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EE-606: Solid State Devices
Lecture 12: Equilibrium Concentrations

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Outline

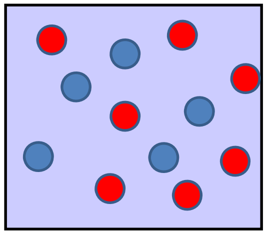
1) **Carrier concentration**
2) Temperature dependence of carrier concentration
3) Multiple doping, co-doping, and heavy-doping
4) Conclusion

Ref. ADF Chapter 4, pages 118-128

Carrier-density with Uniform Doping

A bulk material must be charge neutral over all …

$$\int \left[p - n + N_D^+ + N_A^- \right] dV = 0$$



Further if the doping is spatially homogenous

$$p - n + N_D^+ + N_A^- = 0$$

FD integral vs. FD function ?

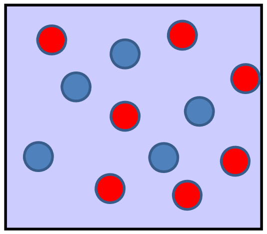
$$N_V \frac{2}{\sqrt{\pi}} F_{1/2}\left[\beta\left(E_F - E_V\right)\right] - N_A \frac{2}{\sqrt{\pi}} F_{1/2}\left[\beta\left(E_C - E_F\right)\right] + \frac{N_D}{1 + 2e^{\beta(E_F - E_D)}} - \frac{N_A}{1 + 4e^{\beta(E_A - E_F)}} = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_A e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_B T}} = 0 \quad \text{(approx.)}$$

Once you know E_F, you can calculate n, p, N_D^+, N_A^-.

Intrinsic Concentration

$$p - n + N_D^+ + N_A^- = 0$$



$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_B T}} = 0$$

$$n - p = 0 \Rightarrow N_C e^{-\beta(E_c - E_F)} = N_V e^{+\beta(E_v - E_F)}$$

$$E_F \equiv E_i = \frac{E_G}{2} + \frac{1}{2\beta} ln \frac{N_V}{N_C}$$

Outline

1) Intrinsic carrier concentration
2) **Temperature dependence of carrier concentration**
3) Multiple doping, co-doping, and heavy-doping
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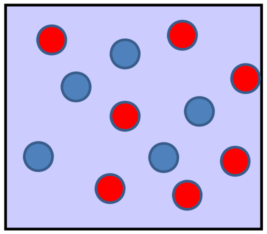
Carrier Density with Donors



In spatially homogenous field-free region …

$$p - n + N_D^+ + N_A^- = 0$$

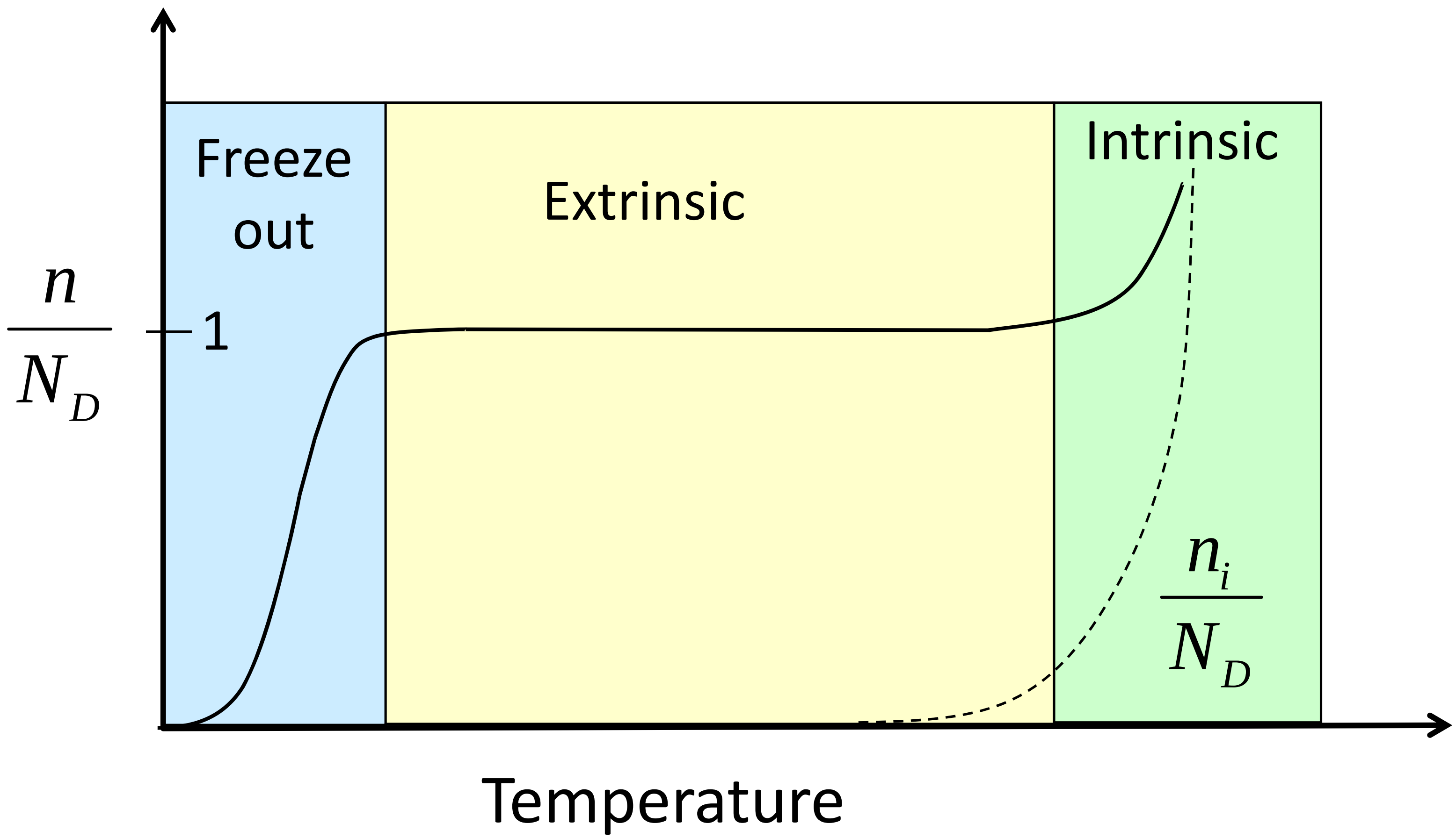
$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} - \cancel{\frac{N_A}{1 + 4e^{(E_A - E_F)/k_B T}}} = 0$$

n

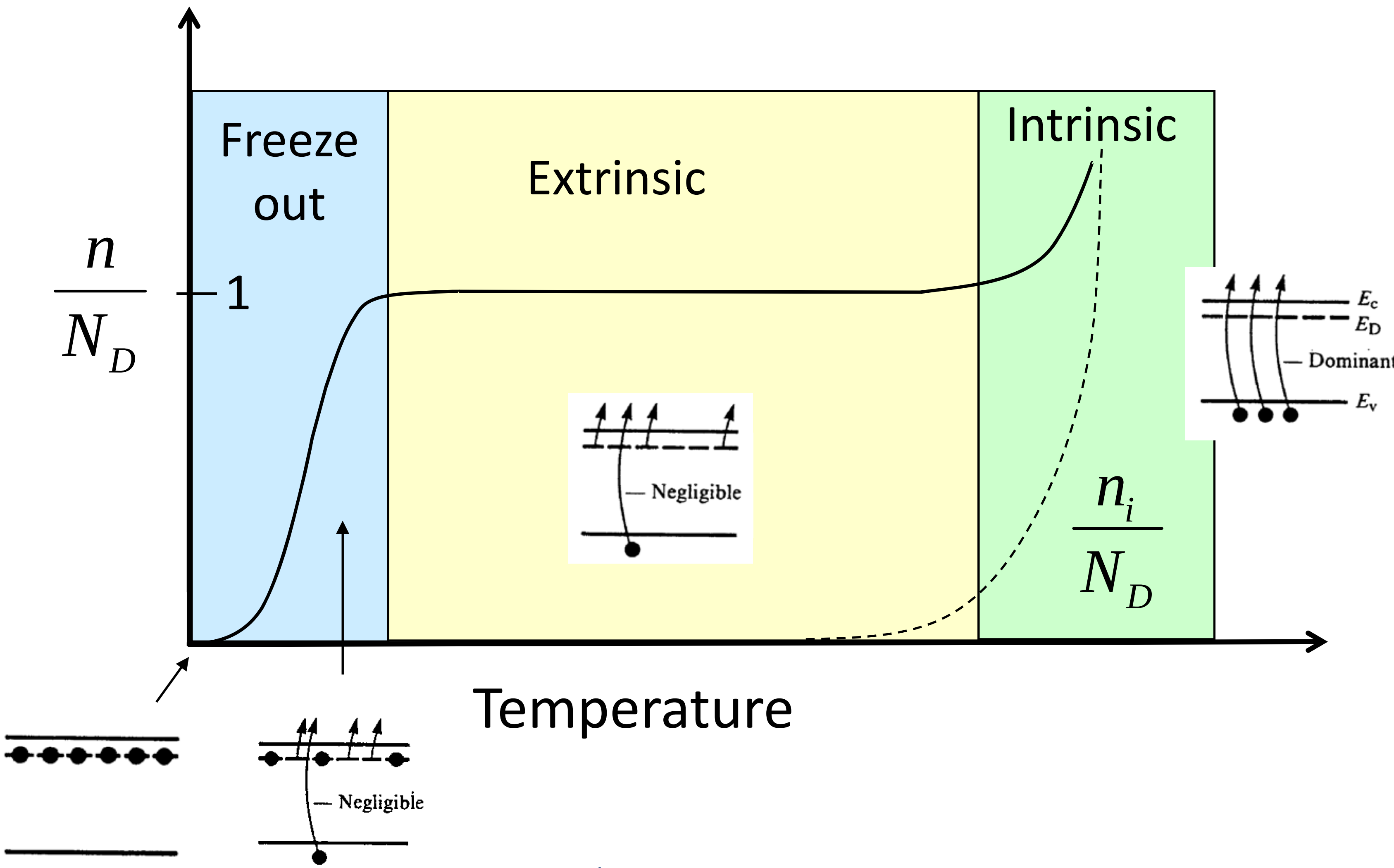
Assume N-type doping …

(will plot in next slide)

Temperature-dependent Concentration



Physical Interpretation



Electron Concentration with Donors

$$n = N_C e^{-\beta(E_C - E_F)} \Rightarrow \frac{n}{N_C} e^{\beta E_C} = e^{\beta E_F}$$

$$N_D^+ = \frac{N_D}{1 + 2e^{\beta(E_F - E_D)}} = \frac{N_D}{1 + 2\left[\frac{n}{N_C} e^{\beta(E_c - E_D)}\right]} \equiv \frac{N_D}{1 + \frac{n}{N_\xi}}$$

Electron concentration with Donors

$$p - n + N_D^+ = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} = 0$$

$$p \times n = n_i^2$$

$$\frac{n_i^2}{n} - n + \frac{N_D}{1 + \frac{n}{N_\xi}} = 0$$

No approximation so far ….

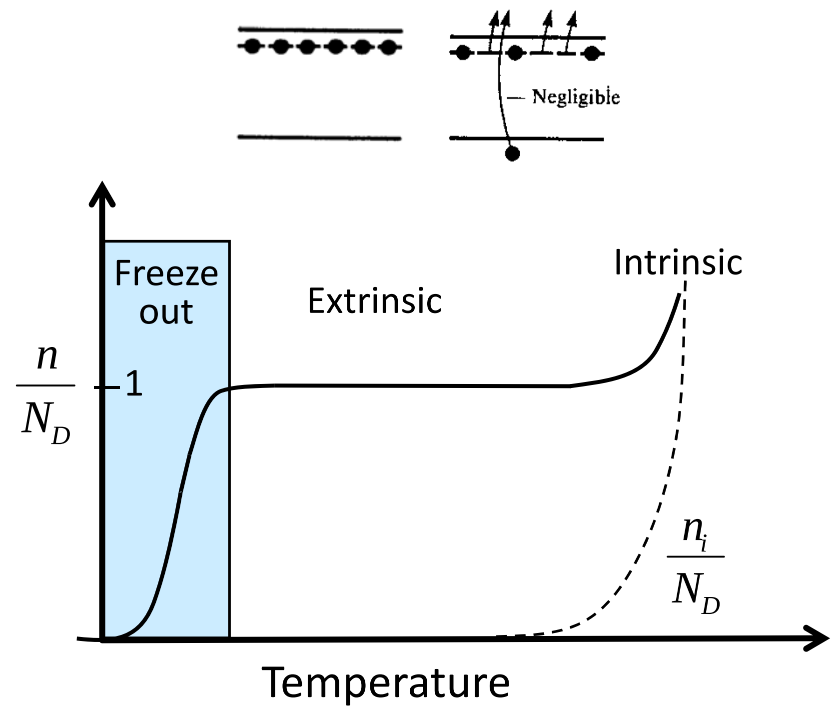
High Donor density/Freeze out T

$$\frac{n_i^2}{n} - n + \frac{N_D}{1+\frac{n}{N_\xi}} = 0$$

$$N_D \gg n_i$$

$$\Rightarrow -n + \frac{N_D}{1+\frac{n}{N_\xi}} \approx 0$$

$$\Rightarrow n^2 + N_\xi n - N_\xi N_D = 0$$

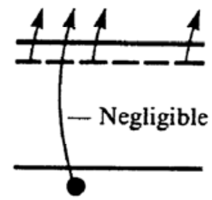


$$N_\xi \equiv \frac{N_C}{2} e^{-\beta(E_C - E_D)}$$

$$n = \frac{N_\xi}{2}\left[\left(1+\frac{4N_D}{N_\xi}\right)^{1/2} - 1\right]$$

Freeze-out/Extrinsic T

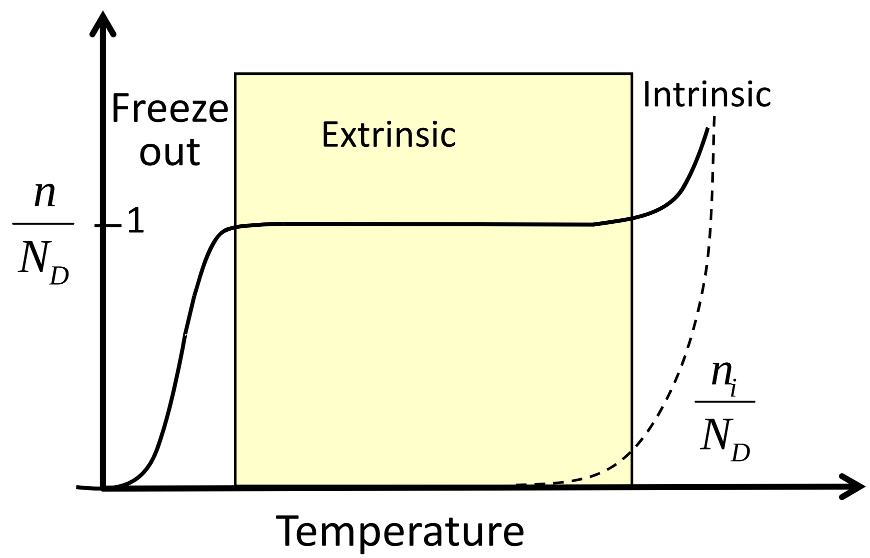
$$N_\xi \equiv \frac{N_C}{2} e^{-(E_C - E_D)/kT} \gg N_D$$



$$n = \frac{N_\xi}{2}\left[\left(1+\frac{4N_D}{N_\xi}\right)^{1/2} - 1\right]$$

$$\approx \frac{N_\xi}{2}\left[\left(1+\frac{1}{2}\frac{4N_D}{N_\xi}\right) - 1\right]$$

$$\approx N_D$$

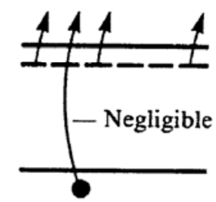


Electron concentration equals donor density
hole concentration by $n \times p = n_i^2$

Extrinsic/Intrinsic T

$$N_D^+ = \frac{N_D}{1+2e^{(E_F-E_D)/k_BT}} \approx N_D \quad \text{for } E_F < E_D$$

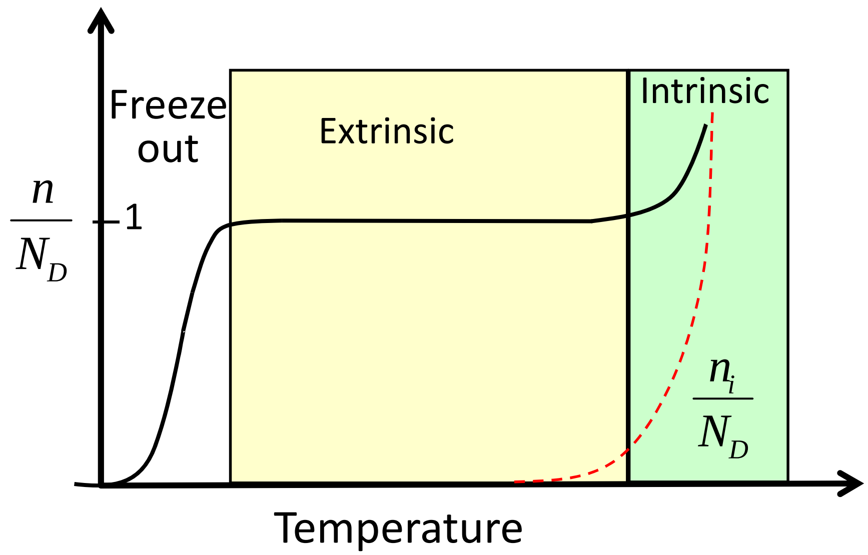
0



$$\frac{n_i^2}{n} - n + \frac{N_D}{1+\frac{n}{N_\xi}} = 0$$

$$\frac{n_i^2}{n} - n + N_D \approx 0$$

$$\Rightarrow -n_i^2 + n^2 - N_D n = 0$$



$$n = \frac{N_D}{2} + \left[\frac{N_D{}^2}{4} + n_i^2\right]^{1/2}$$

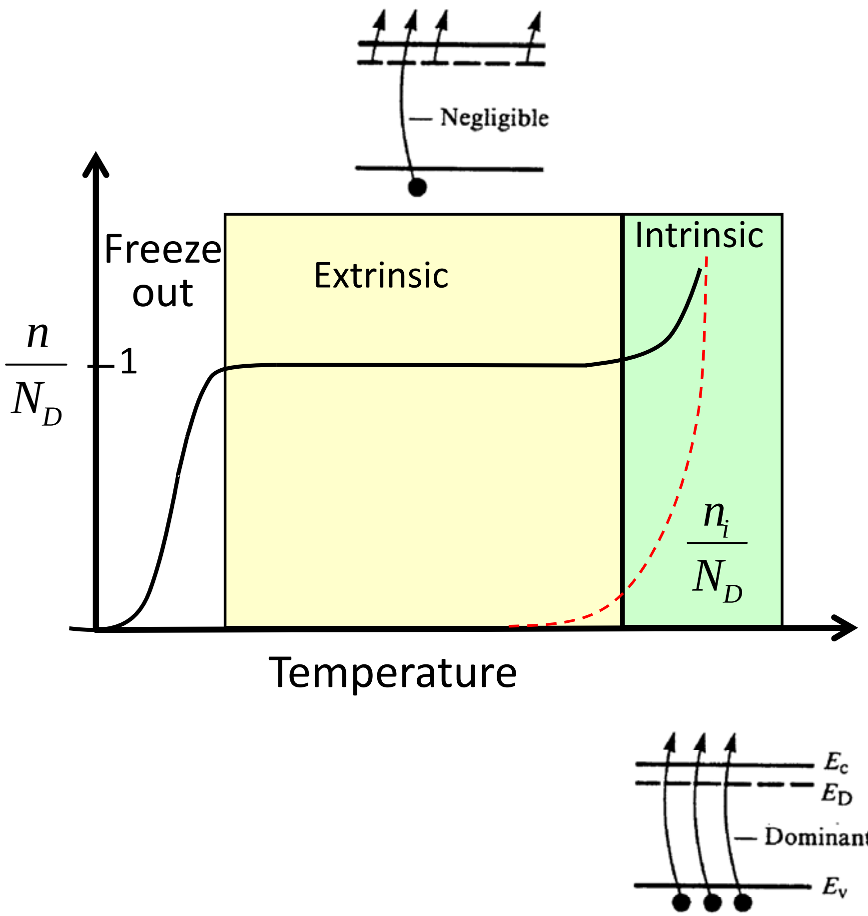
Extrinsic/Intrinsic T

For $N_D \gg n_i$

$$n = \frac{N_D}{2} + \left[\frac{N_D^{\ 2}}{4} + n_i^2 \right]^{1/2} \approx N_D$$

For $n_i \gg N_D$

$$n = \frac{N_D}{2} + \left[\frac{N_D^{\ 2}}{4} + n_i^2 \right]^{1/2} \approx n_i$$



Implications at High Temperature

What will happen if you use silicon circuits at very high temperatures ?

Determination of Fermi-level

$$n = N_C e^{-\beta(E_c - E_F)} \Rightarrow E_F = E_C + \frac{1}{\beta} ln\left(\frac{n}{N_C}\right)$$

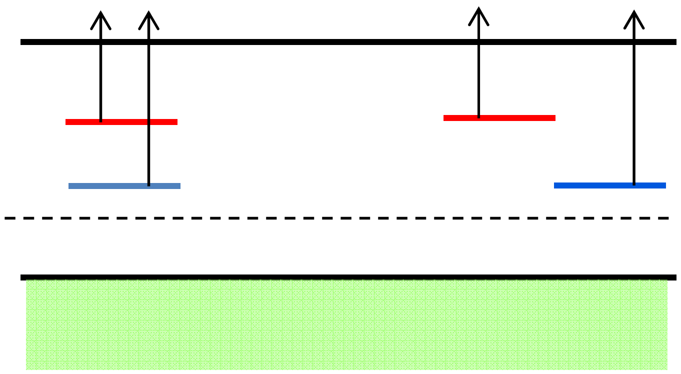
$$p - n + N_D^+ = 0$$

$$N_V e^{-(E_F - E_V)/k_B T} - N_C e^{-(E_C - E_F)/k_B T} + \frac{N_D}{1 + 2e^{(E_F - E_D)/k_B T}} = 0$$

Outline

1) Intrinsic carrier concentration
2) Temperature dependence of carrier concentration
3) **Multiple doping levels, co-doping, and heavy-doping**
4) Conclusion

Multiple Donor Levels



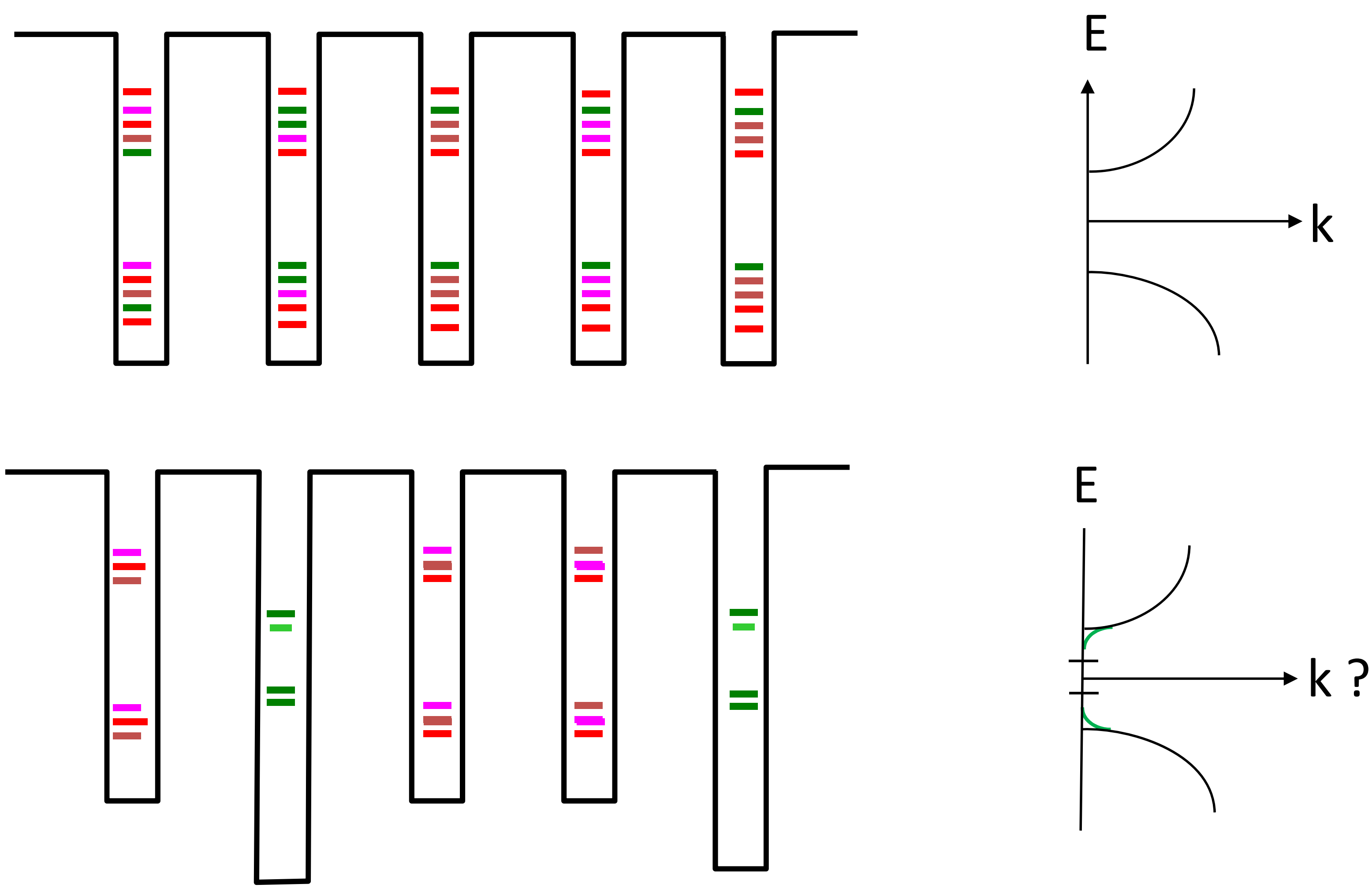
Multiple levels of same donor ...

$$p - n + \frac{N_D}{1 + 2e^{(E_F - E_{D1})/k_BT}} + \frac{N_D}{1 + 2e^{(E_F - E_{D2})/k_BT}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_BT}} = 0$$

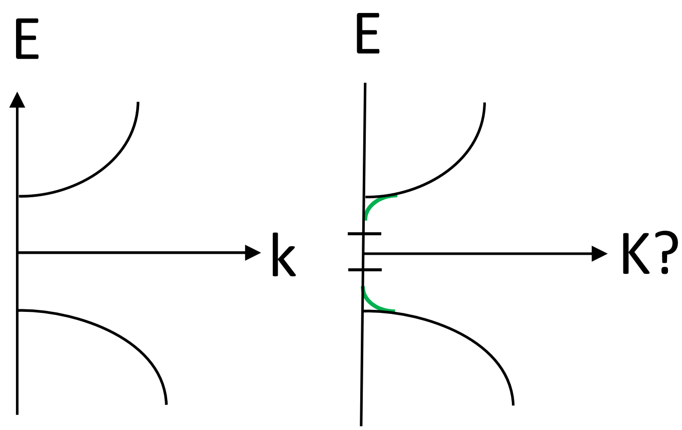
Codoping...

$$p - n + \frac{N_{D1}}{1 + 2e^{(E_F - E_{D1})/k_BT}} + \frac{N_{D2}}{1 + 2e^{(E_F - E_{D2})/k_BT}} - \frac{N_A}{1 + 4e^{(E_A - E_F)/k_BT}} = 0$$

Heavy Doping Effects: Bandtail States



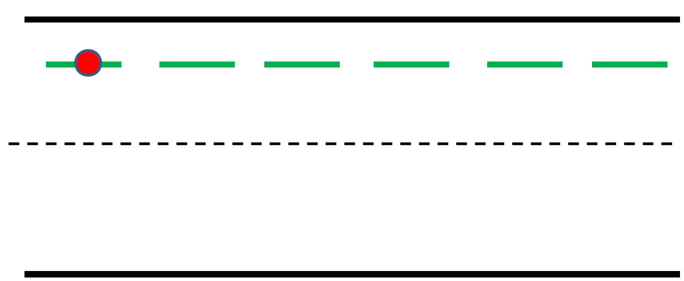
Heavy Doping Effects: Hopping Conduction



Bandgap narrowing

$$p \times n = N_C N_V e^{-\beta E_G^*}$$

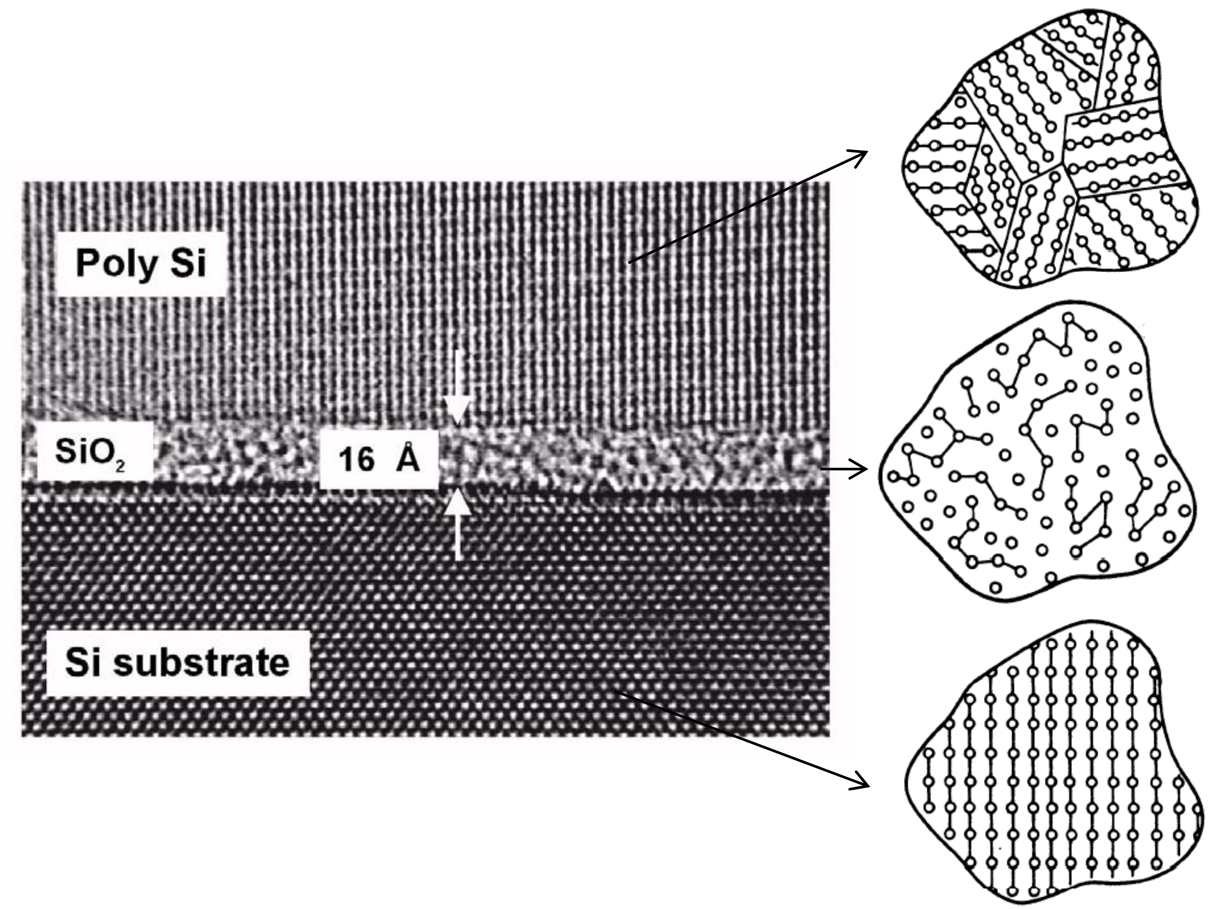
e.g. Base of HBTs



Band transport
vs.
hopping-transport

e.g. a-silicon, OLED

Arrangement of Atoms

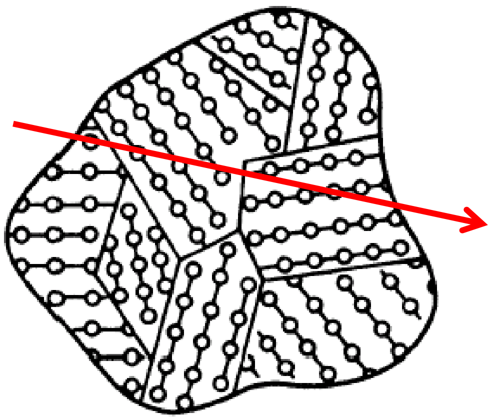


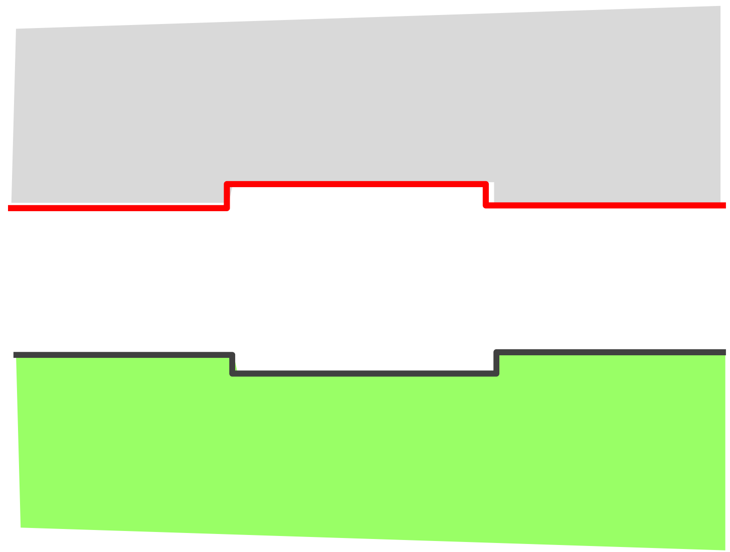
Poly-crystalline
Thin Film
Transistors

Amorphous
Oxides

Crystalline

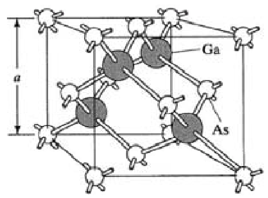
Poly-crystalline material



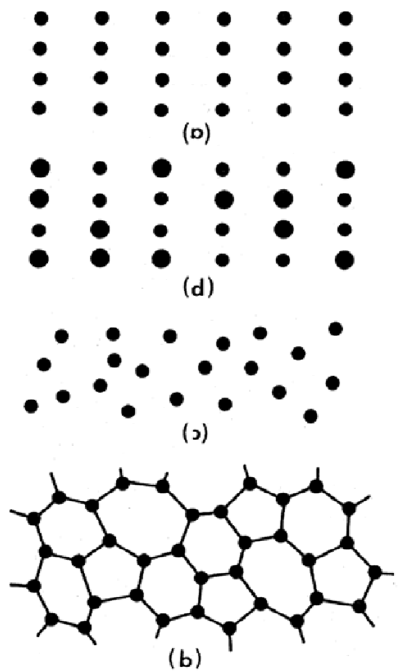


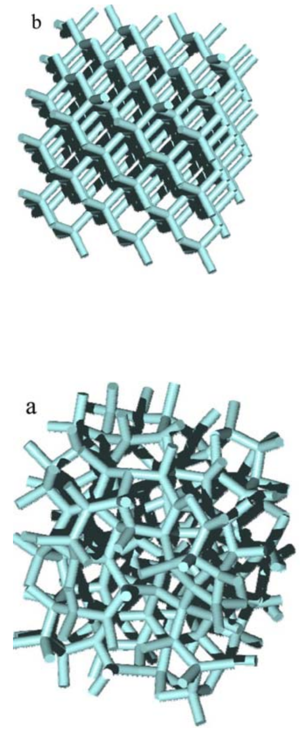
Isotropic bandgap and increase in scattering

Band-structure and Periodicity



PRB, 4, 2508, 1971





Edagawa, PRL, 100,013901, 2008

Periodicity is sufficient, but not necessary for bandgap.
Many amorphous material show full isotropic bandgap

Conclusions

1. Charge neutrality condition and law of mass-action allows calculation of Fermi-level and all carrier concentration.

2. For semiconductors with field, charge neutrality will not hold and we will need to use Poisson equation.

3. Heaving doping effects play an important role in carrier transport.