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ECE606: Solid State Devices
Lecture 17: Hall Effect, Diffusion

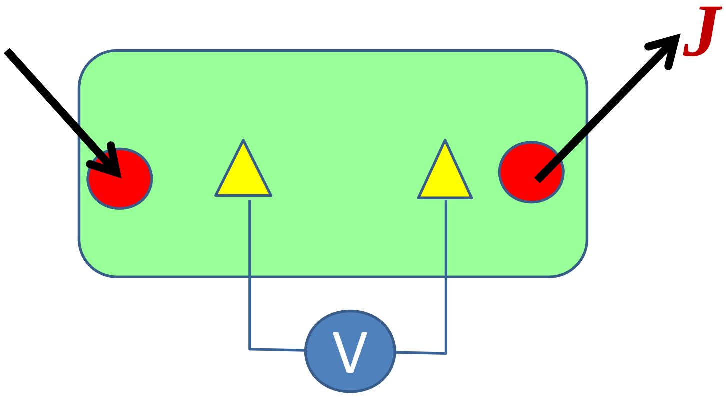
Muhammad Ashraful Alam
alam@purdue.edu

Outline

1) **Measurement of mobility**
2) Hall Effect for determining carrier concentration
3) Physics of diffusion
4) Conclusions

REF: ADF, Chapter 5, pp. 190-202

Problem of mobility measurement …



$$\mathcal{E} = \rho J$$

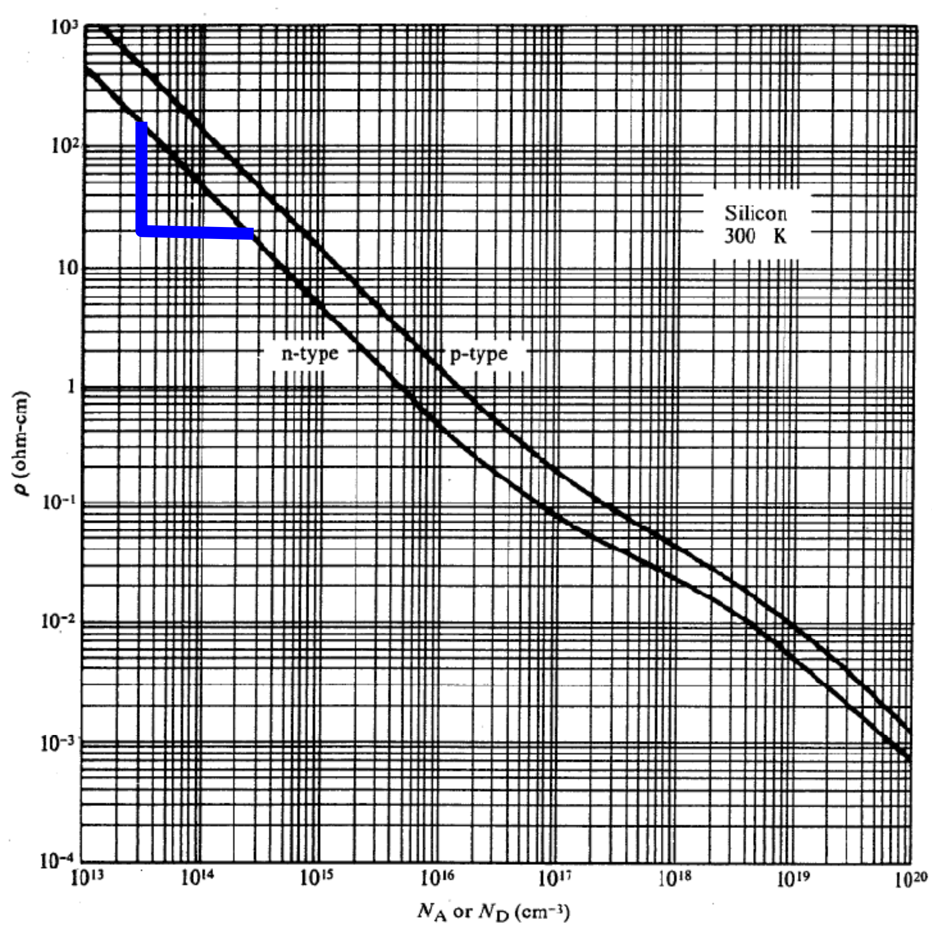
$$J = q(\mu_n n + \mu_p p)\mathcal{E}$$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$$= \frac{1}{q\mu_n N_D} \ldots \text{for n-type}$$

$$= \frac{1}{q\mu_p N_A} \ldots \text{for p-type}$$

?



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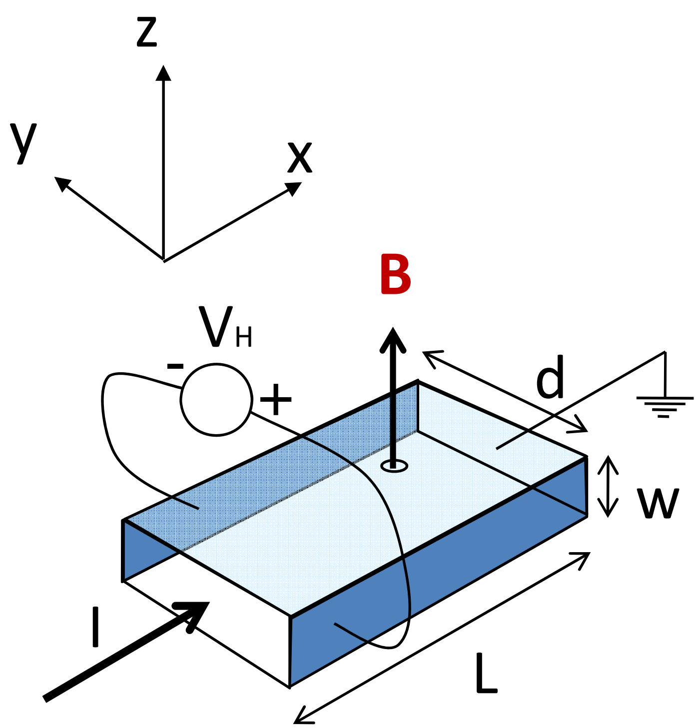
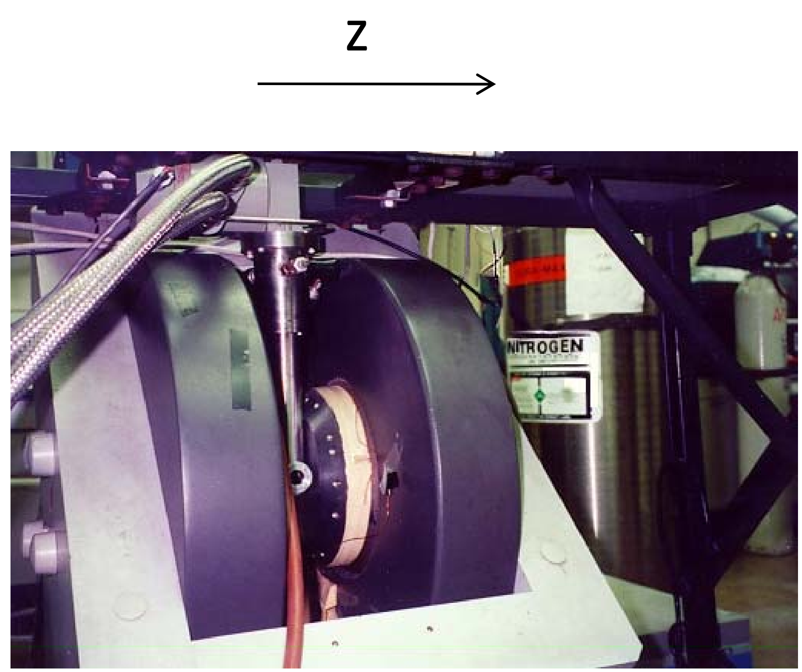
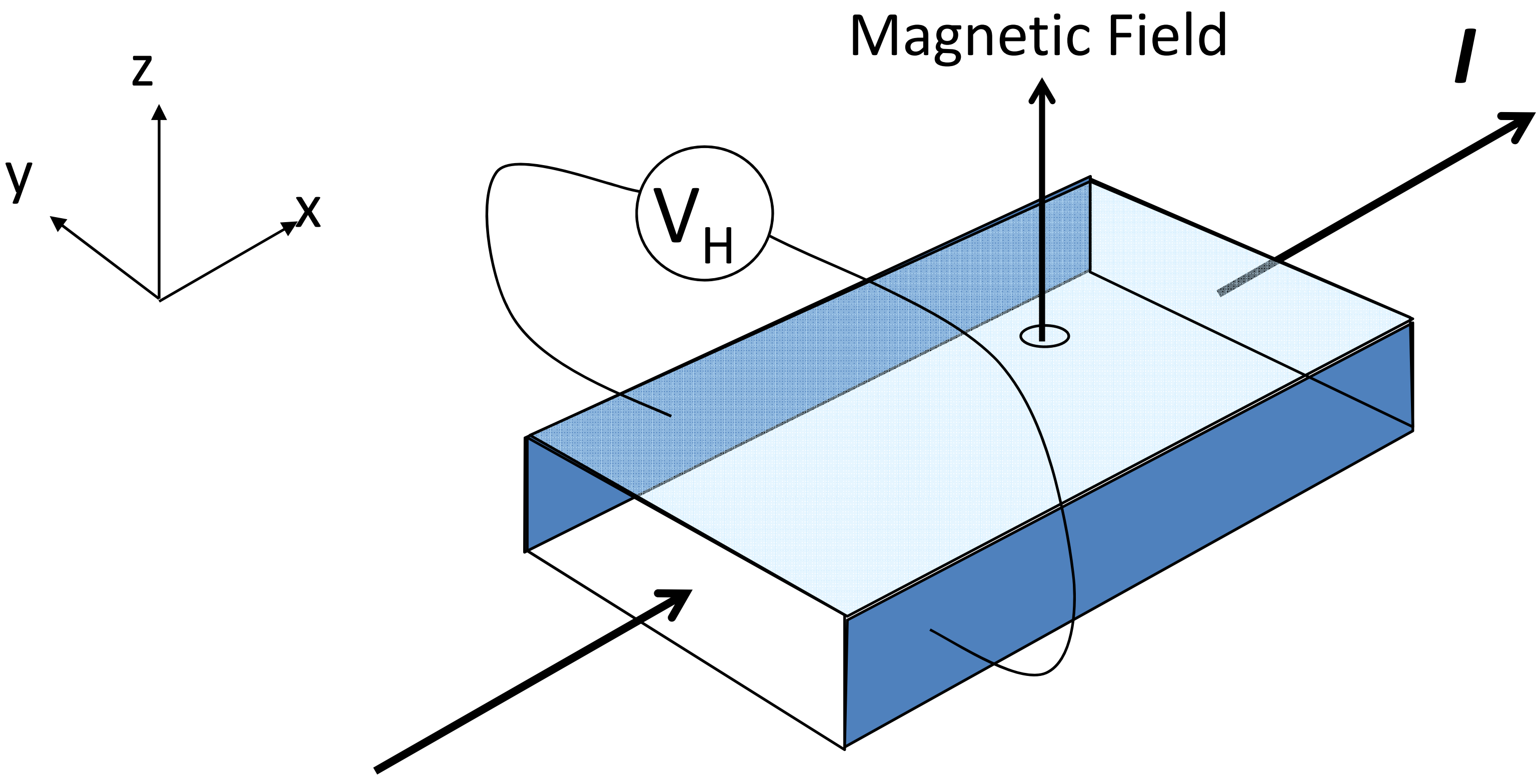
Set up for Hall Measurement





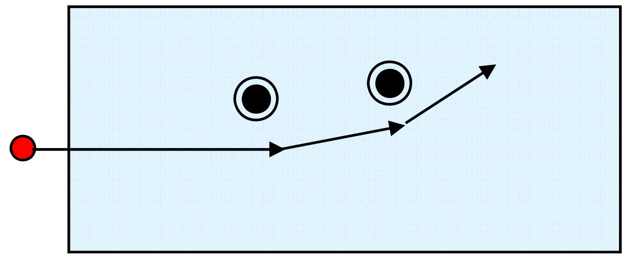
UIC system: 4-300K, 0-1.5 T

Mobility Measurement



Magneto-electric effect
Several Nobel prizes ….

Drude Model



$$-q\mathcal{E} - q\boldsymbol{\upsilon}\times\mathbf{B} - \frac{m^*\boldsymbol{\upsilon}}{\tau} = 0$$

$$m^*\boldsymbol{\upsilon} = -q\tau\mathcal{E} - q\tau\boldsymbol{\upsilon}\times\mathbf{B}$$

$$\approx -q\tau\mathcal{E} - q\tau\left(-\frac{q\tau\mathcal{E}}{m^*}\right)\times\mathbf{B}$$

$$= -q\tau\mathcal{E} + \frac{q^2\tau^2}{m^*}\mathcal{E}\times\mathbf{B}$$

$$\upsilon = -\frac{q\tau\mathcal{E}}{m^*} + \frac{q^2\tau^2}{m^{*2}}\mathcal{E}\times\mathbf{B}$$

Weak **B** field …

$$-q\mathcal{E} - \frac{m^*\boldsymbol{\upsilon}}{\tau} \approx 0$$

$$\upsilon' = \frac{-q\tau\mathcal{E}}{m^*}$$

Perturbation works iff ….

$$\frac{q^2\tau^2 B}{m^* q\tau} = \frac{q\tau B}{m^*} \equiv \tau\omega_c << 1$$

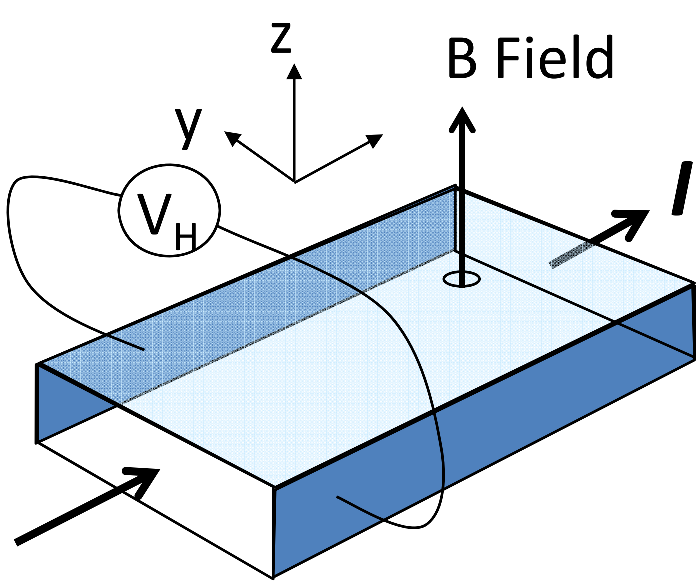
Drude Model & Hall Effect ...

$$\mathbf{J}_n = -qn\boldsymbol{\upsilon}$$

$$= \frac{q^2 n\tau}{m^*}\mathcal{E} - \frac{q^2 n\tau}{m^*}\frac{q\tau}{m^*}\mathcal{E}\times\mathbf{B}$$

$$= \sigma_0\mathcal{E} - \sigma_0\mu\mathcal{E}\times\mathbf{B}$$

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \sigma_0 & -\sigma_0\mu B_z \\ \sigma_0\mu B_z & \sigma_0 \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

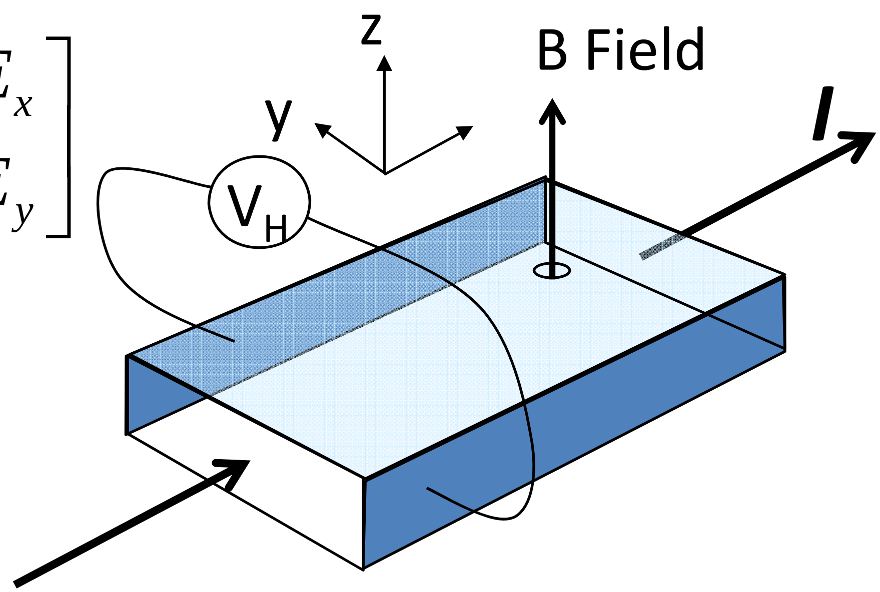


Hall Resistance

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} = \begin{bmatrix} \sigma_0 & -\sigma_0 \mu B_z \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} J_x \\ J_y \end{bmatrix} \approx \begin{bmatrix} \sigma_0 & 0 \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

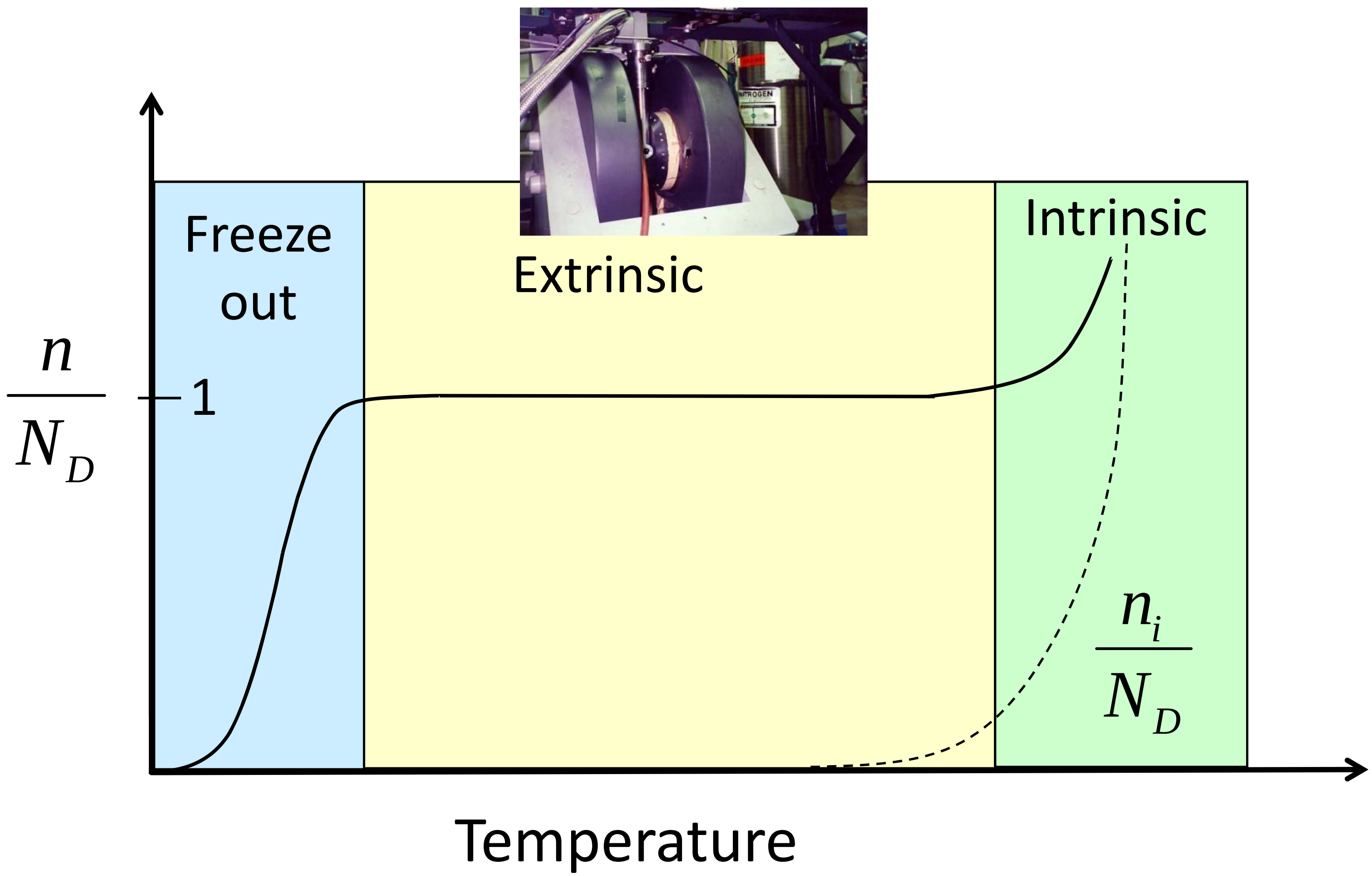
$$\begin{bmatrix} J_x \\ 0 \end{bmatrix} = \begin{bmatrix} \sigma_0 & 0 \\ \sigma_0 \mu B_z & \sigma_0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$



$$R_H = \frac{E_y / B_z}{J_x} = -\frac{1}{qn}$$

between 0.5 and 2 in practice

Temperature-dependent Concentration



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Now the Diffusion term…

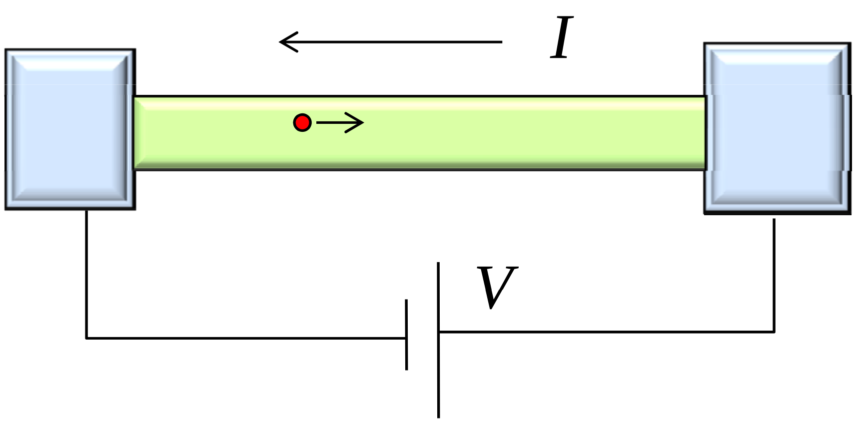
$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

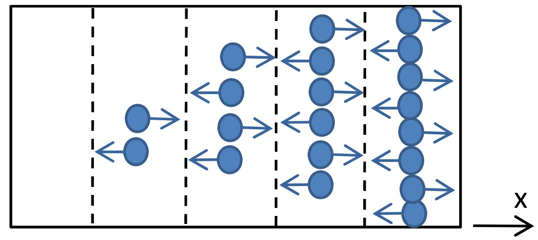
$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

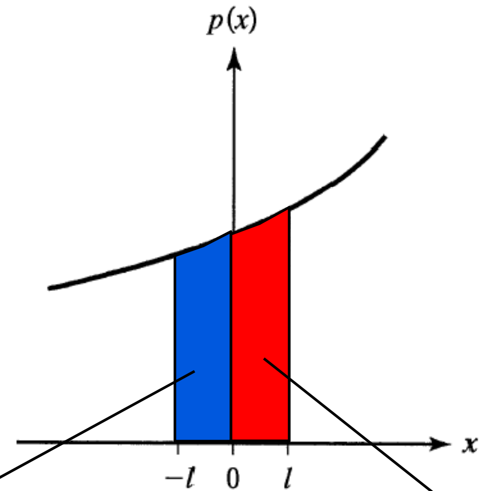
$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$

$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$



Diffusion Flux ...





$$J = \left[-\frac{q}{2}\left(\frac{p(0) + p(0) - \frac{dp}{dx} l}{2} \right) \times l + \frac{q}{2}\left(\frac{p(0) + p(0) + \frac{dp}{dx} l}{2} \right) \times l \right] \Bigg/ \frac{l}{\upsilon_{th}}$$

$$= q \frac{l\upsilon}{2} \frac{dp}{dx} \equiv qD \frac{dp}{dx}$$

Einstein Relationship …

$$\frac{D}{\mu} = \frac{\frac{l\upsilon}{2}}{\frac{q\tau}{m_0^*}} = \frac{\frac{(\upsilon\tau)\times\upsilon}{2}}{\frac{q\tau}{m_0^*}} = \frac{\frac{1}{2}m_0^*\upsilon^2}{q} = \frac{k_B T}{q}$$

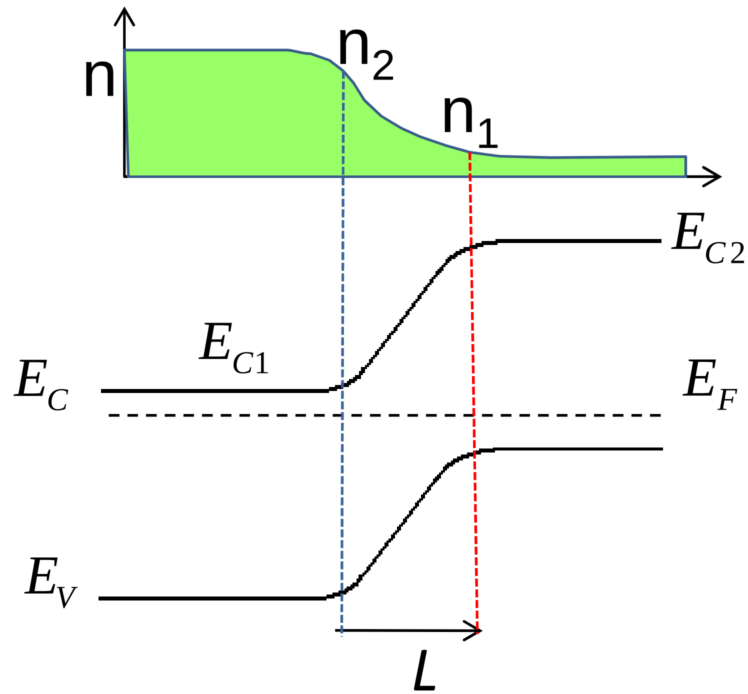
… because scattering dominates both phenomena

Flat Fermi-level defines Equilibrium …

$$J_n = 0 = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$

$$\Rightarrow \frac{1}{n}\frac{dn}{dx} = -\frac{\mu_n\mathcal{E}}{D_n}$$

$$n_2 = n_1 e^{-\int_0^L \frac{\mu_n\mathcal{E}}{D_n}} = n_1 e^{\frac{\mu_n V}{D_n}}$$



$$\frac{n_2}{n_1} = \frac{N_C e^{-(E_{C2}-E_F)/kT}}{N_C e^{-(E_{C1}-E_F)/kT}} = e^{-(E_{C2}-E_{C1})/kT} = e^{qV/kT}$$

$$\frac{qV}{kT} = \frac{\mu_n V}{D_n} \Rightarrow \frac{q}{kT} = \frac{\mu_n}{D_n}$$

Similar to relationship between c_n and e_n discussed in Chapter 5

Conclusion

1) Measurement of mobility and carrier concentration is particularly important for analysis of semiconductor devices.
2) Drift, diffusion, and recombination-generation constitute the elemental processes in semiconductor device physics.
3) We will put the pieces together in the next class.