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# ECE606: Solid State Devices
## Lecture 16: Carrier Transport

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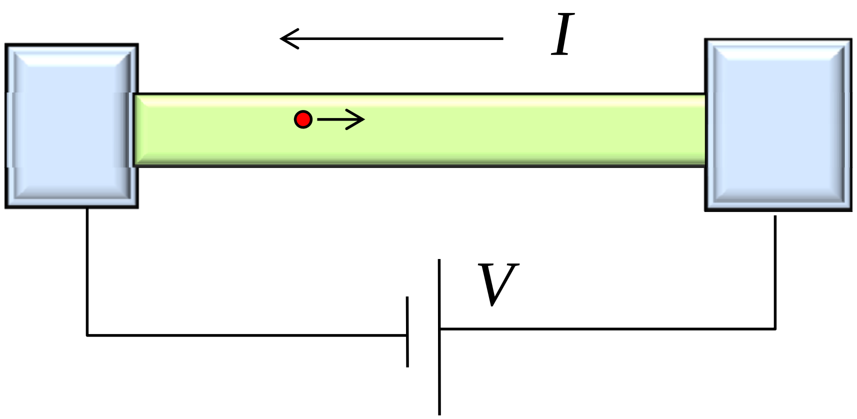
# Outline

1) **Overview**
2) Drift Current
3) Physics of Mobility
4) High field effects
5) Conclusion

REF: Advanced Device Fundmentals, Pages 175- 192

# Current Flow Through Semiconductors




$$I = G \times V$$

$$= q \times n \times v \times A$$

Carrier Density (n)

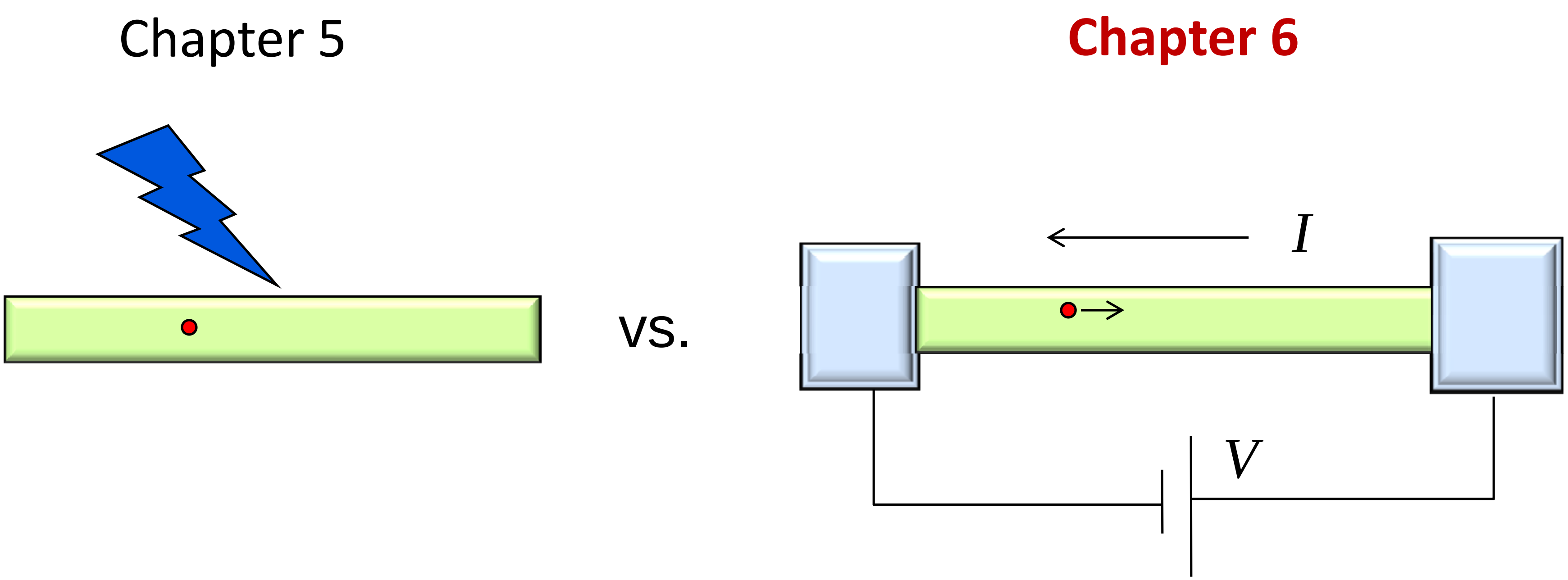
velocity (v)

Depends on chemical composition, crystal structure, temperature, doping, etc.

**Quantum Mechanics + Equilibrium Statistical Mechanics**
$\Rightarrow$ Encapsulated into concepts of effective masses and occupation factors (Ch. 1-4)

**Transport with scattering, non-equilibrium Statistical Mechanics**
$\Rightarrow$ Encapsulated into drift-diffusion equation with recombination-generation (Ch. 5 & 6)

# Non-equilibrium Systems



# Summary of Transport Equations …

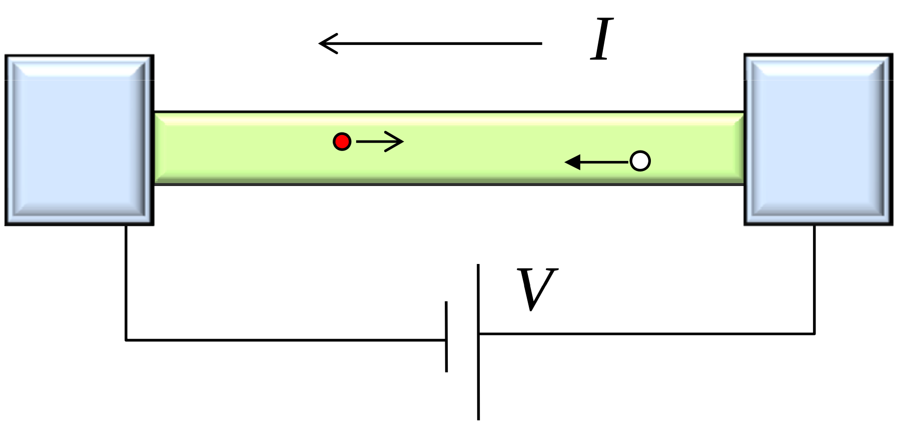
$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$



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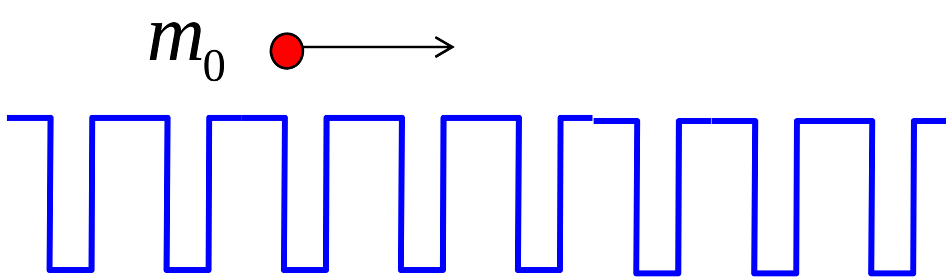
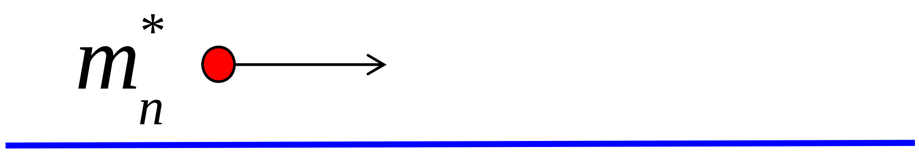
# Meaning of Effective Mass …




$$\left(-\frac{\hbar^2}{2m_0}\frac{d^2}{dx^2}+U_{crys}(x)+U_{ext}(x)\right)\psi = E\psi$$



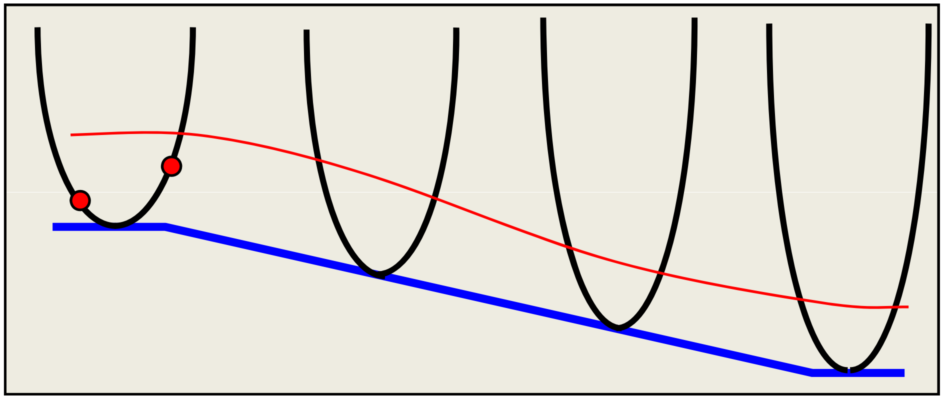

$$\left(-\frac{\hbar^2}{2m_0^*}\frac{d^2}{dx^2}+U_{ext}(x)\right)\phi = E\phi$$

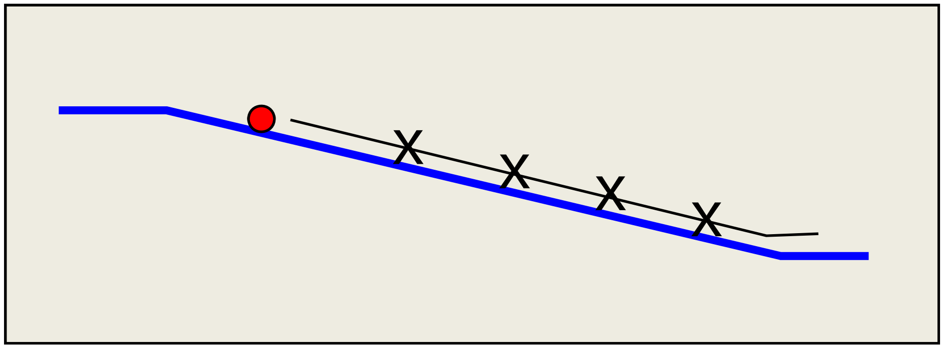
# Drift by Electric field ….

$$J_n = qn\mu_n\mathcal{E}$$

$$\frac{d(m_n^*\upsilon)}{dt} = -q\mathcal{E} - \frac{m_n^*\upsilon}{\tau_n}$$

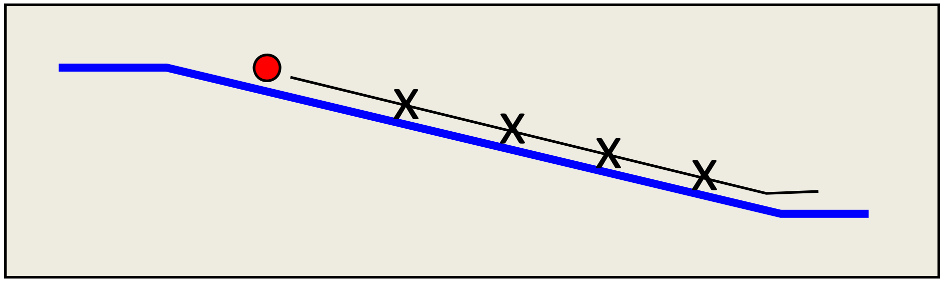
$$\upsilon(t) = -\frac{q\tau_n}{m_n^*}\mathcal{E}\left[1 - e^{-\frac{t}{\tau_n}}\right]$$





# Drift by Electric field ….

$$\upsilon(t) = -\frac{q\tau_n}{m_n^*}\mathcal{E}\left[1 - e^{-\frac{t}{\tau_n}}\right]$$

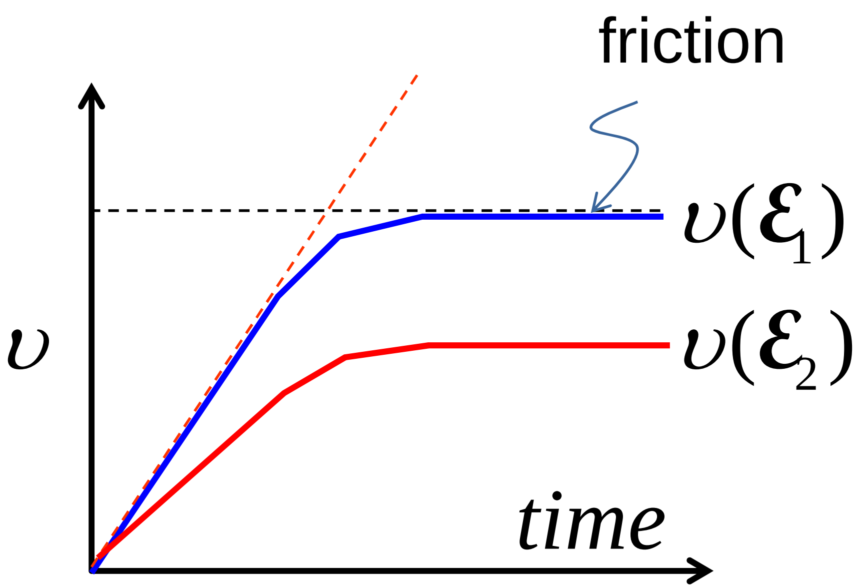


$$= -\frac{q\tau_n}{m_n^*}\mathcal{E} \quad (t \rightarrow \infty,\ \text{1-2 ps})$$

$$\equiv \mu_n \mathcal{E}$$

$$J_n = qn\mu_n\mathcal{E}$$

(Theory valid once t > 1-2 ps)



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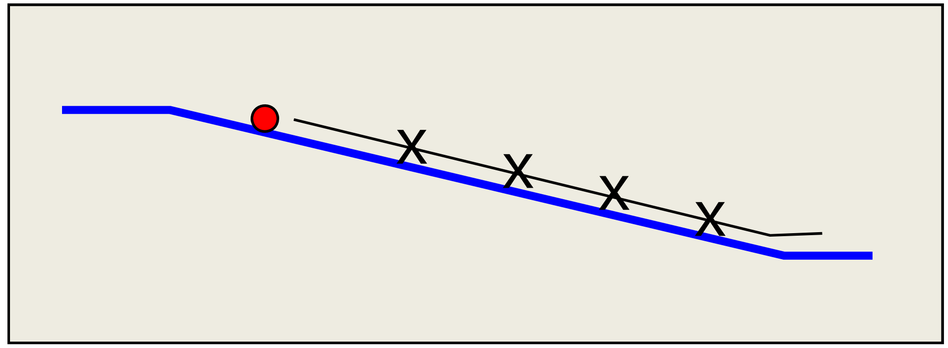
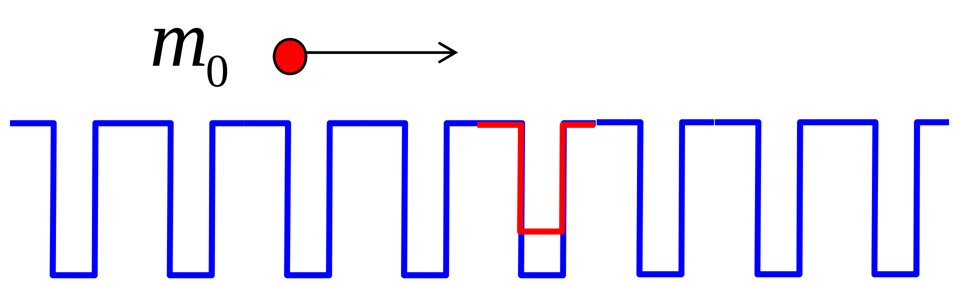
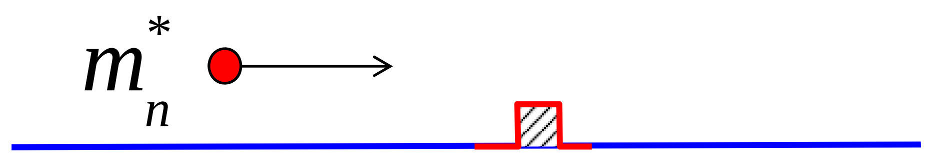
# Mobility and Physics of Scattering Time



$$\mu_n = \frac{q\tau_n}{m_n^*}$$




Fermi's Golden rule …

$$\tau_n^{-1} \sim \left| \frac{2\pi}{\hbar} \int_{-\infty}^{\infty} \psi^*(x) U(x) \psi(x) dx \right|^2$$

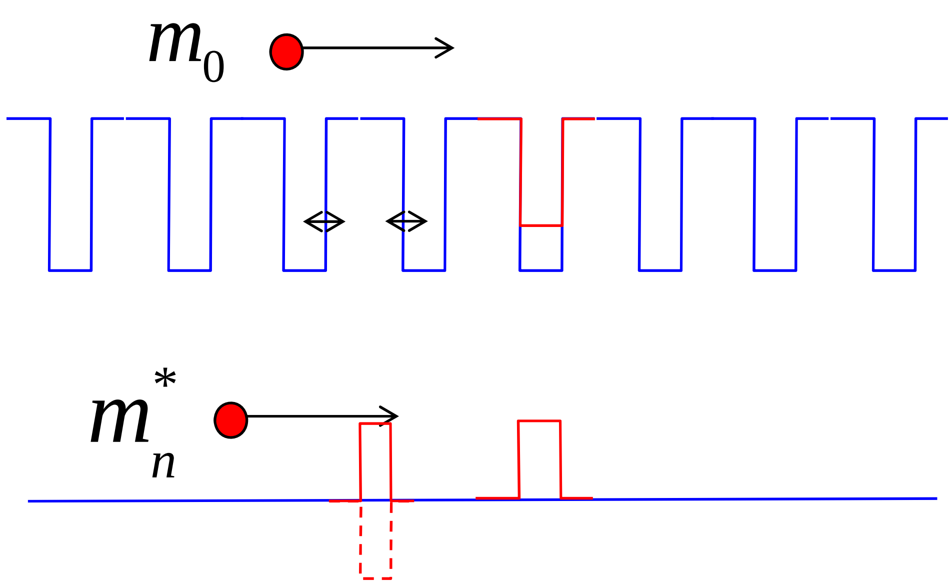
# Phonon and Ionized Impurity Scattering

Ionized impurity

$$\tau_n \sim \frac{T^{3/2}}{N_D}$$

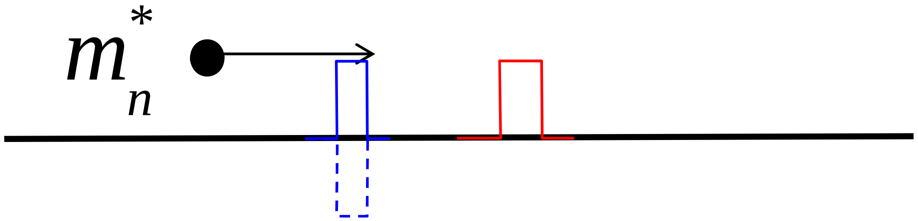
Higher temperature,
more phonon scattering

$$\tau_n \sim T^{-3/2}$$



# Multiple Scattering Events

- Ionized impurity
- Phonon scattering
- others ….



$$\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}$$

$$\Rightarrow \mu_n = \frac{\mu_{ph}\mu_{II}}{\mu_{ph} + \mu_{II}}$$

$$= \mu_{\min} + \left( \frac{\mu_{ph}\mu_{II}}{\mu_{ph} + \mu_{II}} - \mu_{\min} \right)$$

$$= \mu_{\min} + \left( \frac{\mu_0}{1 + \left( N_I / N_0 \right)^{\alpha}} \right)$$

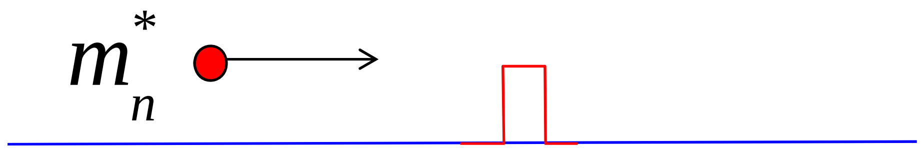
$$\frac{1}{\tau_n} = \frac{1}{\tau_{II}} + \frac{1}{\tau_{ph}} + \frac{1}{\tau_s} + \cdots$$

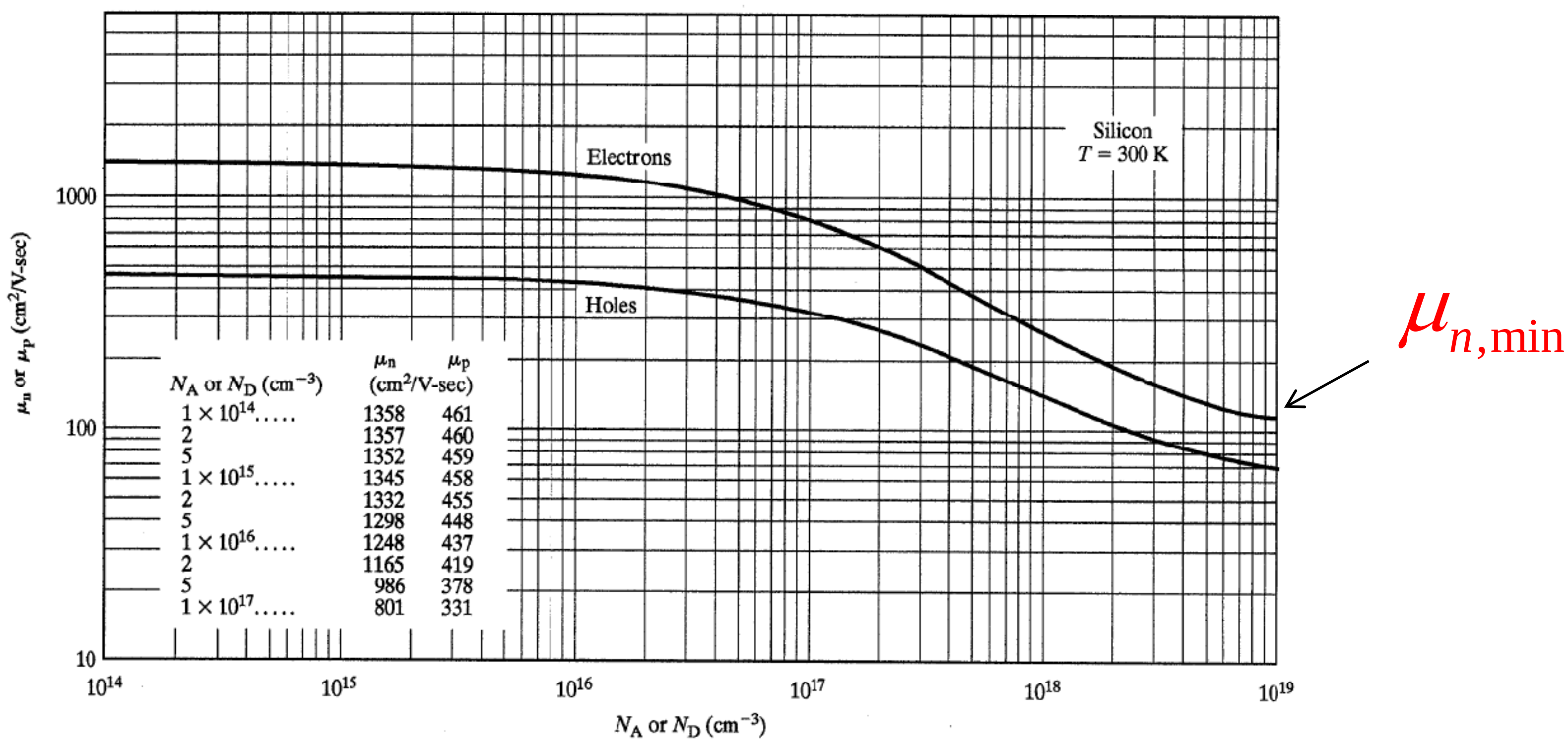
$$\frac{1}{\mu_n} = \frac{m_n^*}{q\tau_n}$$

Matthession Rule ….

# Model for Ionized impurity Scattering

$$\mu_n = \mu_{n,\min} + \left( \frac{\mu_{0,n}}{1 + \left( N_I / N_{0,n} \right)^{\alpha_n}} \right)$$

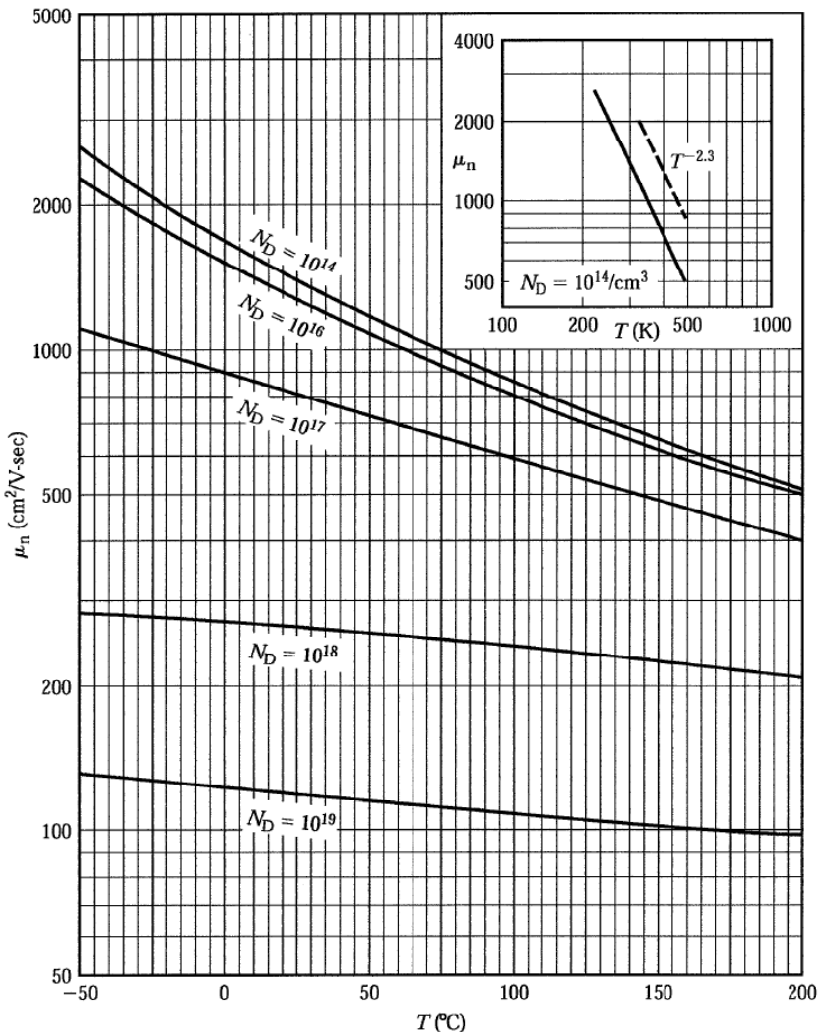



| $N_A$ or $N_D$ (cm$^{-3}$) | $\mu_n$ (cm$^2$/V-sec) | $\mu_p$ (cm$^2$/V-sec) |
|---|---|---|
| $1 \times 10^{14}$..... | 1358 | 461 |
| 2 | 1357 | 460 |
| 5 | 1352 | 459 |
| $1 \times 10^{15}$..... | 1345 | 458 |
| 2 | 1332 | 455 |
| 5 | 1298 | 448 |
| $1 \times 10^{16}$..... | 1248 | 437 |
| 2 | 1165 | 419 |
| 5 | 986 | 378 |
| $1 \times 10^{17}$..... | 801 | 331 |

# Temperature-dependent Mobility

$$\mu_n \sim \tau_n \sim T^{-3/2}$$



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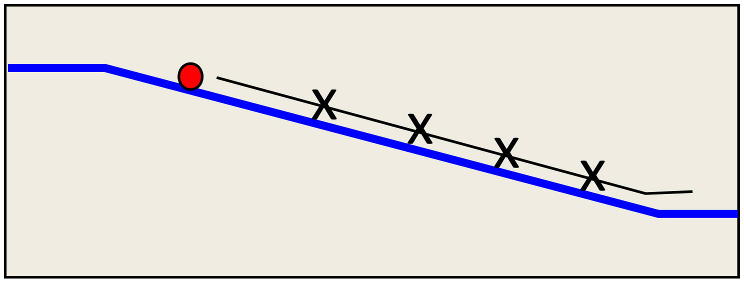
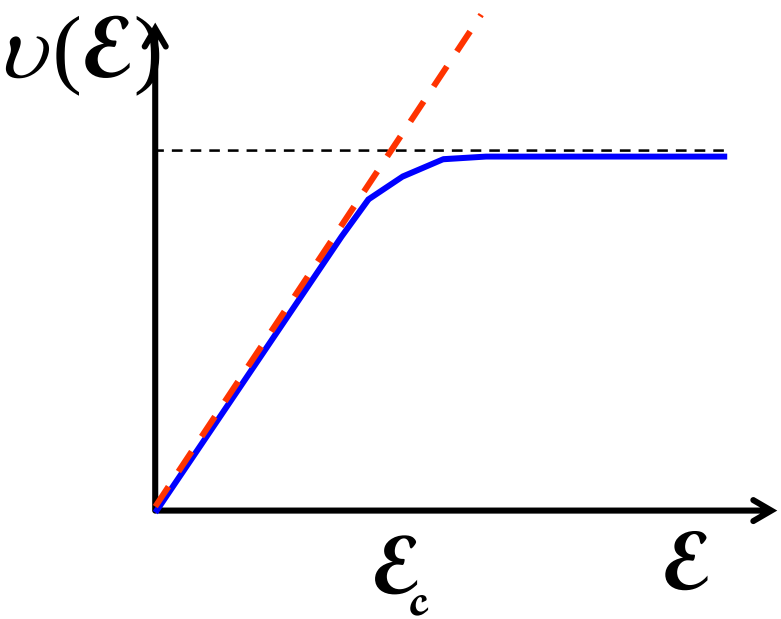
# Mobility at High Fields?



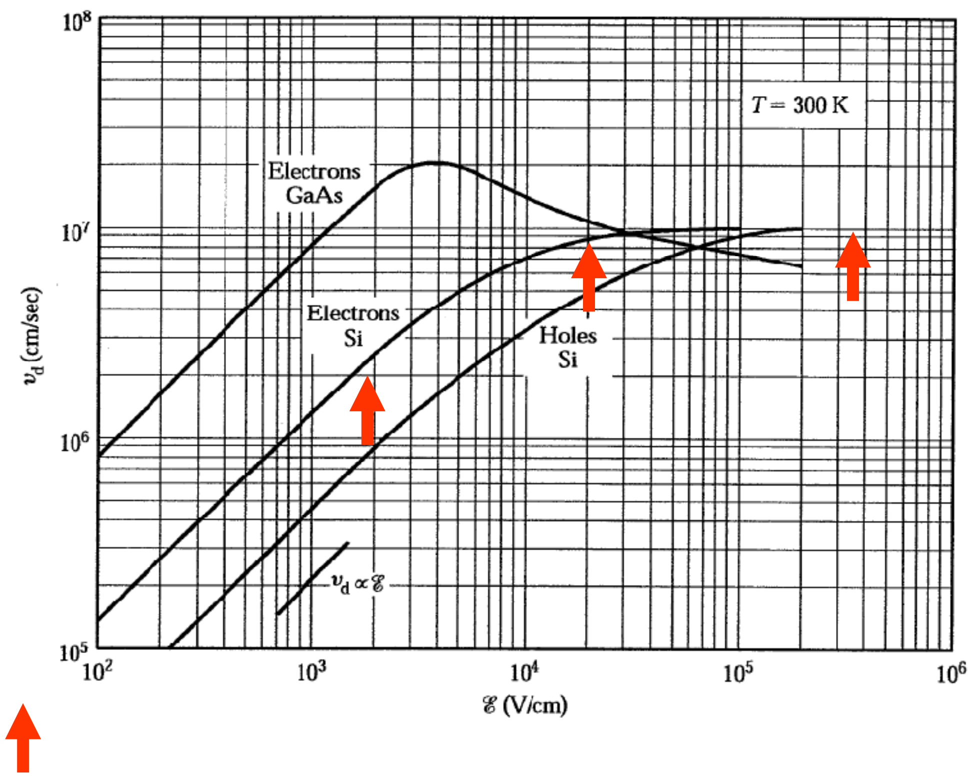
$$\upsilon = \frac{q\tau_N}{m_N^*}\mathcal{E}$$

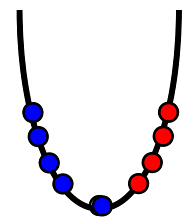



What causes velocity saturation at high fields?

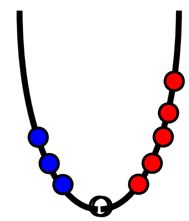
Where does all the mobility formula in device simulator come from?

# Velocity Saturation in Si/Ge

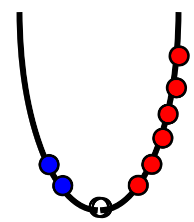


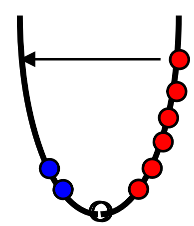
$\mathcal{E} = 0 \quad J_1 = J^+ - J^- = 0$



$\mathcal{E} \ll \mathcal{E}_c \quad J_2 = J^+ - J^- > J_1$

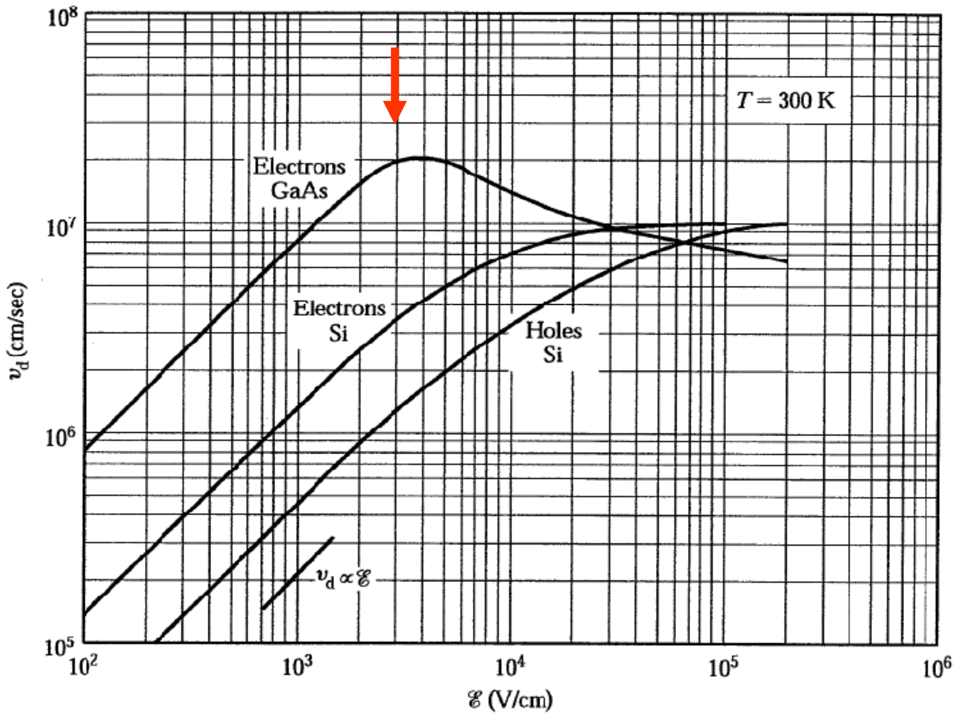


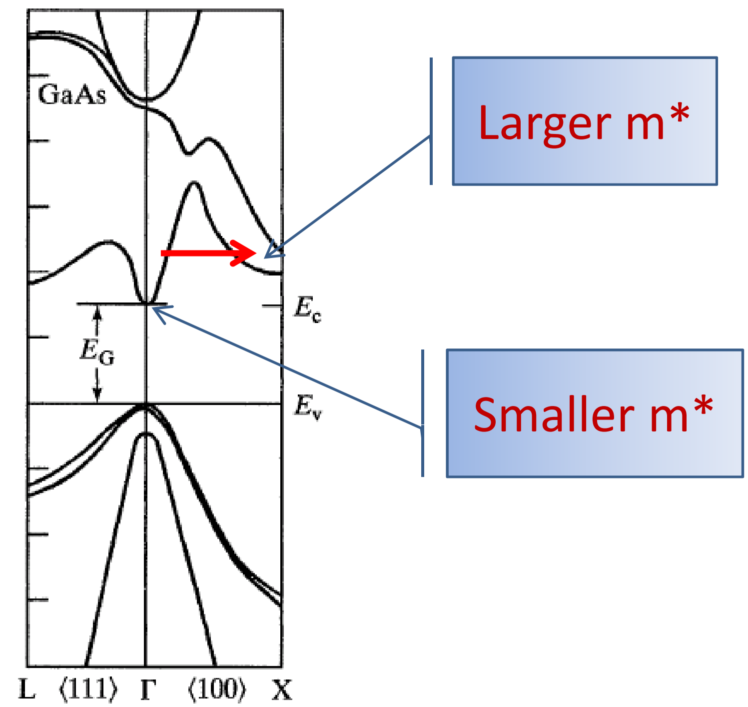
$\mathcal{E} \approx \mathcal{E}_c \quad J_3 = J^+ - J^- > J_2$



$\mathcal{E} \gg \mathcal{E}_c \quad J_4 = J^+ - J^- \approx J_3$

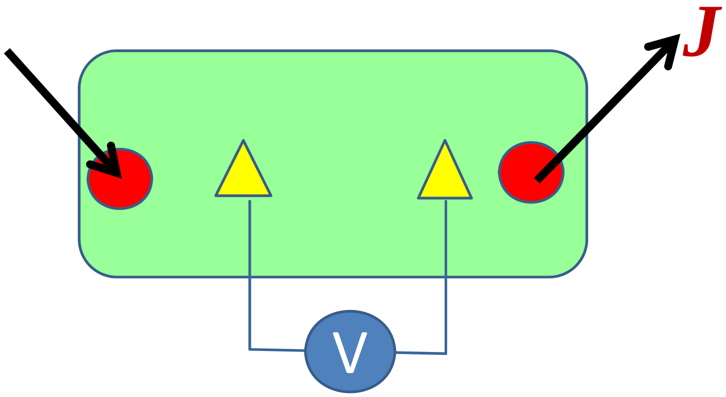
# Velocity Overshoot & Inter-valley Transfer




What type of scattering would you need for inter-valley transfer?

# Doping dependent Resistivity



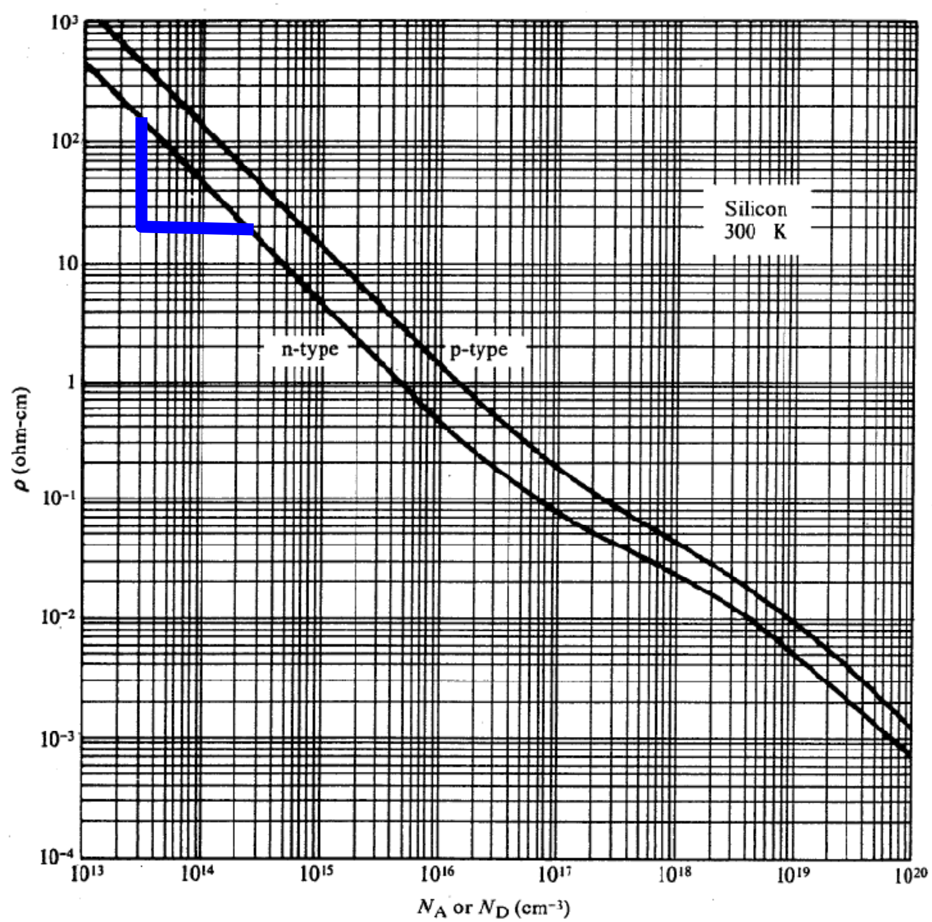

$$\mathcal{E} = \rho J$$

$$J = q(\mu_n n + \mu_p p)\mathcal{E}$$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$$= \frac{1}{q\mu_n N_D} \ldots \text{for n-type}$$

$$= \frac{1}{q\mu_p N_A} \ldots \text{for p-type}$$



# Conclusion

1) Poisson and drift-diffusion equations form a complete semi-classical transport model that can explain wide variety of device phenomena.

2) Drift current results from response of electrons/holes to electric field. The physics of mobility is complex and material dependent.

3) Constancy of low-field mobility can be checked by experiments.