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ECE606: Solid State Devices
Lecture 14: Bulk Recombination

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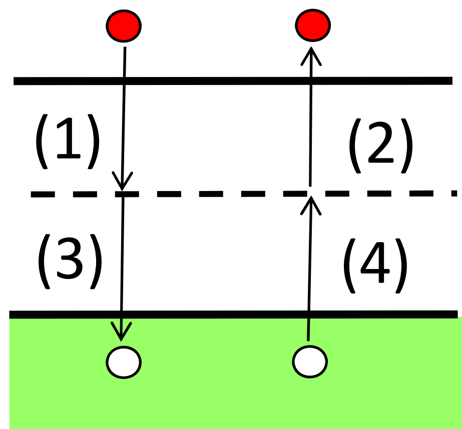
Outline

1) **Derivation of SRH formula**
2) Application of SRH formula for special cases
3) Direct and Auger recombination
4) Conclusion

Ref. ADF, Chapter 5, pp. 141-154

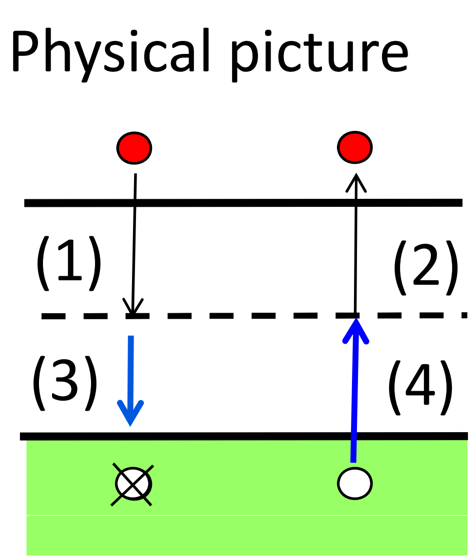
Sub-processes of SRH Recombination

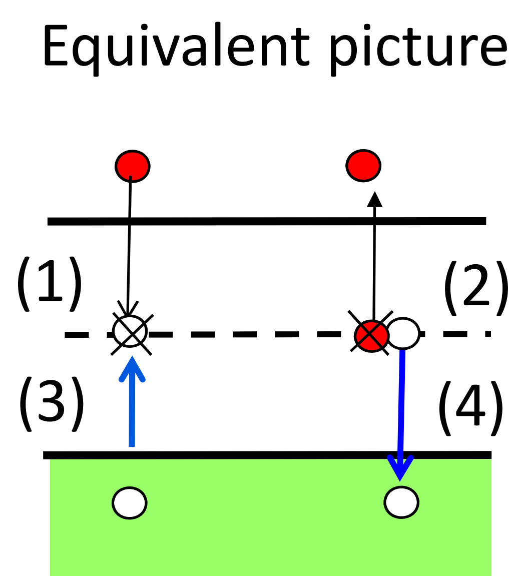


(1)+(3): one electron reduced from Conduction-band & one-hole reduced from valence-band

(2)+(4): one hole created in valence band and one electron created in conduction band

SRH Recombination

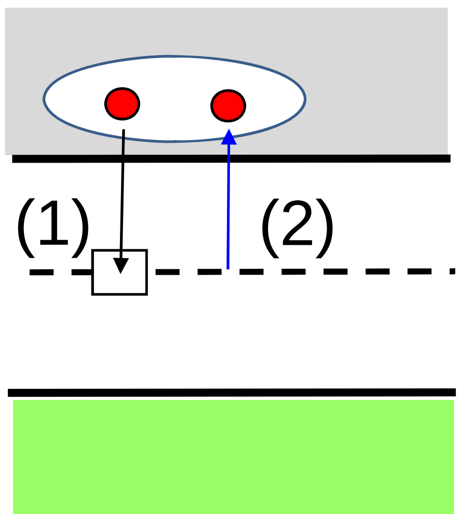


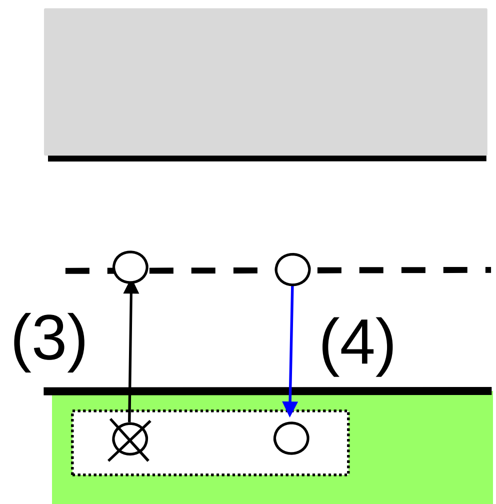


(1)+(3): one electron reduced from C-band &
one-hole reduced from valence-band

(2)+(4): one hole created in valence band &
one electron created in conduction band

Changes in electron and hole Densities

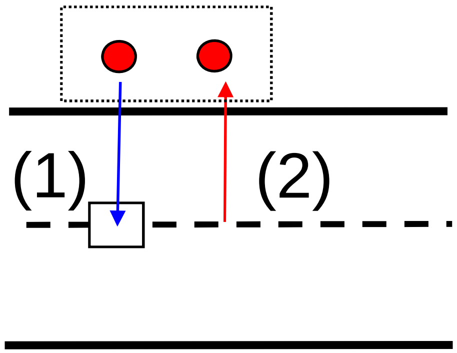


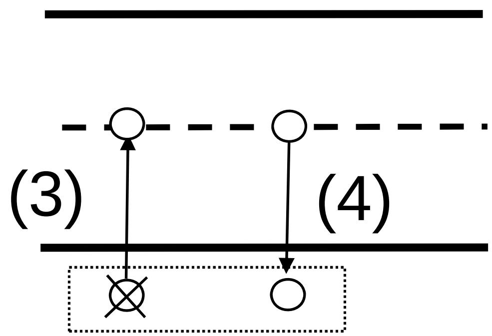


$$\left.\frac{\partial n}{\partial t}\right|_{1,2} = -c_n\, n\, p_T + e_n n_T \left(1 - f_c\right)$$

$$\left.\frac{\partial p}{\partial t}\right|_{3,4} = -c_p\, p\, n_T + e_p\, p_T\, f_\upsilon$$

Detailed Balance in *Equilibrium*





$$\left.\frac{\partial n}{\partial t}\right|_{1,2} = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$0 = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$e_n = c_n \frac{n_0 p_{T0}}{n_{T0}} \equiv c_n n_1$$

$$0 = -c_n \left(n_0 p_{T0} - n_{T0} n_1 \right)$$

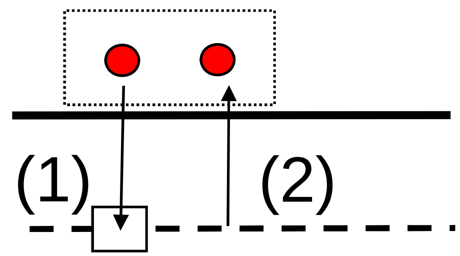
$$\left.\frac{\partial p}{\partial t}\right|_{3,4} = -c_p \, p \, n_T \quad + e_p p_T$$

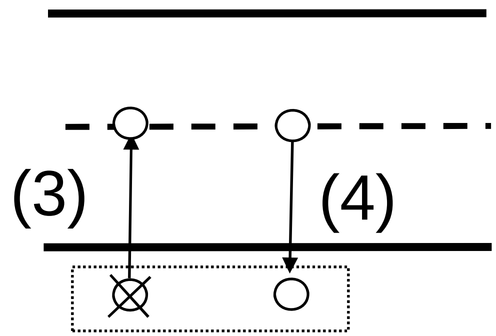
$$0 = -c_p p_0 n_{T0} + p_{T0} e_p$$

$$e_p \equiv \frac{c_p p_0 n_{T0}}{p_{T0}} = c_p p_1$$

$$0 = -c_p \left(p_0 n_{T0} - p_{T0} p_1 \right)$$

Expressions for (n_1) and (p_1)



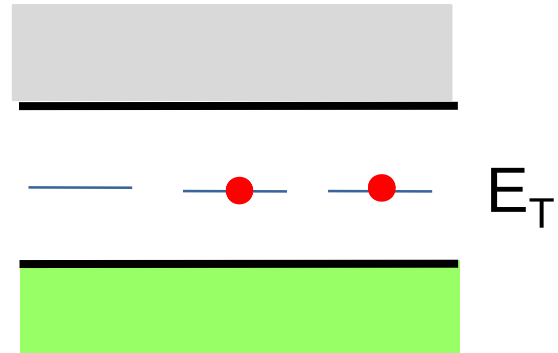


$$n_1 = \frac{n_0 p_{T0}}{n_{T0}} \qquad p_1 = \frac{p_0 n_{T0}}{p_{T0}}$$

$$n_1 p_1 = \frac{n_0 p_{T0}}{n_{T0}} \times \frac{p_0 n_{T0}}{p_{T0}} = n_0 p_0 = n_i^2$$

Expressions for (n_1) and (p_1)

$$n_{T0} = N_T\left(1 - f_{00}\right) = \frac{N_T}{1 + g_D e^{\beta(E_T - E_F)}}$$



$$n_1 = \frac{n_0 p_{T0}}{n_{T0}} \quad = n_0 \frac{\left(N_T f_{00}\right)}{N_T\left(1 - f_{00}\right)}$$

$$n_1 = n_i e^{\beta(E_F - E_i)}\left[1 + g_D e^{\beta(E_T - E_F)} - 1\right]$$

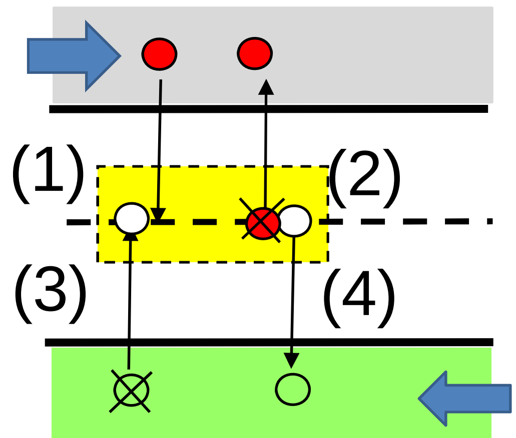
$$= n_i g_D e^{\beta(E_T - E_i)}$$

$$p_1 n_1 = n_i^2$$

$$p_1 = n_i^2 / n_1$$

$$= n_i g_D^{-1} e^{\beta(E_i - E_T)}$$

Dynamics of Trap Population

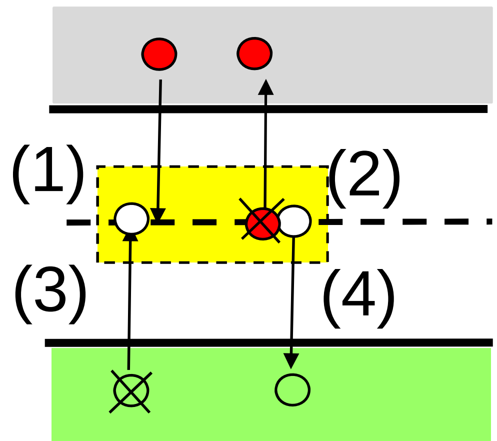


$$\frac{\partial n_T}{\partial t} = -\left.\frac{\partial n}{\partial t}\right|_{1,2} + \left.\frac{\partial p}{\partial t}\right|_{3,4}$$

$$= c_n n\, p_T - e_n n_T - c_p\, p\, n_T + e_p p_T$$

$$= c_n \left(n p_T - n_T n_1 \right) - c_p \left(p\, n_T - p_T p_1 \right)$$

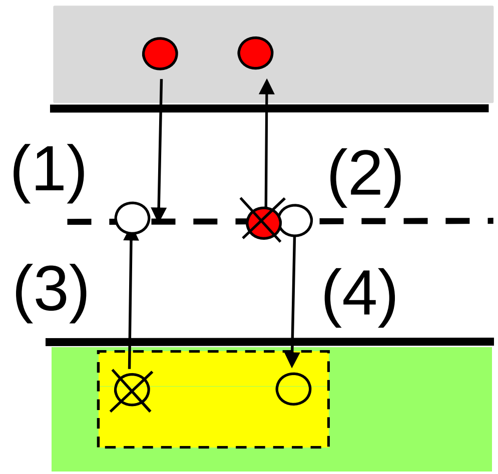
Steady-state Trap Population



$$\frac{\partial n_T}{\partial t} = 0 = c_n\left(np_T - n_T n_1\right) - c_p\left(p\, n_T - p_T\, p_1\right)$$

$$n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n\left(n + n_1\right) + c_p\left(p + p_1\right)} = c_n\left(np_T - n_T n_1\right)$$

Net Rate of Recombination-Generation



$$R = -\frac{dp}{dt} = c_p \left(p\, n_T - p_T\, p_1 \right)$$

$$= \frac{np - n_i^2}{\underbrace{\left(\frac{1}{c_p N_T}\right)}_{\tau_n}(n + n_1) + \underbrace{\left(\frac{1}{c_n N_T}\right)}_{\tau_p}(p + p_1)}$$

Outline

1) Derivation of SRH formula
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Case 1: Low-level Injection in p-type

$$R = \frac{np - n_i^2}{\tau_p\left(n + n_1\right) + \tau_n\left(p + p_1\right)}$$

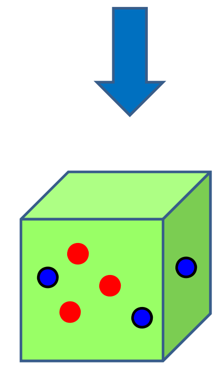
$$= \frac{\left(n_0 + \Delta n\right)\left(p_0 + \Delta n\right) - n_i^2}{\tau_p\left(n_0 + \Delta n + n_1\right) + \tau_n\left(p_0 + \Delta p + p_1\right)}$$

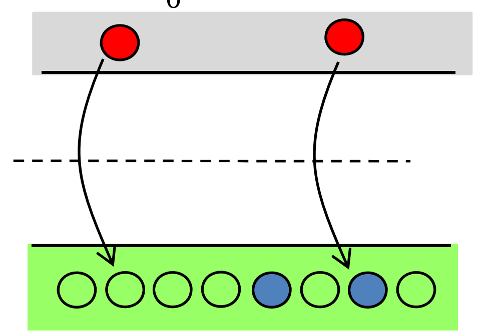
$$= \frac{\Delta n\left(n_0 + p_0\right) + \cancel{\Delta n^2}}{\tau_p\left(\cancel{n_0} + \Delta n + n_1\right) + \tau_n\left(p_0 + \Delta p + p_1\right)}$$

$$= \frac{\Delta n\left(p_0\right)}{\tau_n\left(p_0\right)} = \frac{\Delta n}{\tau_n}$$

$$\Delta n^2 \approx 0$$

$$p_0 \gg \Delta n \gg n_0$$





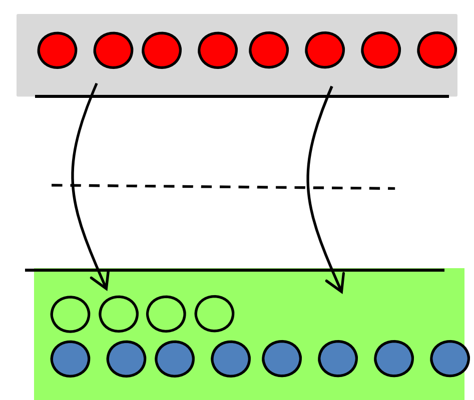
Case 2: High-level Injection

$$R = \frac{np - n_i^2}{\tau_p\left(n + n_1\right) + \tau_n\left(p + p_1\right)}$$

e.g. organic solar cells

$$= \frac{\left(n_0 + \Delta n\right)\left(p_0 + \Delta p\right) - n_i^2}{\tau_p\left(n_0 + \Delta n + n_1\right) + \tau_n\left(p_0 + \Delta p + p_1\right)}$$

$n_0 + \Delta n$



$p_0 + \Delta n$

$$= \frac{\cancel{\Delta n}\left(n_0 + p_0\right) + \Delta n^2}{\tau_p\left(\cancel{n_0} + \Delta n + n_1\right) + \tau_n\left(\cancel{p_0} + \Delta n + p_1\right)}$$

$$= \frac{\Delta n^2}{\left(\tau_n + \tau_p\right)\Delta n} = \frac{\Delta n}{\left(\tau_n + \tau_p\right)}$$

$$\Delta n \gg p_0 \gg n_0$$

High/Low Level Injection …

$$R_{high} = \frac{\Delta n}{\left(\tau_n + \tau_p\right)} \qquad \Delta n \gg p_0 \gg n_0$$

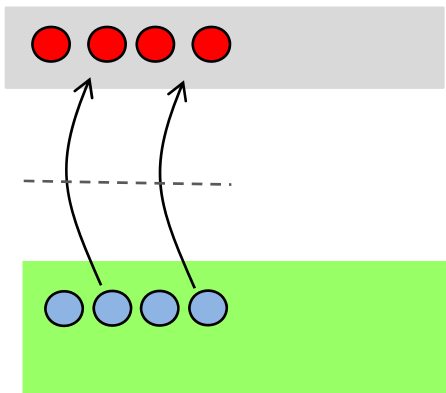
$$R_{low} = \frac{\Delta n}{\tau_p} \qquad p_0 \gg \Delta n \gg n_0$$

which one is larger and why?

Case 3: Generation in Depletion Region

$$R = \frac{\cancel{n}p - n_i^2}{\tau_p\left(\cancel{n} + n_1\right) + \tau_n\left(\cancel{p} + p_1\right)}$$

$$= \frac{-n_i^2}{\tau_p\left(n_1\right) + \tau_n\left(p_1\right)}$$



$$n \ll n_1 \qquad p \ll p_1$$

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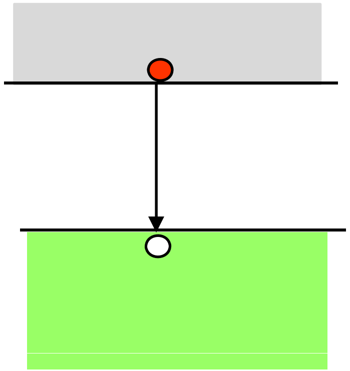
Band-to-band Recombination

$$R = B\left(np - n_i^2\right)$$



Direct recombination at low-level injection

$$n_0 \ll \left(\Delta n = \Delta p\right) \ll p_0$$

$$R = B\left[\left(n_0 + \Delta n\right)\left(p_0 + \Delta p\right) - n_i^2\right] \approx Bp_0 \times \Delta n$$

Direct generation in depletion region

$$n, p \sim 0$$

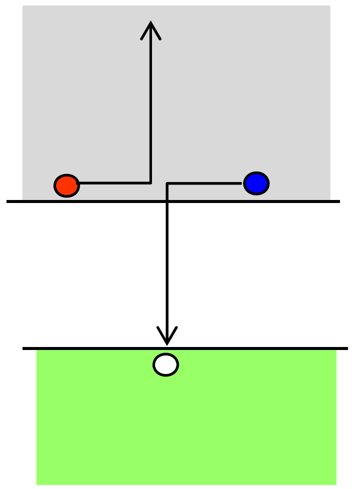
$$R = B\left(np - n_i^2\right) \approx -Bn_i^2$$

Auger Recombination

2 electron & 1 hole

$$R = c_n\left(n^2 p - n_i^2 n\right) + c_p\left(np^2 - n_i^2 p\right)$$

$$c_n, c_p \sim 10^{-29}\ \text{cm}^6/\text{sec}$$

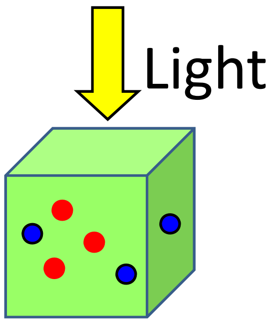


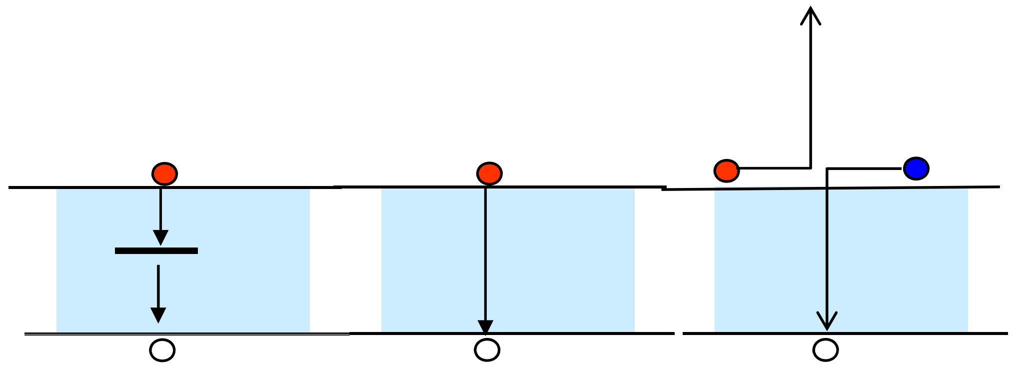
Auger recombination at low-level injection

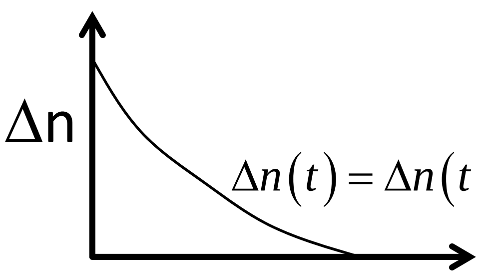
$$n_0 \ll \left(\Delta n = \Delta p\right) \ll \left(p_0 = N_A\right)$$

$$R \approx c_p N_A^2 \Delta n = \frac{\Delta n}{\tau_{auger}} \qquad \tau_{auger} = \frac{1}{c_p N_A^2}$$

Effective Carrier Lifetime







$$\Delta n(t) = \Delta n(t=0)\, e^{-\frac{t}{\tau_{eff}}}$$

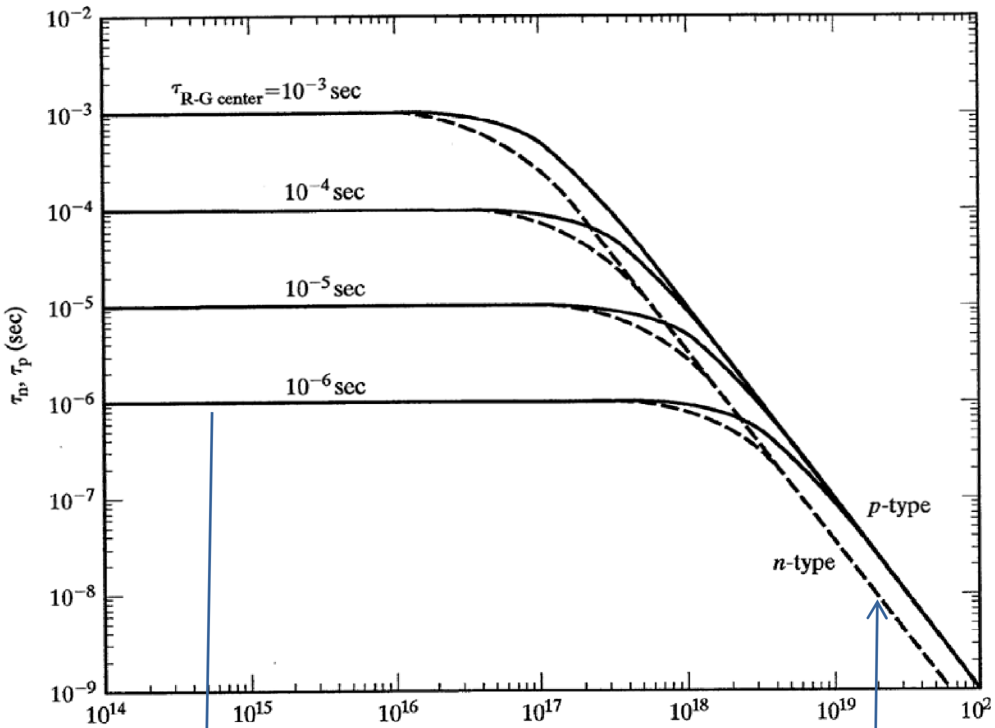
$$R = R_{SRH} + R_{direct} + R_{Auger}$$

$$= \Delta n\left(\frac{1}{\tau_{SRH}} + \frac{1}{\tau_{direct}} + \frac{1}{\tau_{Auger}}\right)$$

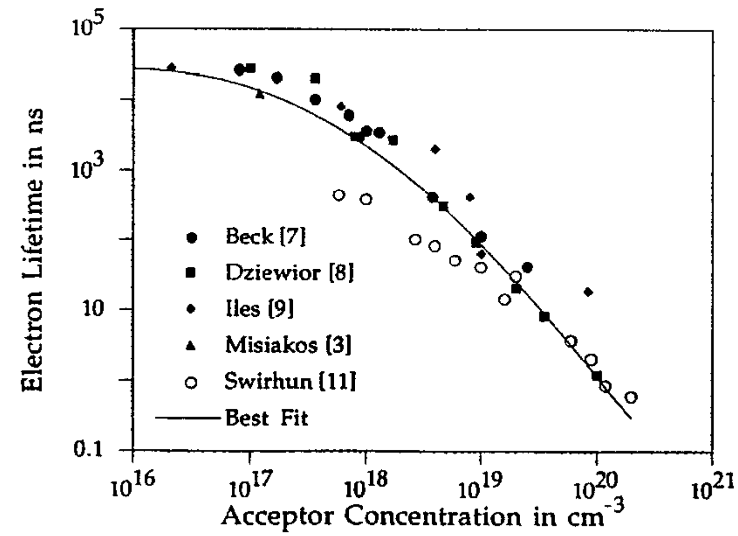
$$= \Delta n\left(c_n N_T + B N_D + c_{n,auger} N_D^2\right)$$

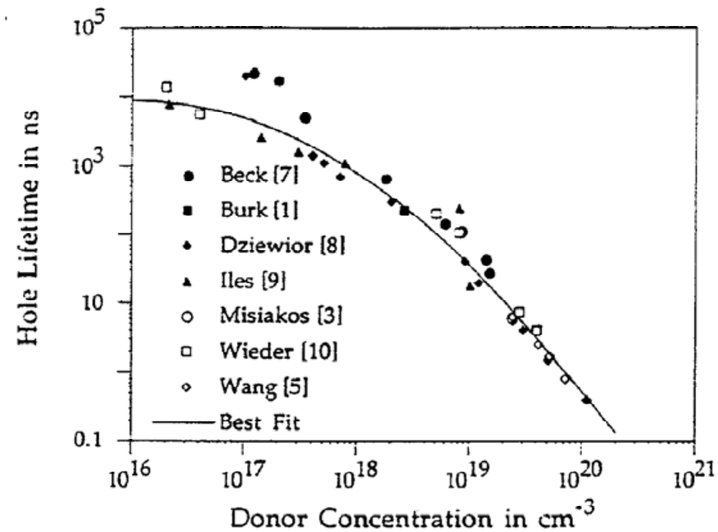
$$\tau_{eff} = \left(c_n N_T + B N_D + c_{n,auger} N_D^2\right)^{-1}$$

Effective Carrier Lifetime with all Processes



N_D





$$\tau_{eff} \approx c_{n,auger} N_D^{-2}$$

$$\tau_{eff} = \left(c_n N_T + BN_D + c_{n,auger} N_D^2\right)^{-1}$$

Elec. Dev. Lett., 12(8), 1991.

Conclusion

SRH is an important recombination mechanism in important semiconductors like Si and Ge.

SRH formula is complicated, therefore simplification for special cases are often desired.

Direct band-to-band and Auger recombination can also be described with simple phenomenological formula.

These expressions for recombination events have been widely validated by measurements.