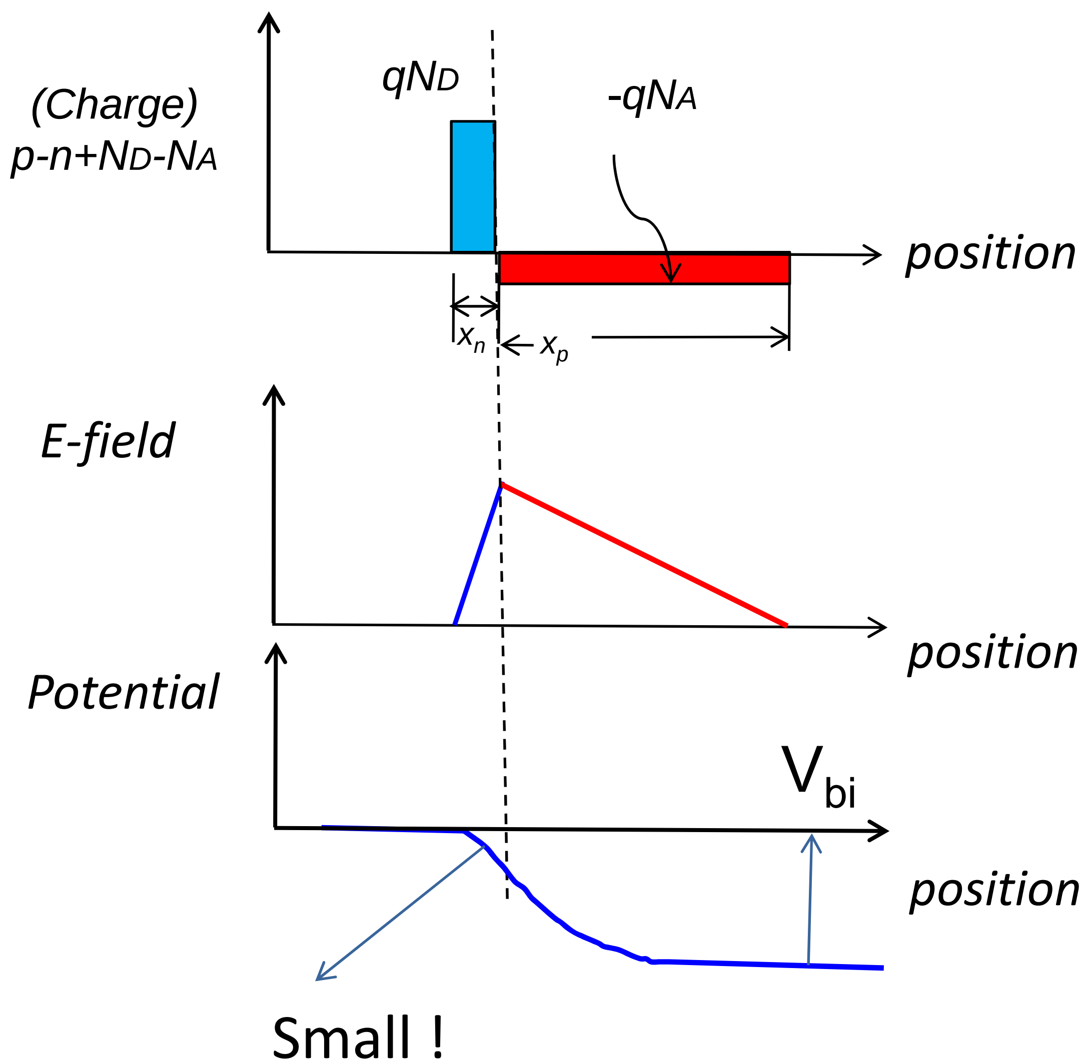
$$qV_{bi} = k_B T \ln \frac{N_A N_D}{N_{V,2} N_{C,1} e^{-E_{g,2}/k_B T}} + (\chi_2 - \chi_1) = k_B T \ln \frac{N_A N_D}{N_V N_C e^{-E_g/k_B T}} = k_B T \ln \frac{N_A N_D}{n_i^2}$$

Analytical Solution of Poisson Equation



$$K_S \varepsilon_0 \frac{d^2V}{dx^2} = -q\left(p - n + N_D^+ - N_A^-\right)$$

Analytical Solution for Homojunctions



$$E\left(0^-\right) = \frac{qN_D x_n}{k_s \varepsilon_0}$$

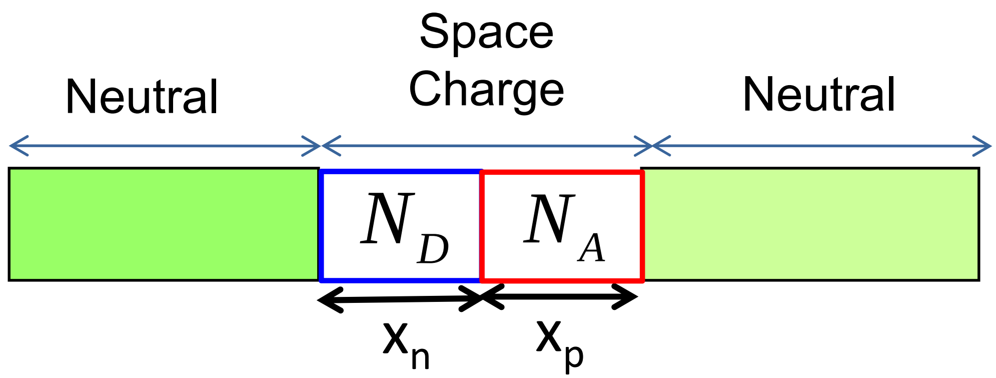
$$E\left(0^+\right) = \frac{qN_A x_p}{k_s \varepsilon_0}$$

$$\Rightarrow N_D x_n = N_A x_p$$

$$qV_{bi} = \frac{E\left(0^-\right) x_n}{2} + \frac{E\left(0^+\right) x_p}{2}$$

$$= \frac{qN_D x_n^{\ 2}}{2k_s \varepsilon_0} + \frac{qN_A x_p^{\ 2}}{2k_s \varepsilon_0}$$

Depletion Regions in Homojunctions



$$N_D x_n = N_A x_p$$

$$qV_{bi} = \frac{qN_D x_n^2}{2k_s\varepsilon_0} + \frac{qN_A x_p^2}{2k_s\varepsilon_0}$$

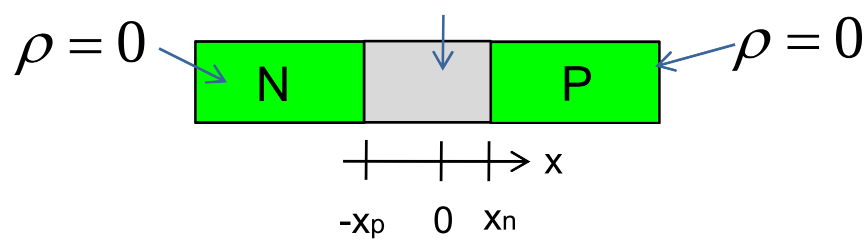
$$x_n = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_A}{N_D\left(N_A+N_D\right)}V_{bi}}$$

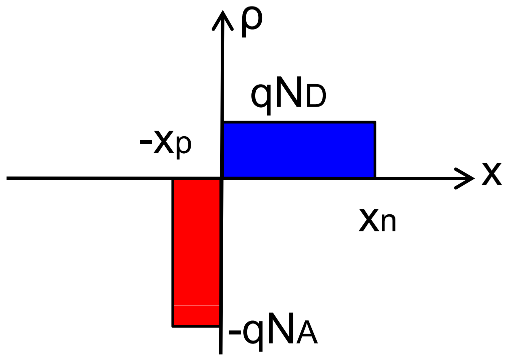
$$x_p = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_D}{N_A\left(N_A+N_D\right)}V_{bi}}$$

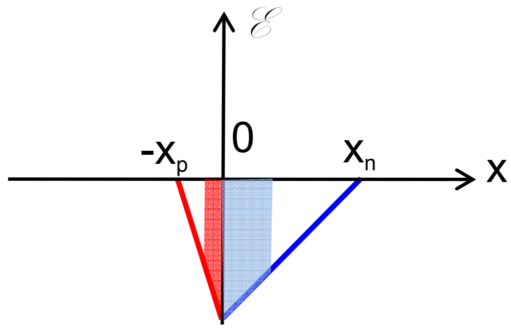
HW: Solve the same problem for a hetero-junction

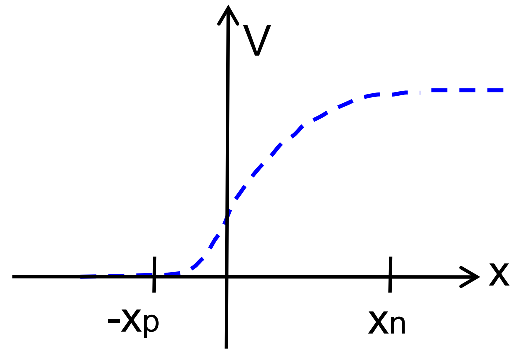
Complete Analytical Solution

$$\rho = q(N_D - N_A)$$









$$\frac{d\mathcal{E}}{dx} = \begin{cases} \frac{-qN_A}{K_s\varepsilon_o} \ldots\ldots\ldots -x_p \le x \le 0 \\ \frac{qN_D}{K_s\varepsilon_o} \ldots\ldots\ldots 0 \le x \le x_n \\ 0 \ldots\ldots\ldots x \le -x_p,\ x \ge x_n \end{cases}$$

$$\int_0^{\mathcal{E}(x)} d\mathcal{E}' = -\int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_0} dx'$$

$$\int_{\mathcal{E}(x)}^{0} d\mathcal{E}' = \int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_0} dx'$$

$$\mathcal{E}(x) = -\frac{qN_A}{K_S\varepsilon_0}(x_p + x) \ldots\ldots\ldots -x_p \le x \le 0$$

$$\mathcal{E}(x) = -\frac{qN_D}{K_S\varepsilon_0}(x_n - x) \ldots\ldots\ldots 0 \le x \le x_n$$

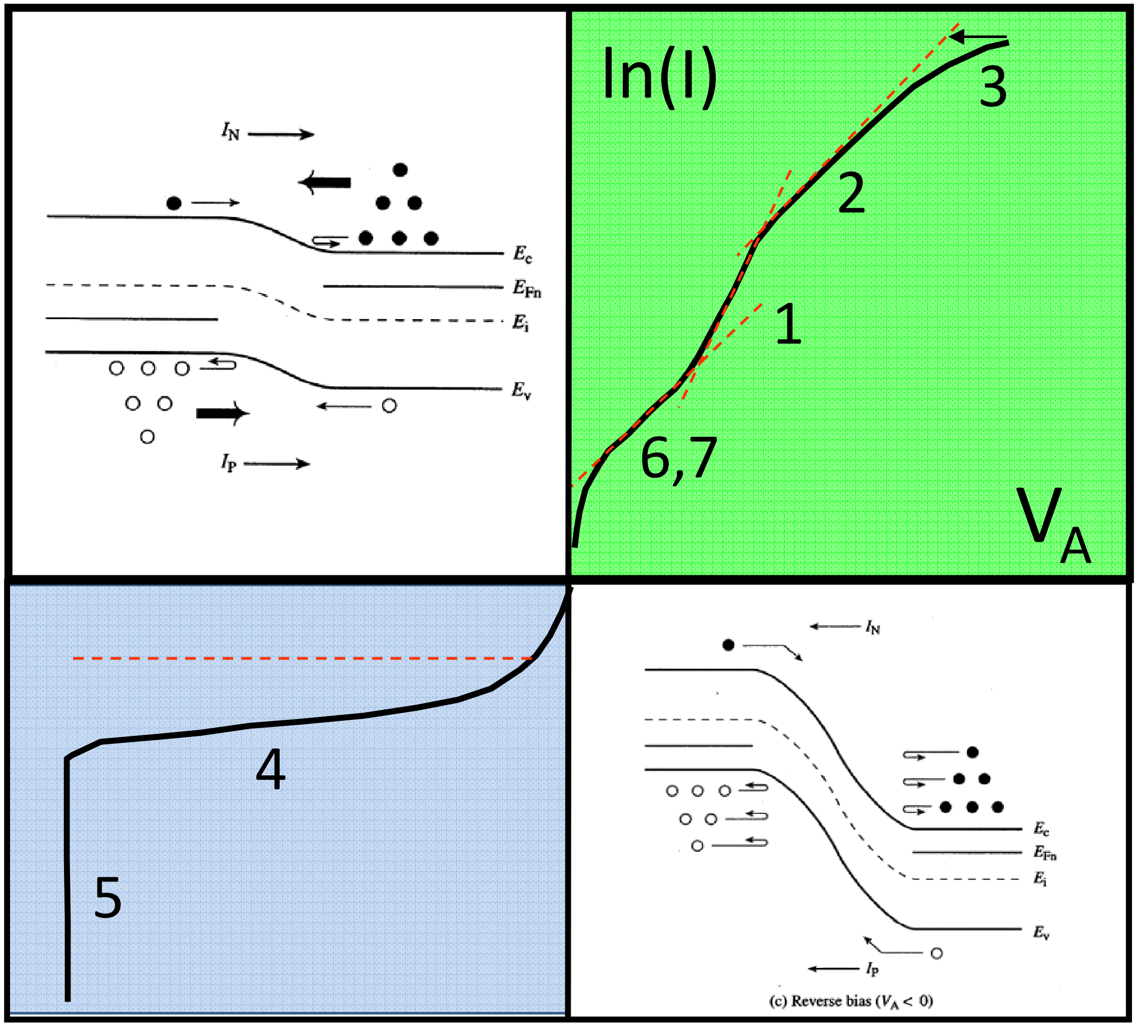
Outline

1) Introduction to p-n junction transistors
2) Drawing band-diagrams
3) Analytical solution in equilibrium
4) **Band-diagram with applied bias**

Topic Map

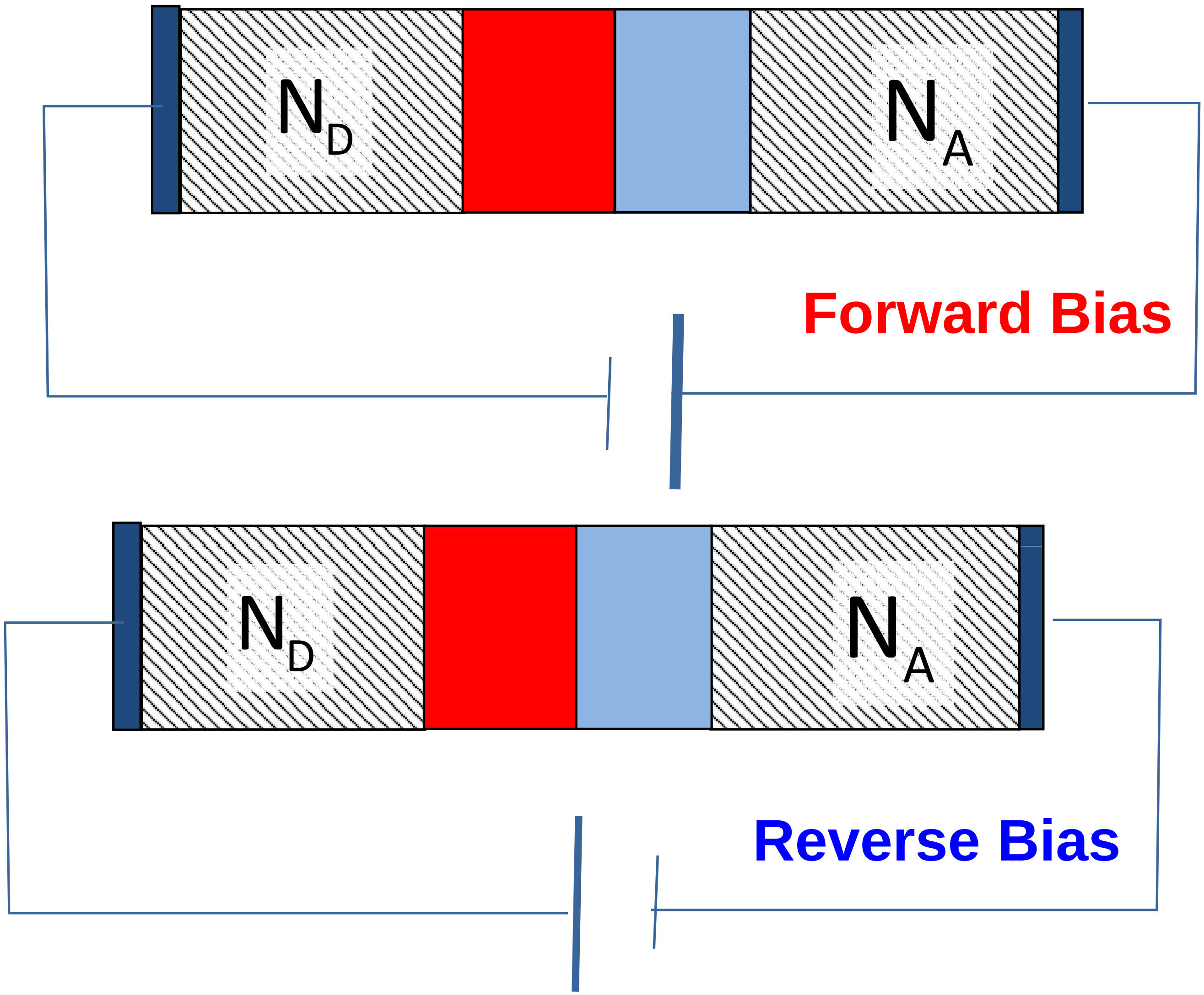
	Equilibrium	DC	Small signal	Large Signal	Circuits
Diode		◆			
Schottky					
BJT/HBT					
MOS					

Applying Bias to p-n Junction



1. *Diffusion limited*
2. *Ambipolar transport*
3. *High injection*
4. *R-G in depletion*
5. *Breakdown*
6. *Trap-assisted R-G*
7. *Esaki Tunneling*

Forward and Reverse Bias



Band Diagram with Applied Bias...

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

← Band diagram (now) ...

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

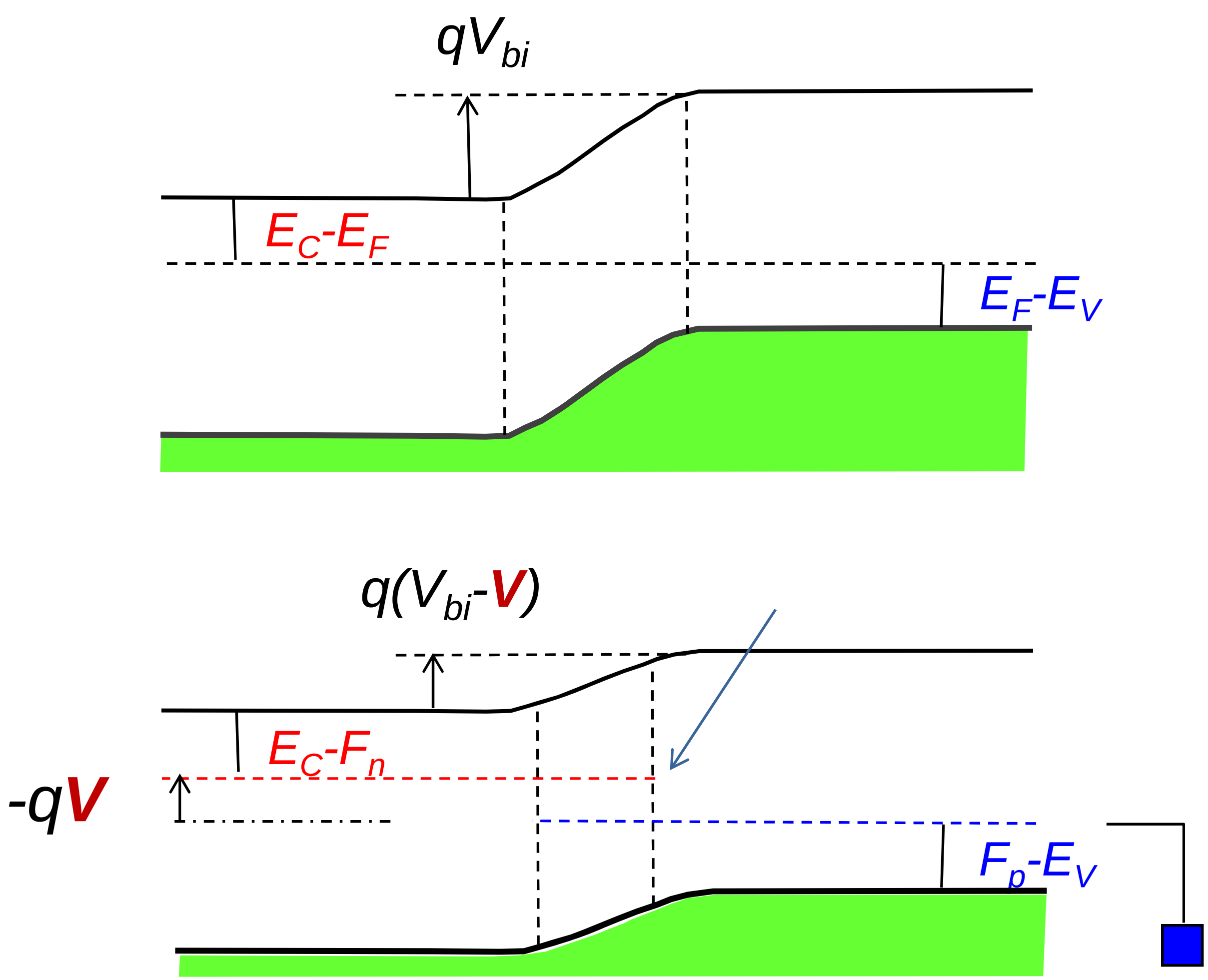
$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

} Next class ...

Applying a Bias: Poisson Equation



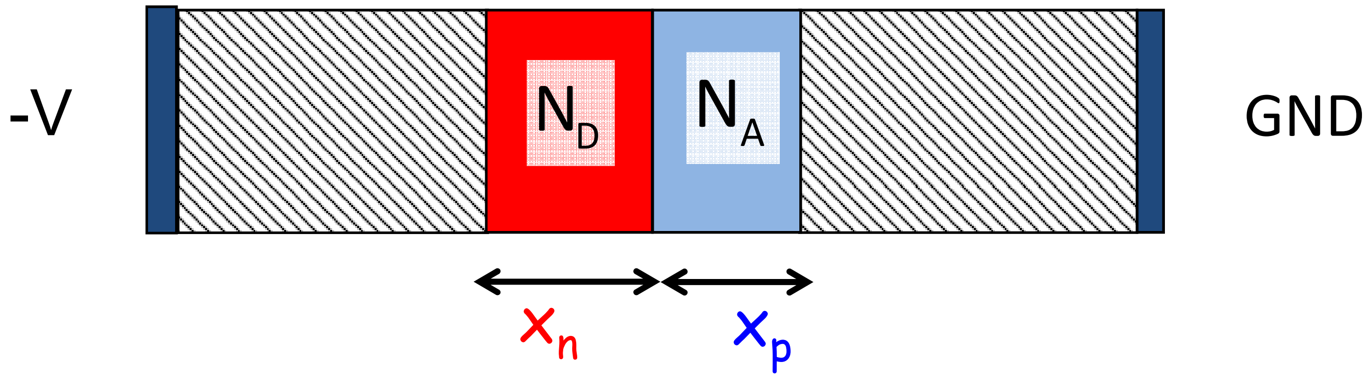
Max value of V_{bi}?

$$n(x) = n_i\, e^{(F_n - E_i)\beta}$$

$$p(x) = n_i\, e^{-(F_p - E_i)\beta}$$

$$n \times p = n_i^{\,2}\, e^{(F_n - F_p)\beta}$$

Depletion Widths



$$N_D x_n = N_A x_p$$

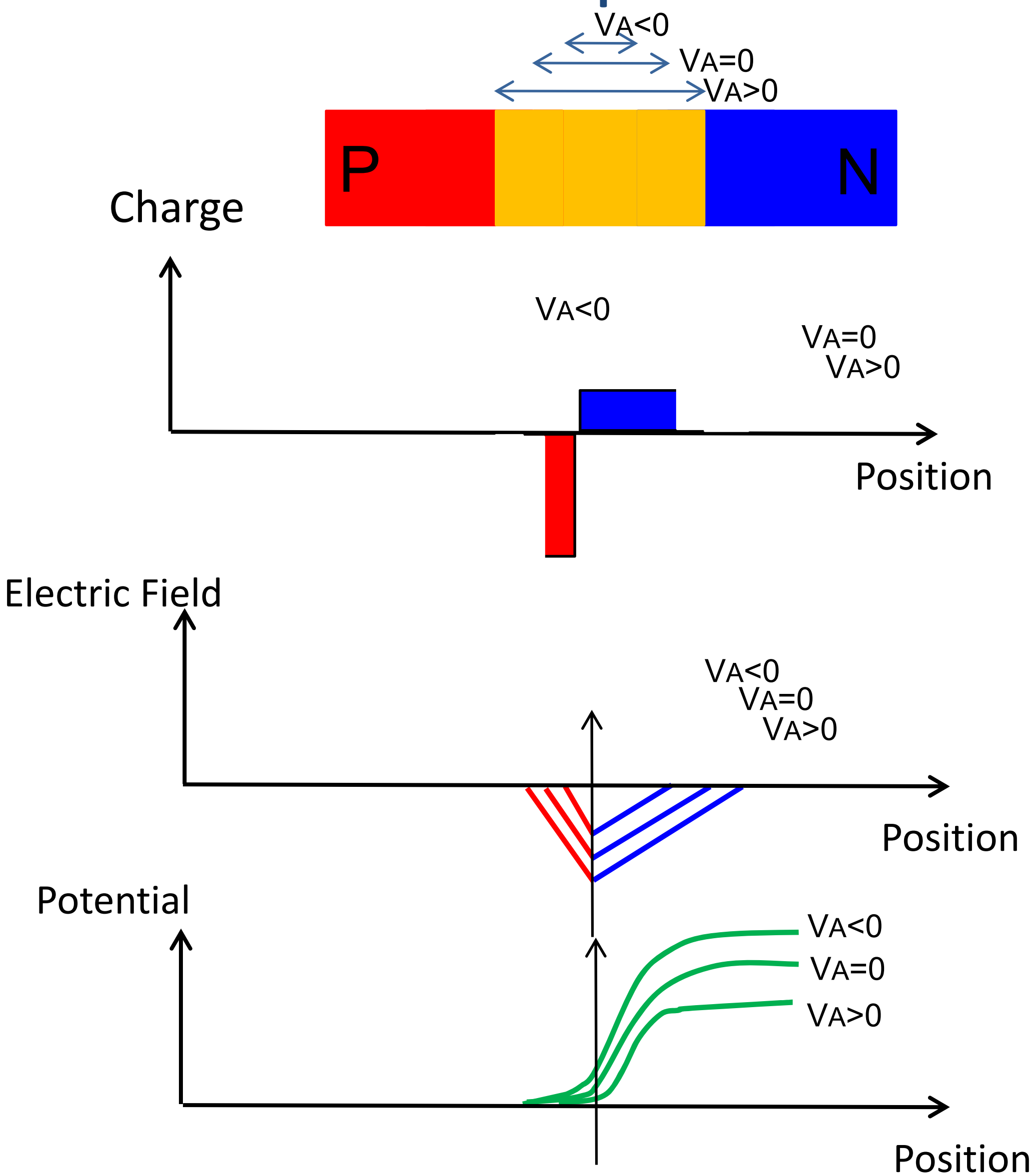
$$q\left(V_{bi} - V\right) = \frac{qN_D x_n^{\,2}}{2k_s\varepsilon_0} + \frac{qN_A x_p^{\,2}}{2k_s\varepsilon_0}$$

$$x_n = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_A}{N_D\left(N_A + N_D\right)}(V_{bi} - V)}$$

$$x_p = \sqrt{\frac{2k_s\varepsilon_0}{q}\frac{N_D}{N_A\left(N_A + N_D\right)}(V_{bi} - V)}$$

What about heterojunctions?

Fields and Depletion at Forward/Reverse Biases



Barrier height is reduced at forward biases

Significant increase of peak field at reverse bias ...

Conclusion

1) Learning to draw band-diagram is one of the most important topic you learn in this course. Band-diagram is a graphical way of quickly solving Poisson equation.

2) If you consistently follow the rules of drawing band-diagrams, you will always get correct results. Try to follow the rules, not guess the final result.