

Engineering Space for Light with Metamaterials

Part 1: Electrical and Magnetic
Metamaterials

*Part 2: Negative-Index Metamaterials, NLO, and
super/hyper-lens*

Part 3: Cloaking and Transformation Optics

Outline

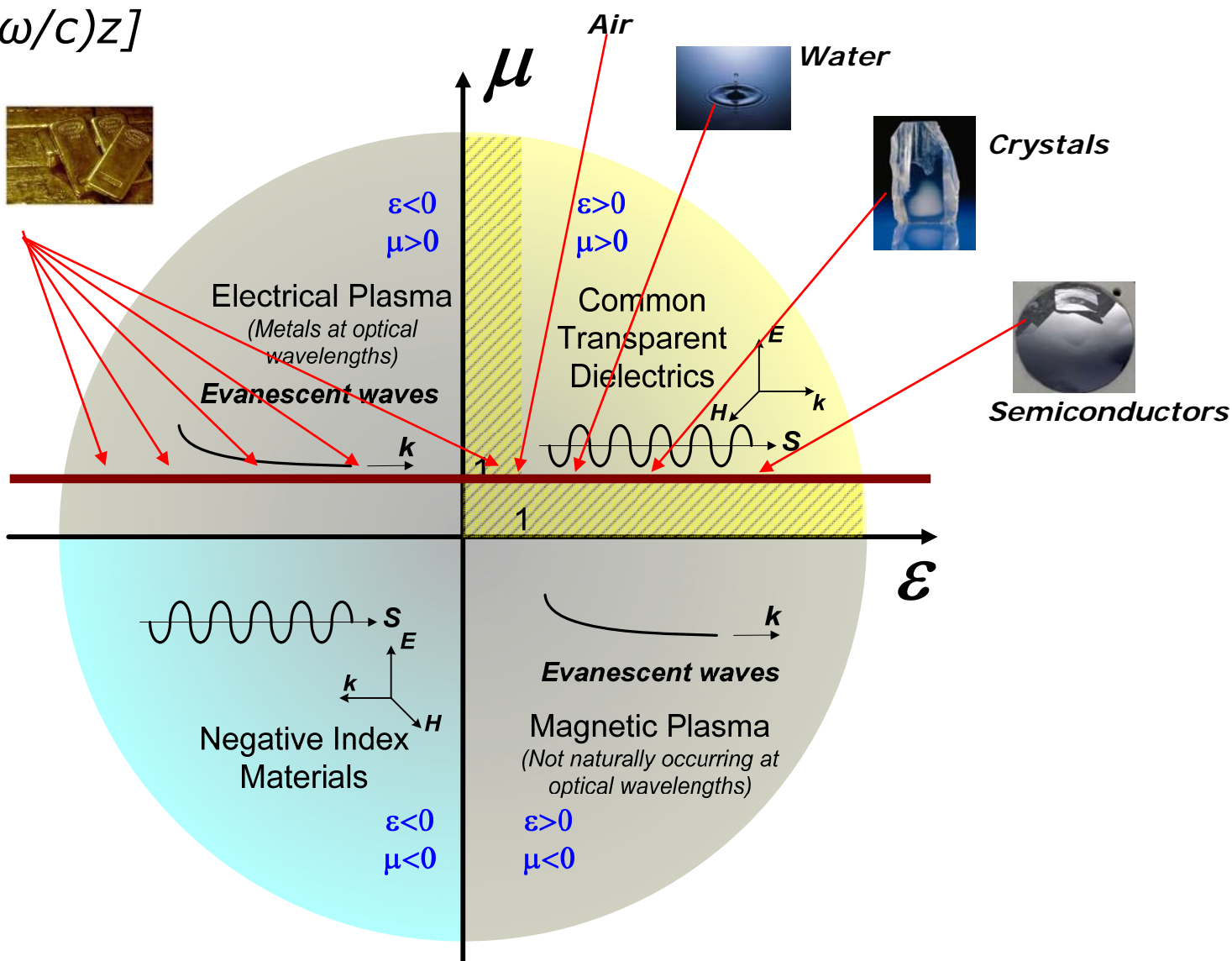
- **What are metamaterials?**
- Early electrical metamaterials
- Magnetic metamaterials
- Negative-index metamaterials
- Chiral metamaterials
- Nonlinear optics with metamaterials
- Super-resolution
- Optical Cloaking and Transformation Optics

Natural Optical Materials

$$E, H \sim \exp[in(\omega/c)z]$$

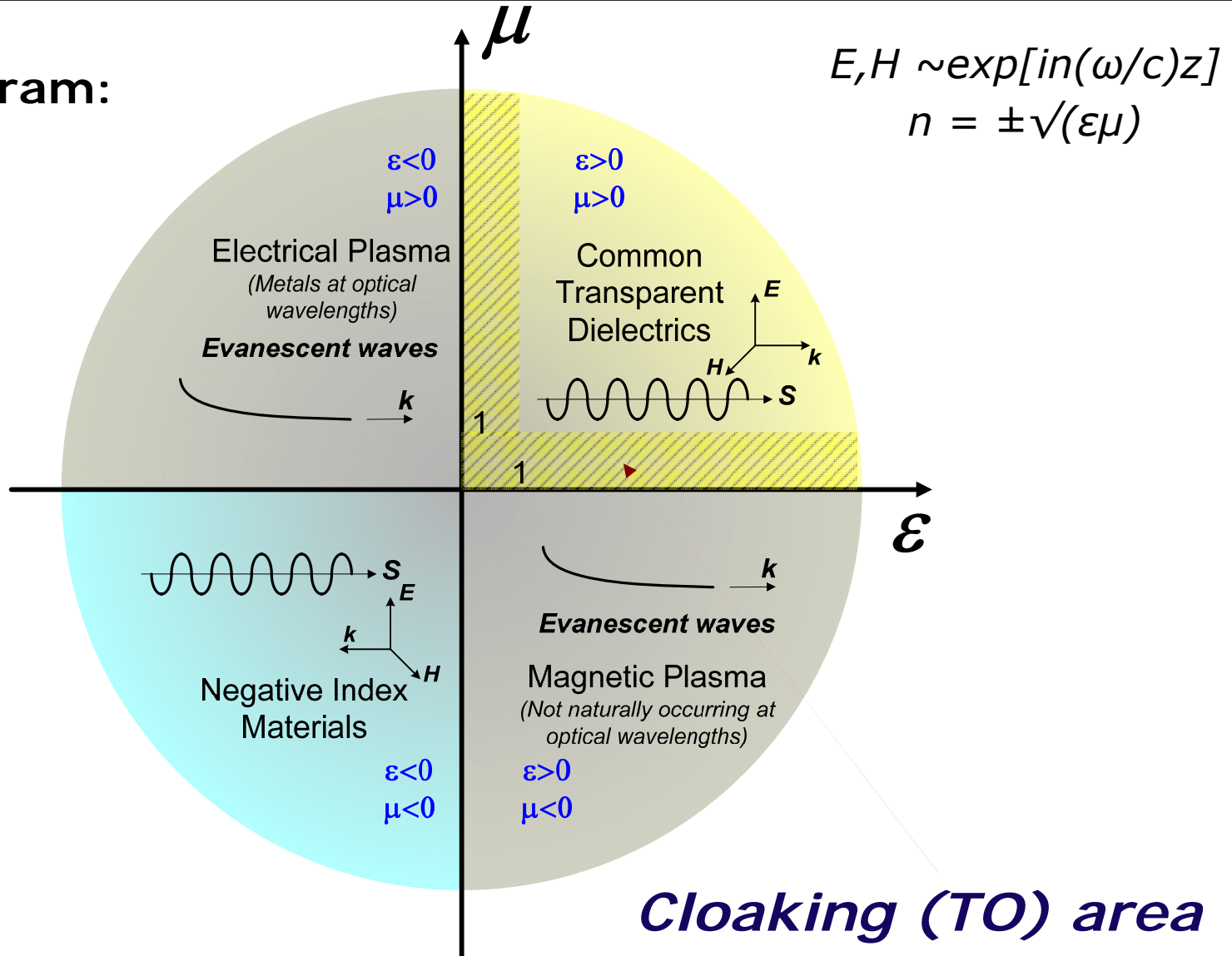
$$n = \pm\sqrt{(\epsilon\mu)}$$

metals



Materials & Metamaterials

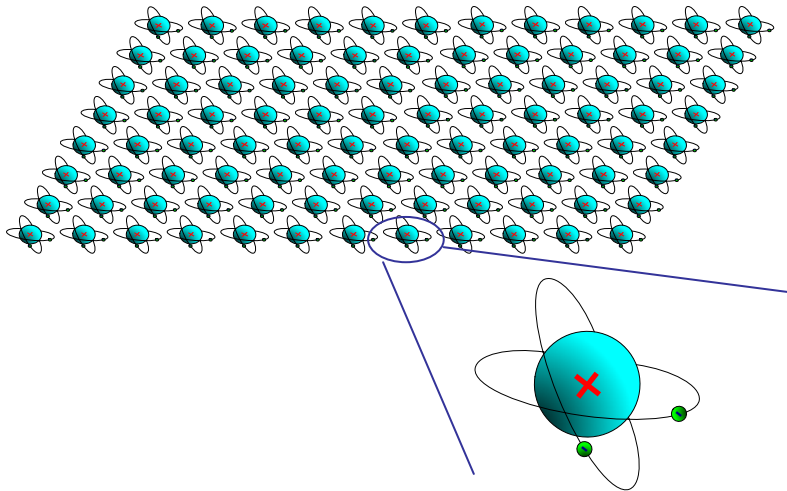
ϵ, μ diagram:



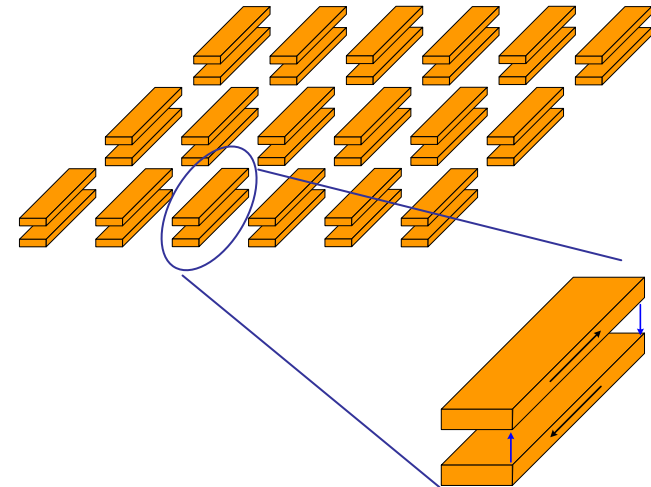
What is a metamaterial?

Metamaterial is an arrangement of artificial structural elements, designed to achieve advantageous and unusual electromagnetic properties.

$\mu\epsilon\tau\alpha$ = meta = beyond (Greek)



A natural material with its atoms



A metamaterial with artificially structured "atoms"

Photonic crystals vs. Optical metamaterials: connections and differences



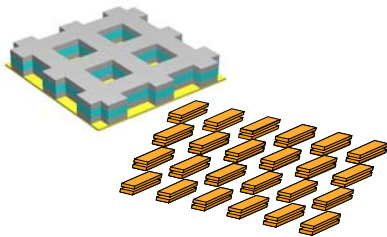
$$a \ll \lambda.$$

Effective medium
description using
Maxwell equations with
 μ, ϵ, n, Z

Example:

Optical crystals

Metamaterials



$$a \sim \lambda.$$

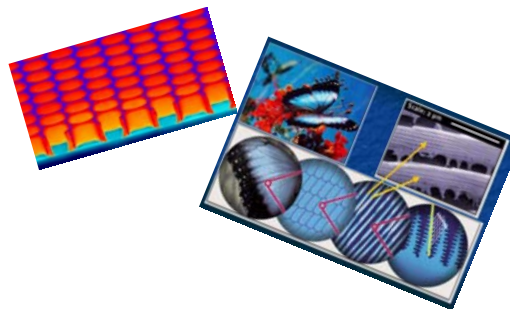
Structure dominates.
Properties determined
by diffraction and
interference

Example:

Photonics crystals

Phased array radar

X-ray diffraction optics



$$\infty$$

$$a/\lambda$$

$$a \gg \lambda.$$

Properties described
using geometrical optics
and ray tracing

Example:

Lens system

Shadows

Natural Crystals



... have lattice constants much smaller than light wavelengths: $a \ll \lambda$

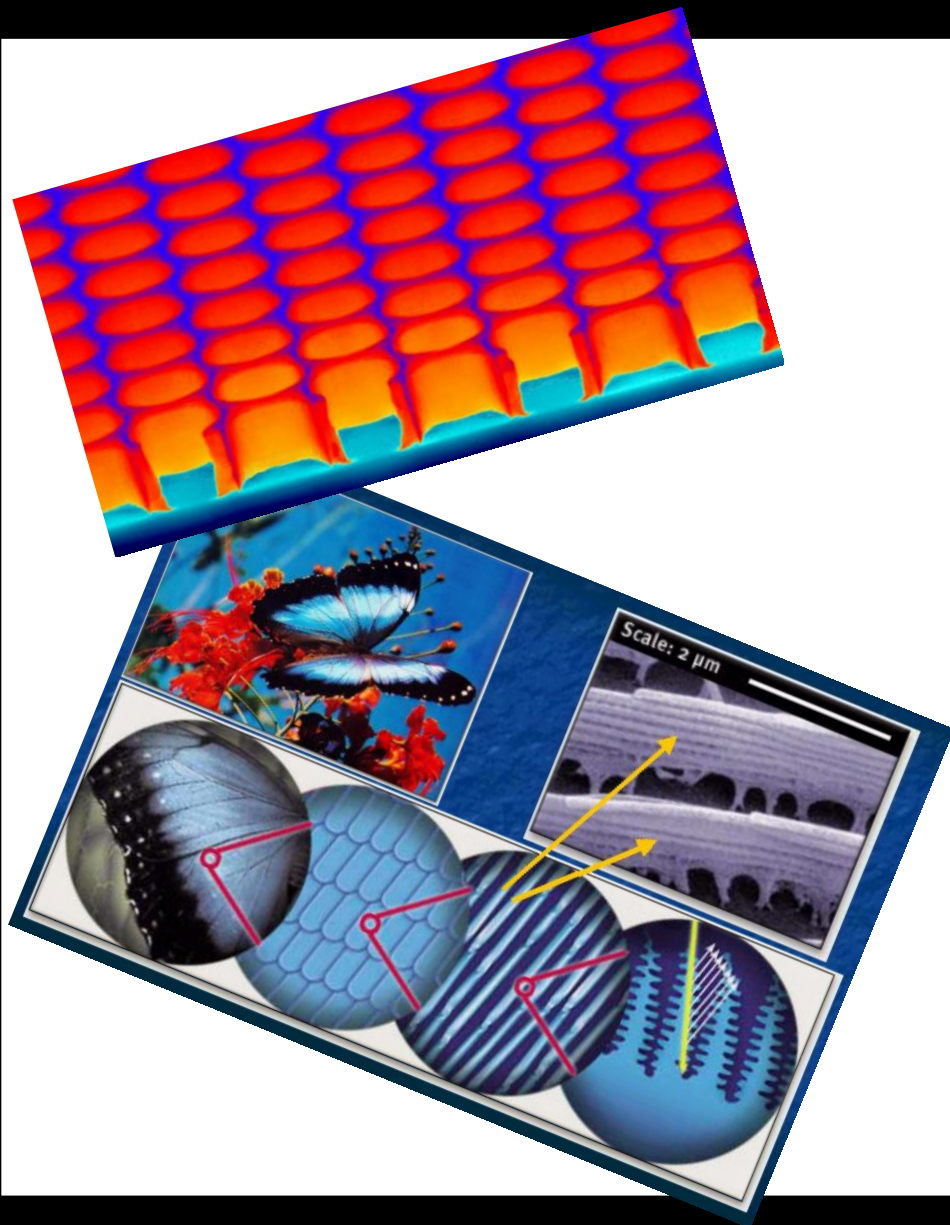
... are treated as homogeneous media with parameters ϵ , μ , n , Z (tensors in anisotropic crystals)

... have a positive refractive index: $n > 1$

... show no magnetic response at optical wavelengths: $\mu = 1$



Photonic crystals



... have lattice constants comparable to light wavelengths: $a \sim \lambda$

... can be artificial or natural

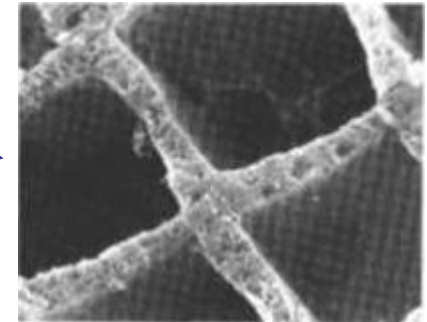
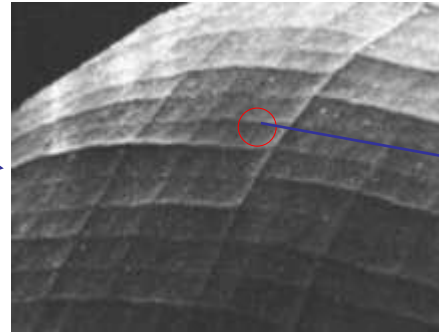
... have properties governed by the diffraction of the periodic structures

... may exhibit a bandgap for photons

... typically are *not* well described using effective parameters ϵ , μ , n , Z

... often behave like but they are *not* true metamaterials

Metamaterials: Properties not found in nature?



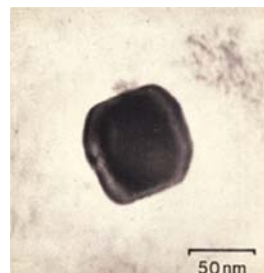
Journal of the European Optical Society - Rapid Publications 1, 06010 (2006)

www.jeos.org

Invertebrate superposition eyes-structures
that behave like metamaterial with
negative ~~refractive index~~
(refraction!)

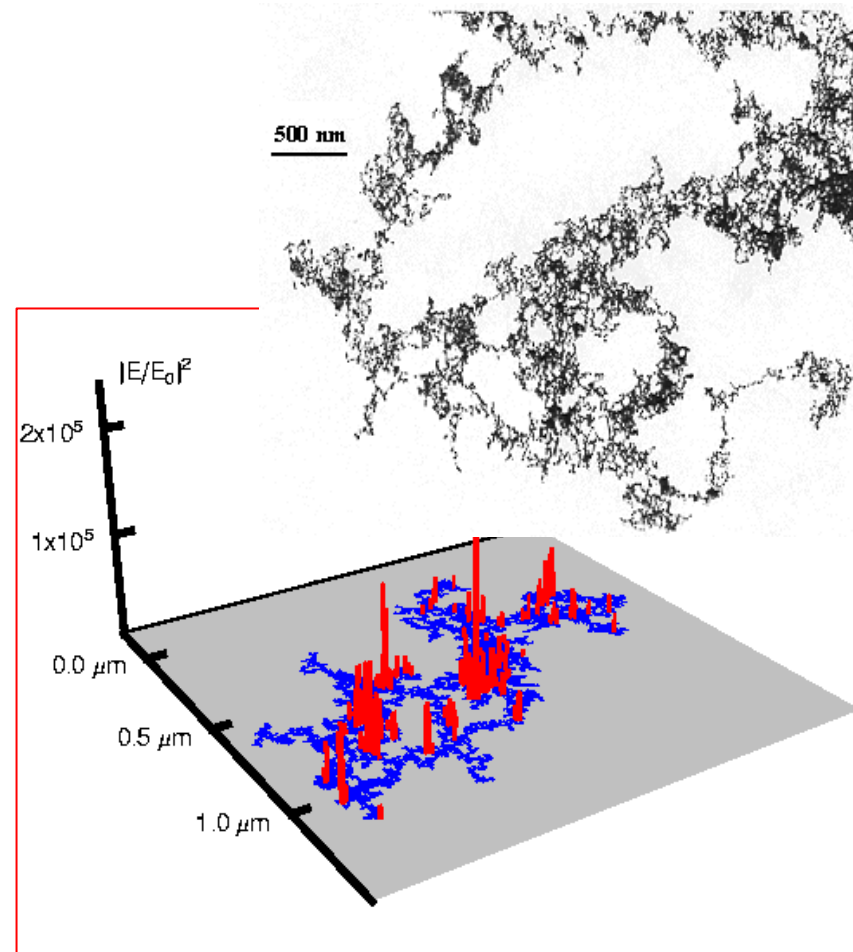
Metamaterials: Artificial periodic structures?

Lycurgus Cup (4th century AD)



Ancient (first?) random metamaterial (carved in Rome) with gold nano particles

"Hot-spots" in fractals



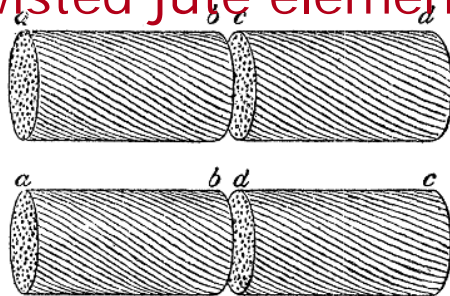
Shalaev, Nonlinear Optics of Random Media, Springer, 2000

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Early (first?) Example of Meta-Atoms

Twisted jute elements



Artificial chiral molecules

In order to imitate the rotation produced by liquids like sugar solutions, I made small elements or “molecules” of twisted jute, of two varieties, one kind being twisted to the right (positive) and the other twisted to the left (negative). I now interposed a number of, say, the positive variety, end to end, between the crossed polariser and analyser; this produced a restoration of the field. The same was the case with the negative variety. *I now mixed equal numbers of the two varieties, and there was now no restoration of the field, the rotation produced by one variety being counteracted by the opposite rotation produced by the other.*

Jagadis C. Bose, *Proceeding of Royal Soc. London*, 1898

“On the Rotation of Plane of Polarization of Electric Waves by a Twisted Structure”

Early Electric Metamaterial: Artificial Dielectrics

Periodic metal-dielectric plates with effective index of less than 1

828

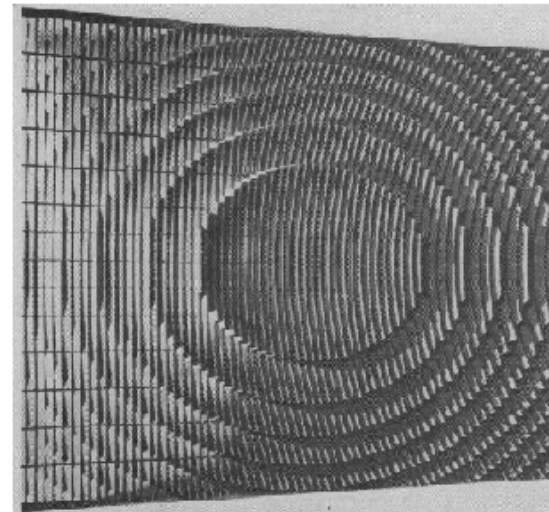
Proceedings of the I.R.E. and Waves and Electrons

November

Metal-Lens Antennas*

WINSTON E. KOCK†, SENIOR MEMBER, I.R.E.

velocity.¹ This same property is acquired by waves confined between conducting plates which are parallel to the electric vector and spaced apart a distance greater than one half wavelength. A row of such parallel plates accordingly constitutes a refractive medium with index of refraction less than unity. Such a medium, when cut to the proper profile, can be used to produce a focusing or lens effect in a manner similar to that of a dielectric lens.

W. E. Kock, *Proc. IRE*, Vol. 34, 1946

Noble metal: $\epsilon < 1$ in nature

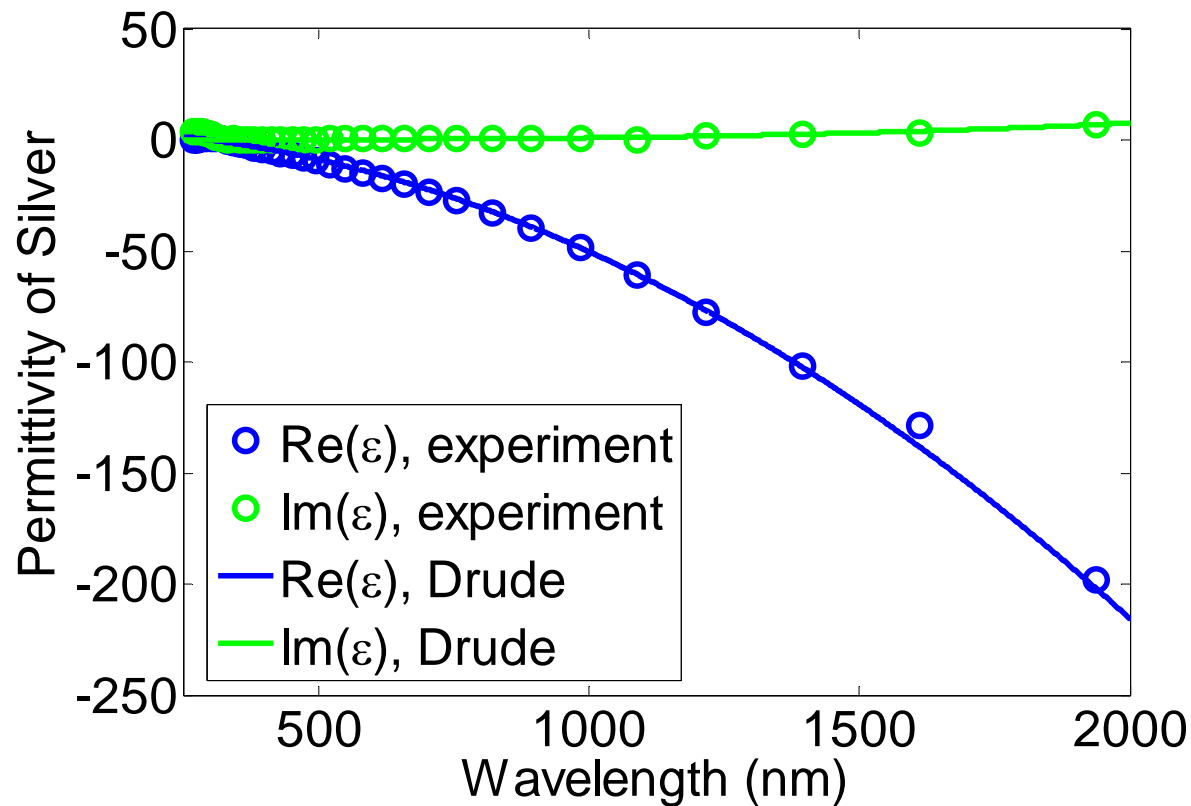
Drude model for permittivity:

$$\epsilon(\omega) = \epsilon_0 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)}$$

Silver parameters: $\epsilon_0 = 5.0$

$$\omega_p = 9.216 \text{ eV}$$

$$\Gamma = 0.0212 \text{ eV}$$



Experimental data from Johnson & Christy, PRB, 1972

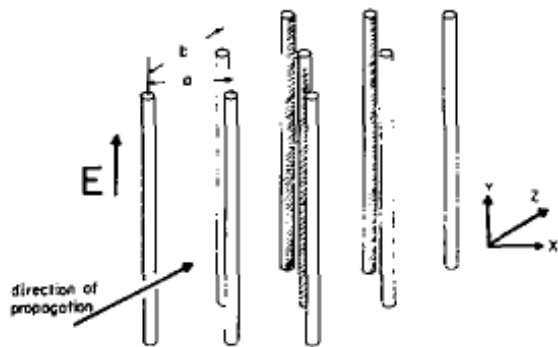
Array of Thin Wires and Tunable Plasma Frequency

1962

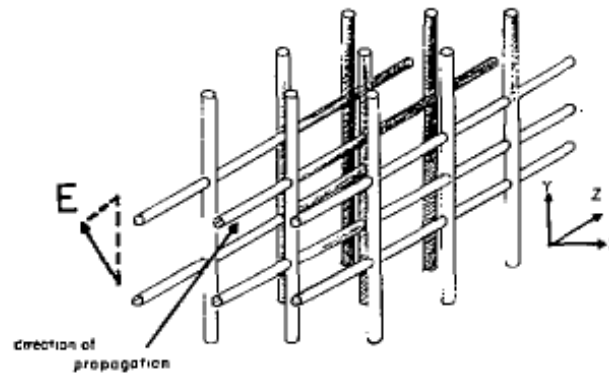
IRE TRANSACTIONS ON ANTENNAS AND PROPAGATION

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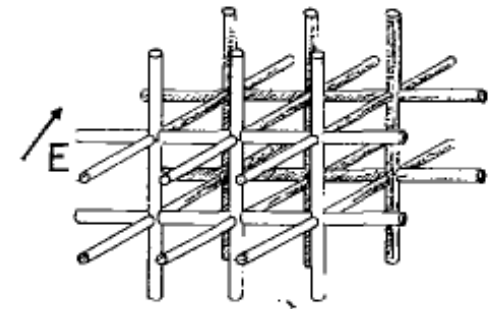
Plasma Simulation by Artificial Dielectrics and Parallel-Plate Media*



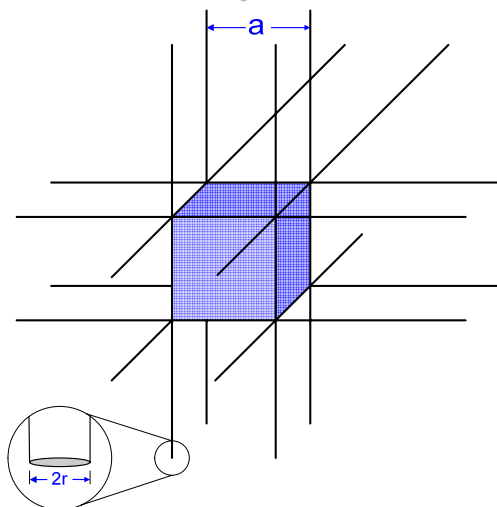
(a)



(b)

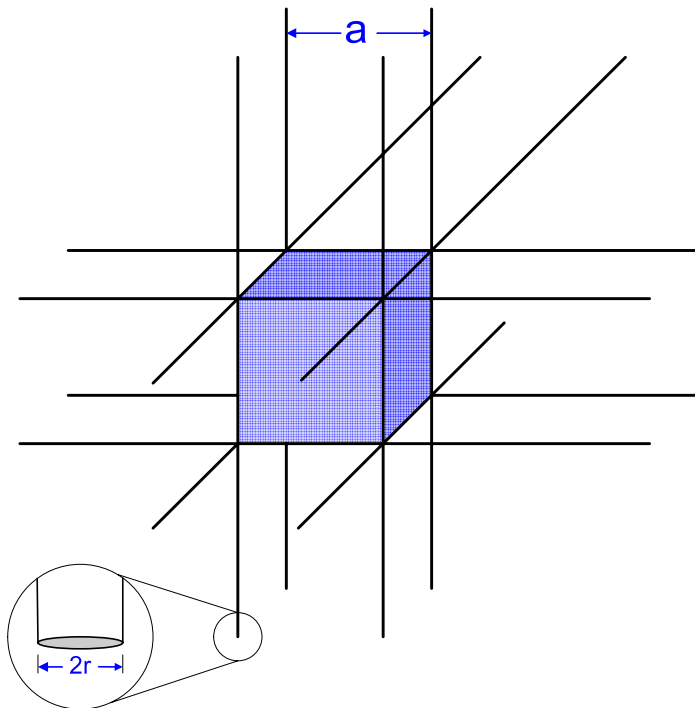


(c)



J. Brown, Proc. IEE 100 (1953)
W. Rotman, Trans. IRE AP 10 (1962)
J.B. Pendry, et al., Phys. Rev. Lett. (1996)

Electrical metamaterials: *metal wires arrays with tunable plasma frequency*



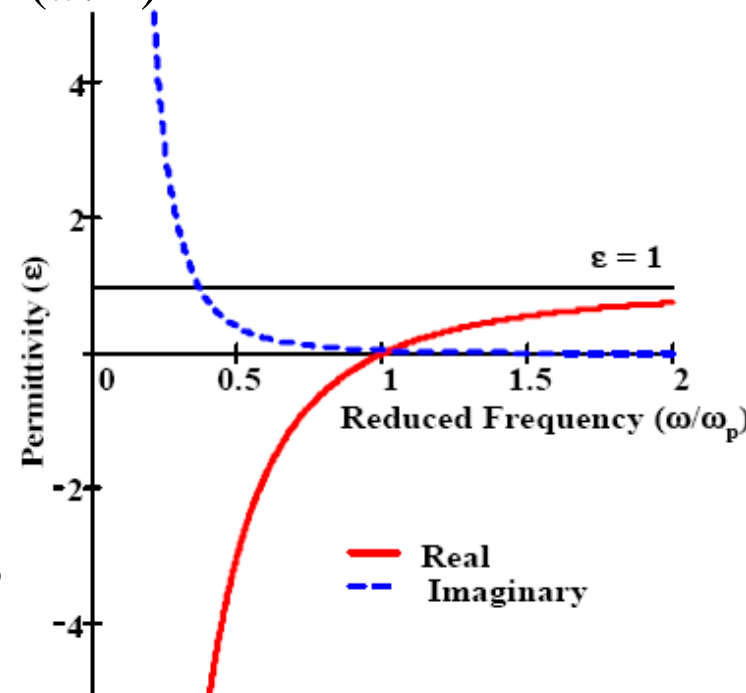
A periodic array of thin metal wires with $r \ll a \ll \lambda$ acts as a low frequency plasma

The effective ϵ is described with modified ω_p

Plasma frequency depends on geometry rather than on material properties

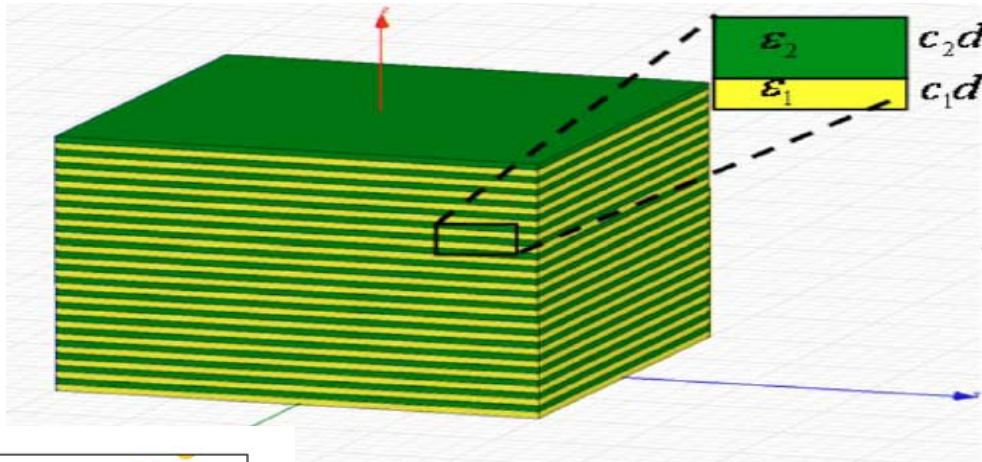
$$\epsilon = \epsilon' + i\epsilon'' = 1 - \frac{\omega_p^2}{\omega(\omega + i\varepsilon_0 a^2 \omega_p^2 / \pi r^2 \sigma)}$$

$$\omega_p^2 = \frac{2\pi c^2}{a^2 \ln(a/r)}$$



Pendry, PRL (1996)

Metal-Dielectric Composites and Mixing Rules



$$\begin{cases} \epsilon_{\square} = c_1\epsilon_1 + c_2\epsilon_2 \\ \epsilon_{\perp} = \epsilon_1\epsilon_2 / (c_1\epsilon_2 + c_2\epsilon_1) \end{cases}$$



Maxwell-Garnett (MG) theory:

$$\frac{\epsilon_{MG}(\omega) - \epsilon_h(\omega)}{\epsilon_{MG}(\omega) + 2\epsilon_h(\omega)} = f \frac{\epsilon_i(\omega) - \epsilon_h(\omega)}{\epsilon_i(\omega) + 2\epsilon_h(\omega)} \quad f \ll 1$$



Effective-Medium Theory (EMT):

$$f \frac{\epsilon_m - \epsilon_{eff}}{\epsilon_m + (d-1)\epsilon_{eff}} + (1-f) \frac{\epsilon_d - \epsilon_{eff}}{\epsilon_d + (d-1)\epsilon_{eff}} = 0$$

Composites with “elongated” inclusions

Depolarization factor:

$$q_i = \int_0^\infty \frac{a_i a_j a_k ds}{2(s + a_i^2)^{3/2} (s + a_j^2)^{1/2} (s + a_k^2)^{1/2}}$$

Screening factor:

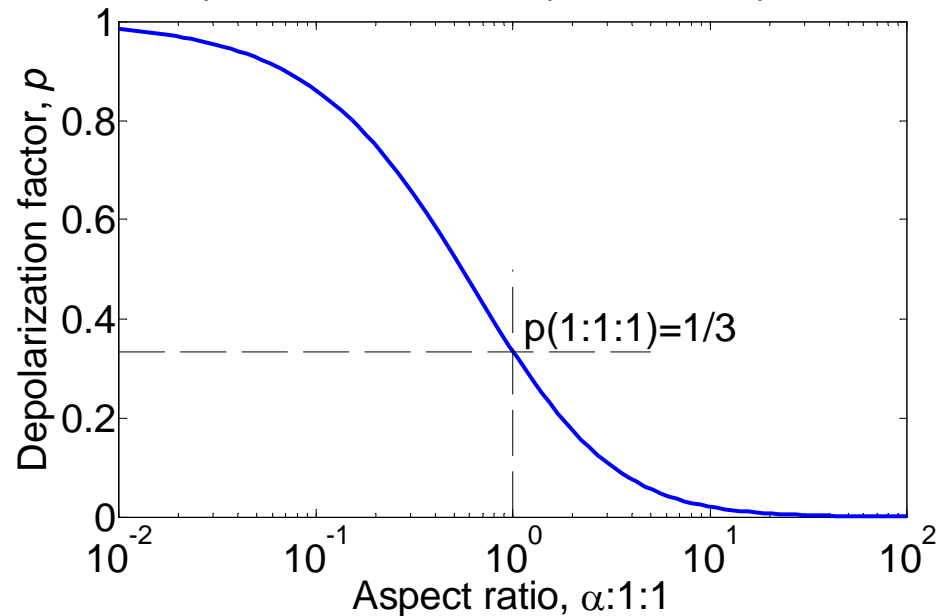
$$\kappa = (1 - q) / q$$

Clausius-Mossotti yields shape-dependent EMT:

$$f \frac{\varepsilon_m - \varepsilon_{eff}}{\varepsilon_m + \kappa \varepsilon_{eff}} + (1 - f) \frac{\varepsilon_d - \varepsilon_{eff}}{\varepsilon_d + \kappa \varepsilon_{eff}} = 0$$

$$\varepsilon_{eff} = \frac{1}{2\kappa} \left\{ \bar{\varepsilon} \pm \sqrt{\bar{\varepsilon}^2 + 4\kappa \varepsilon_m \varepsilon_d} \right\}$$

Lorentz depolarization factor for a spheroid with aspect ratio $\alpha:1:1$



$$\bar{\varepsilon} = [(\kappa + 1)f - 1]\varepsilon_m + [\kappa - (\kappa + 1)f]\varepsilon_d$$

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Absence (or very weak: $\mu \approx 1$) Optical Magnetism in Nature

Magnetic coupling to an atom: $\sim \mu_B = e\hbar / 2m_e c = \alpha e a_0$ (Bohr magneton)

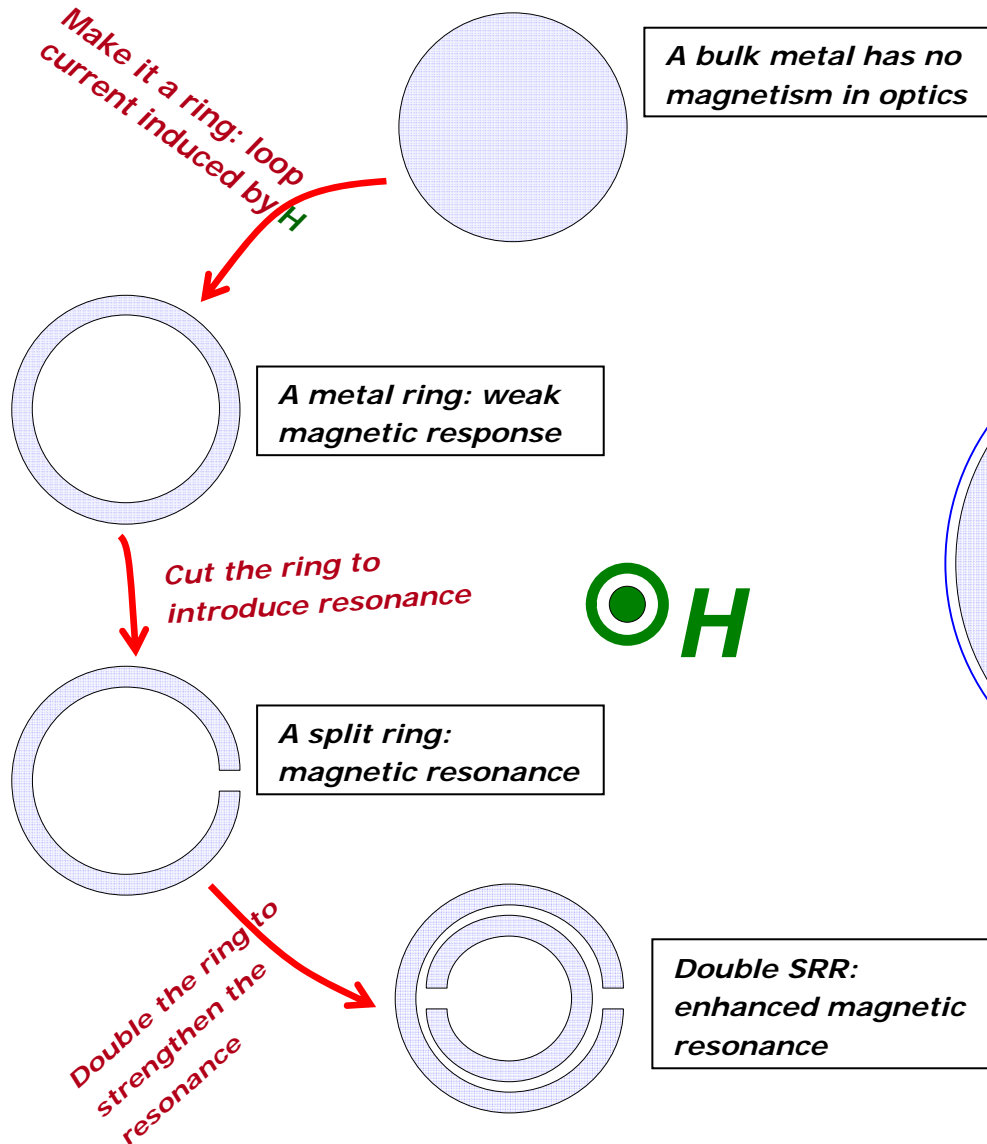
Electric coupling to an atom: $\sim e a_0$

Magnetic effect / electric effect $\approx \alpha^2 \approx (1/137)^2 < 10^{-4}$

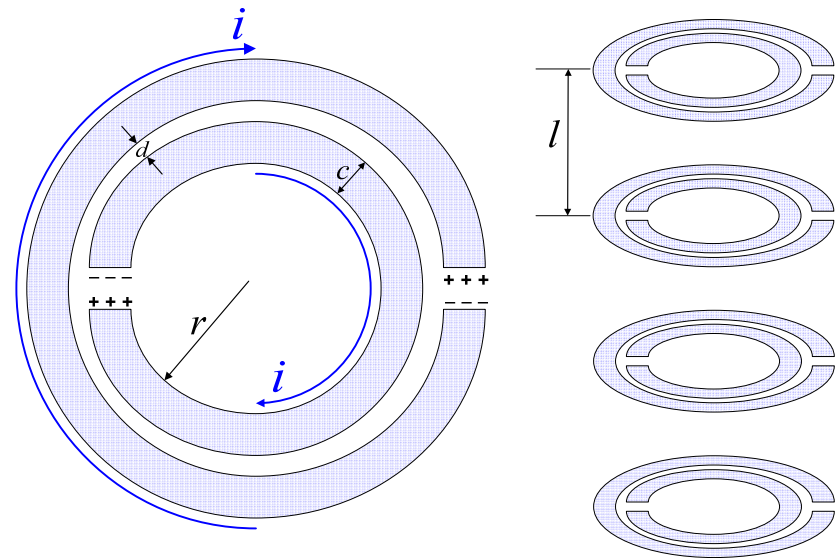
“... the magnetic permeability $\mu(\omega)$ ceases to have any physical meaning at relatively low frequencies...there is certainly no meaning in using the magnetic susceptibility from optical frequencies onwards, and in discussion of such phenomena we must put $\mu=1$.”

Landau and Lifshitz, ECM, Chapter 79.

SRRs: first magnetic metamaterials



Split-ring resonator (SRR)



*Theory: Pendry et al., 1999.
Experiment: Smith et al., 2000.*

Artificial magnetic resonators: Earlier form and Today's design

SRR for GHz magnetic resonance (Hardy et al., 1981):

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Rev. Sci. Instrum. 52(2), Feb. 1981

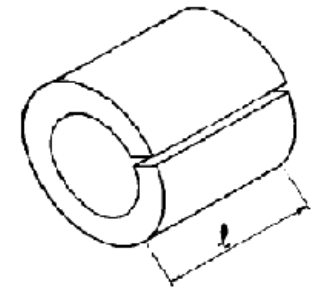
0034-6748/81/020213-04\$00.60

© 1981 American Institute of Physics

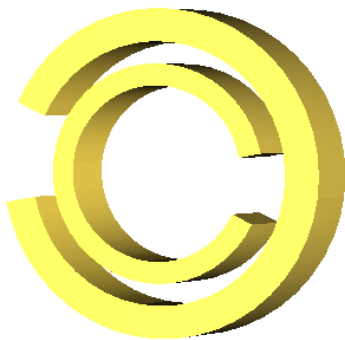
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Split-ring resonator for use in magnetic resonance from 200–2000 MHz

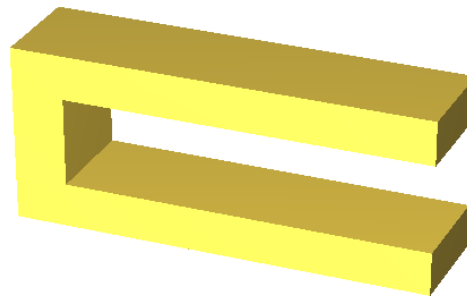
W. N. Hardy and L. A. Whitehead



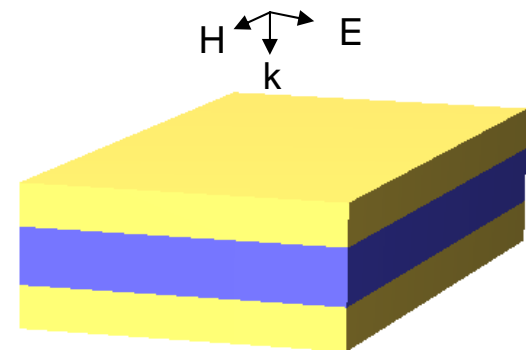
Modern magnetic units for optical metamagnetism:



SRR



C-shaped Rod



Nanostrip (or nanorod) Pair

Limits of size scaling in SRRs

Direct scaling-down the SRR dimensions doesn't help much...

Loss in metal gives kinetic inductance

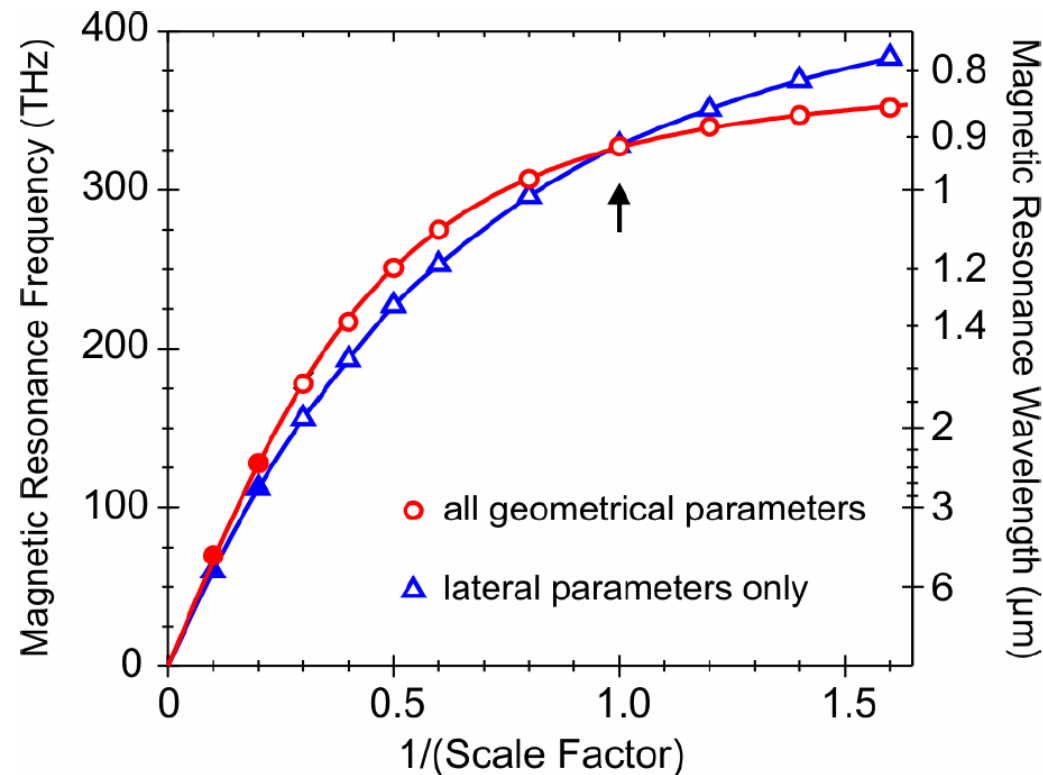
$$L_{coil} \propto size \qquad L_{kinetic} \propto \frac{1}{size}$$

$$L_{total} = L_{coil} + L_{kinetic}$$

$$C_{total} \propto size$$

$$\omega_{res} \propto \frac{1}{\sqrt{L_{total} \times C_{total}}} = \frac{1}{\sqrt{(A \cdot size + B / size) \cdot (C \cdot size)}} \propto \frac{1}{\sqrt{size^2 + const.}}$$

Saturation

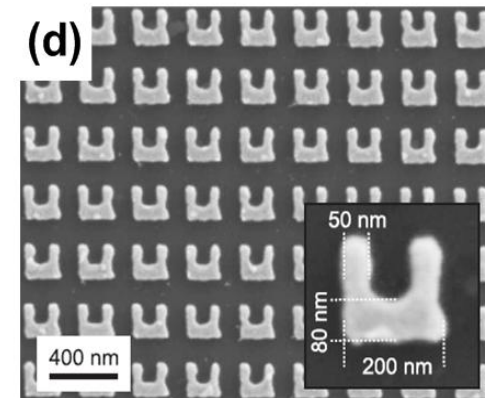
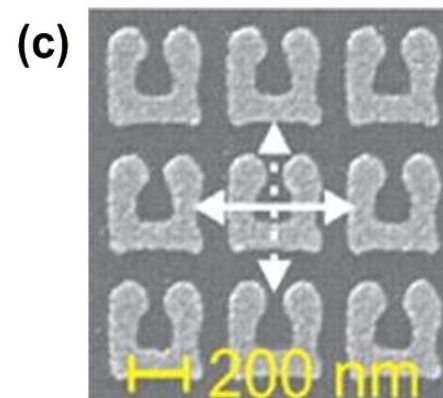
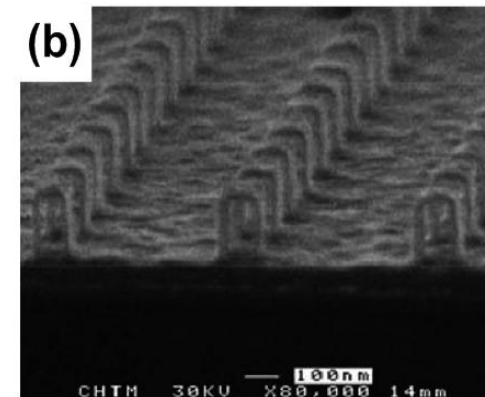
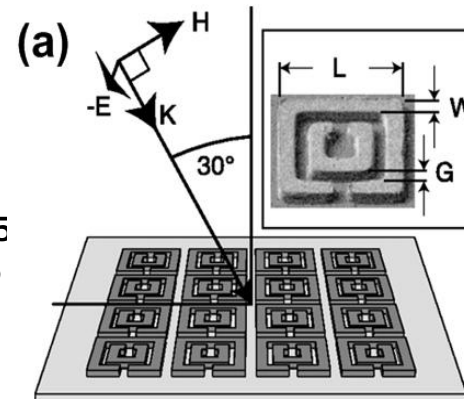


Zhou et al, PRL (2005); Klein, et al., OL (2006)

Progress in Optical Magnetism Metamaterials

Terahertz magnetism

- a) Yen, et al. ~ 1THz (2-SRR) – 2004
Katsarakis, et al (SRR – 5 layers) - 2005
- b) Zhang et al ~50THz (SRR+mirror) - 2005
- c) Linden, et al. 100THz (1-SRR) -2004
- d) Enkrich, et al. 200THz (u-shaped)-2005

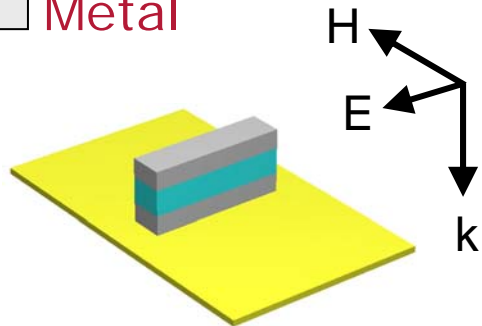


**2004-2007 years:
from 10 GHz to 500 THZ**

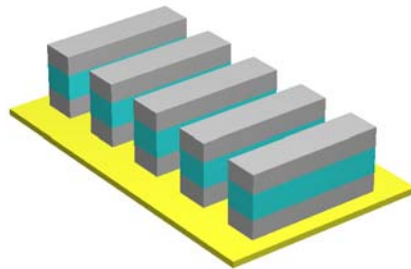
Magnetic Metamaterial: Nanorod to Nanostrip

■ Dielectric

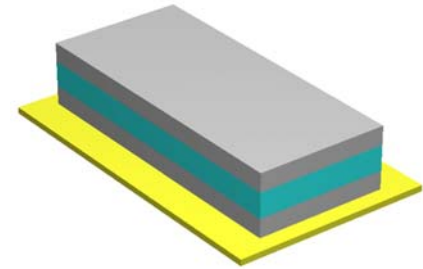
□ Metal



Nanorod pair



Nanorod pair array



Nanostrip pair

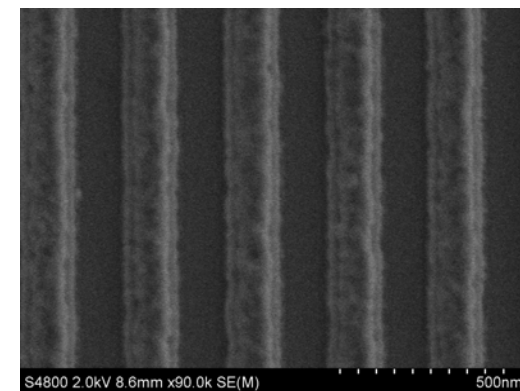
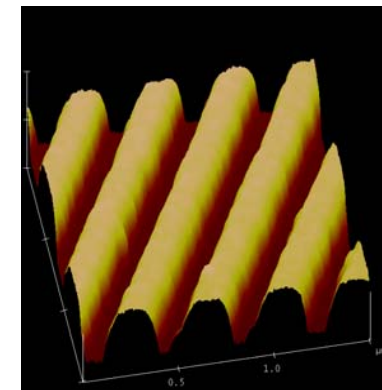
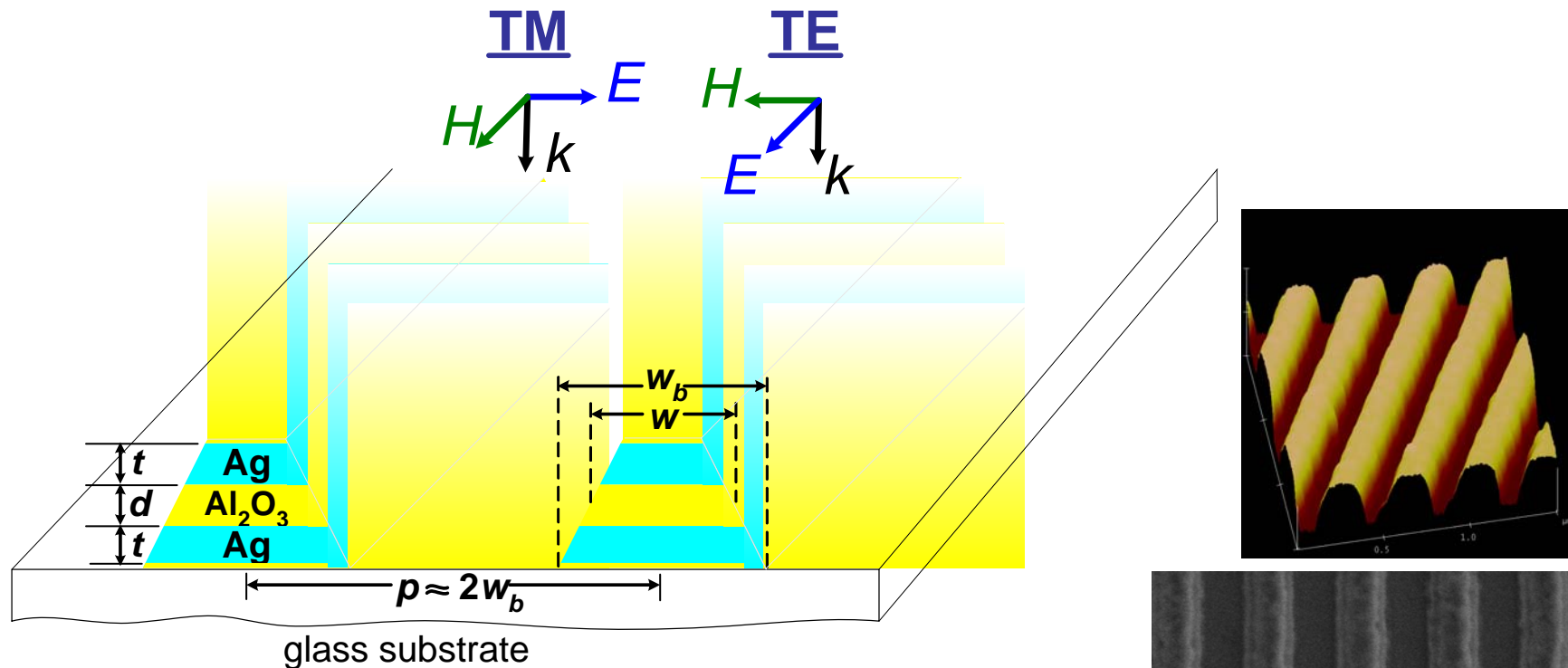
- Nanostrip pair has a much stronger magnetic response

Lagar'kov, Sarychev PRB (1996) - $\mu > 0$

Podolskiy, Sarychev & Shalaev, *JNOPM* (2002) - $\mu < 0$ & $n < 0$

Kildishev et al, *JOSA B* (2006); Shvets et al *JOSA* (2006) – strip pairs
(Svirko, et al, *APL* (2001) - “crossed” rods for chirality)

Visible magnetism: structure and geometries



$$t = 35 \text{ nm} \quad d = 40 \text{ nm} \quad p \approx 2w_b$$

Width varies from 50 nm to 127 nm

Purdue group

Yuan, et al., Opt. Expr., 2007 – red light

Cai, et al., Opt. Expr., 2007 – all the visible

Negative Magnetic Response

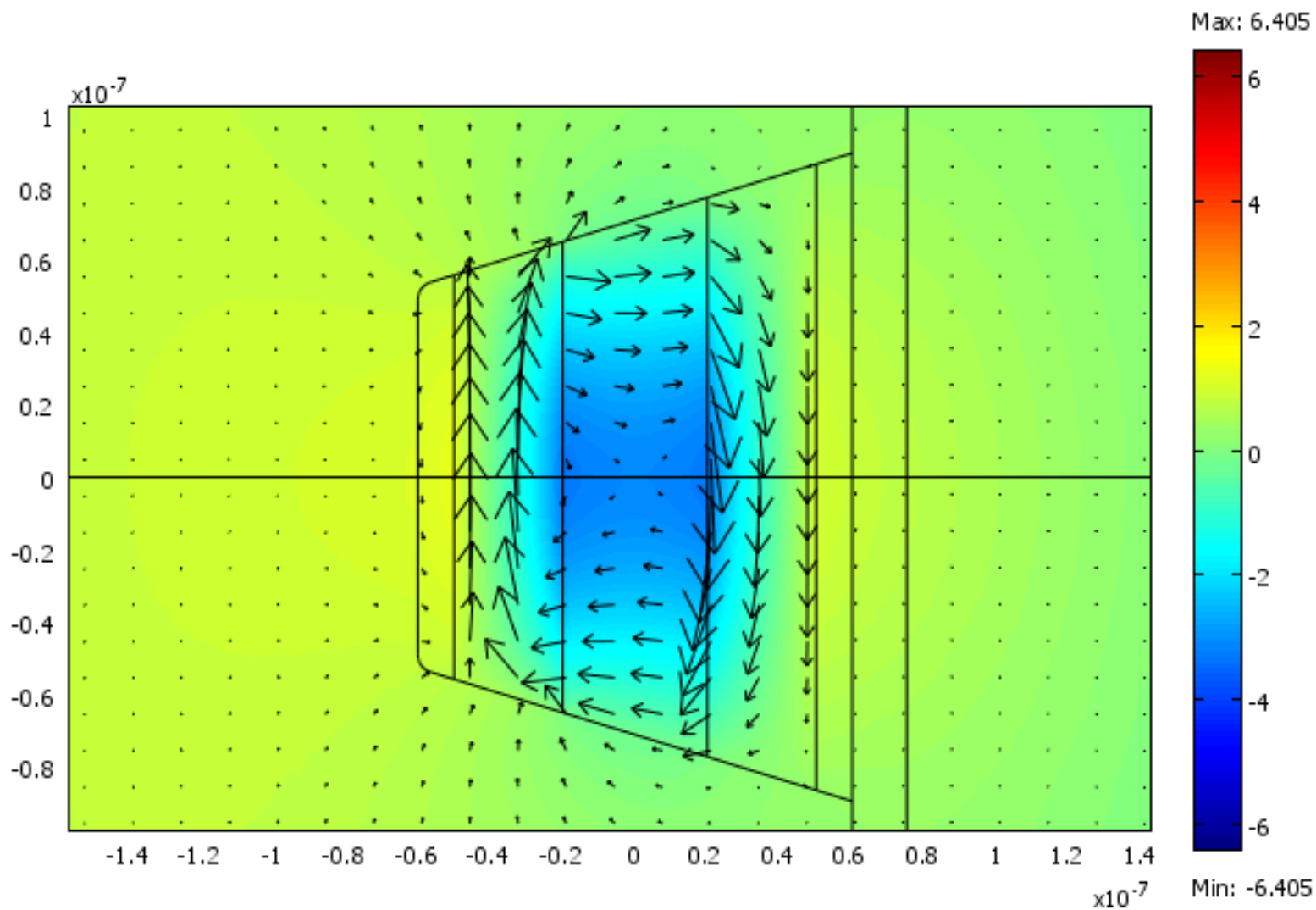


TM

E



k



Negative Magnetic Response

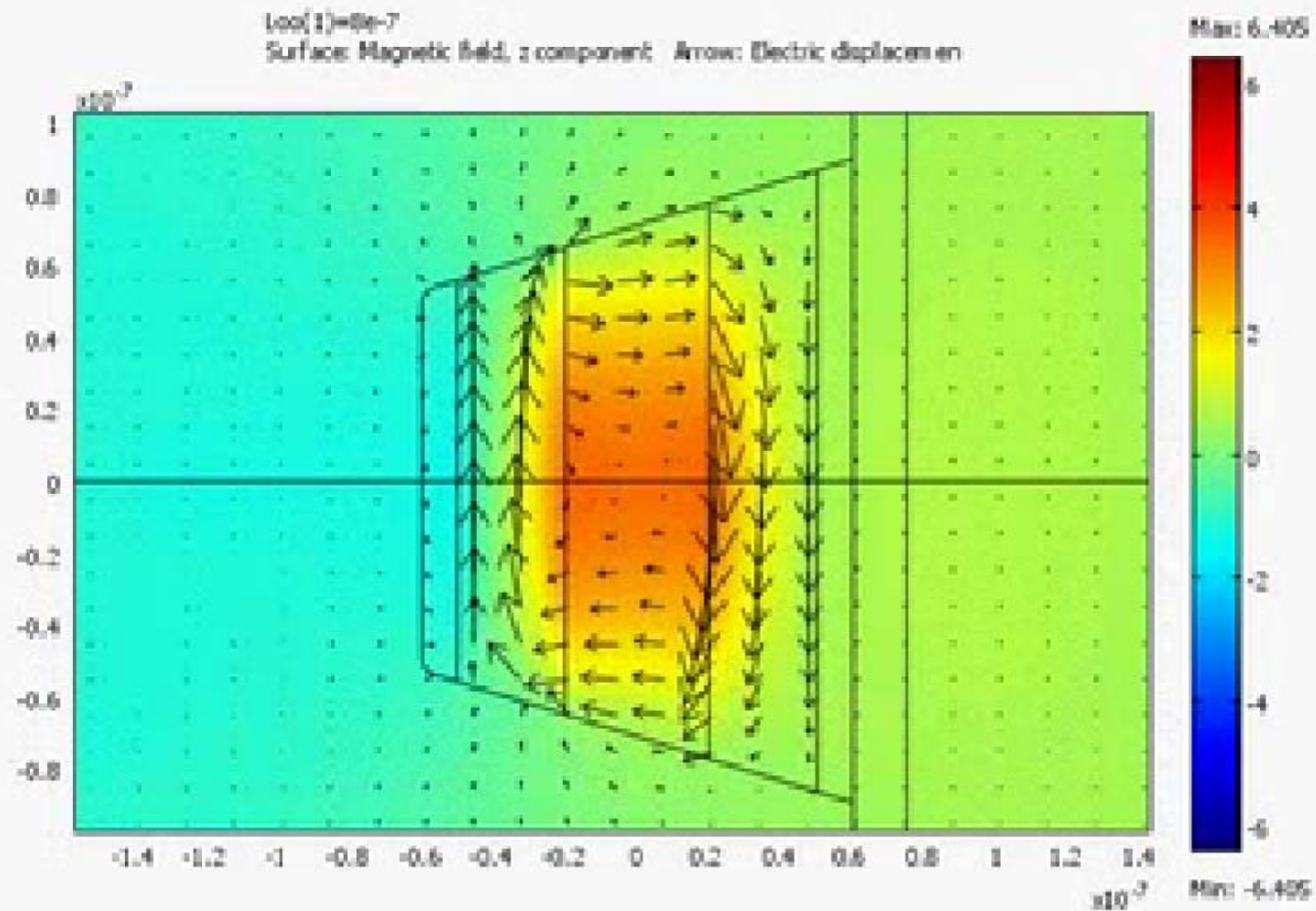


TM

E

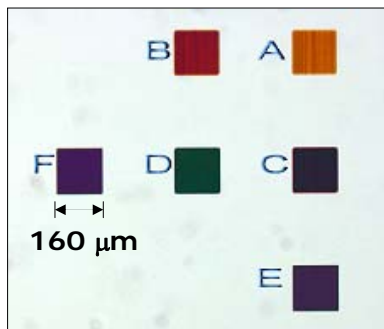


k

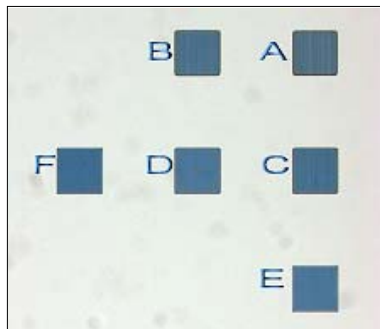


Magnetic Colors: visualizing magnetism

Resonant TM Transmission



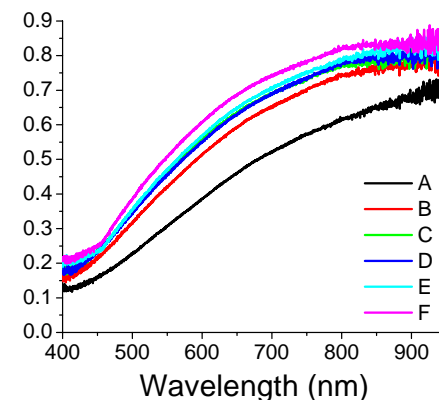
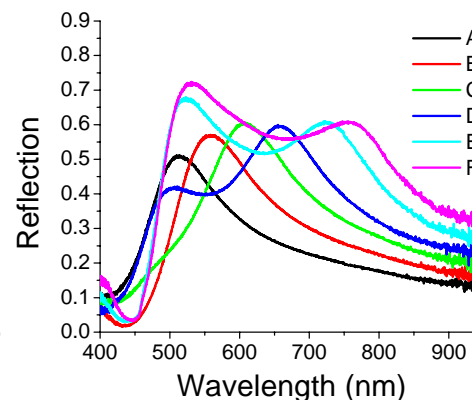
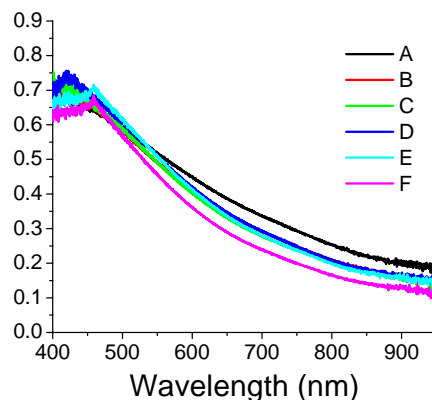
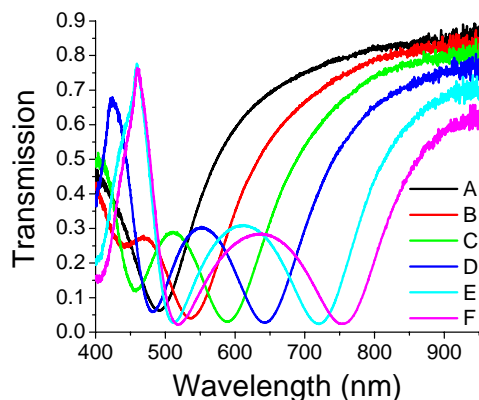
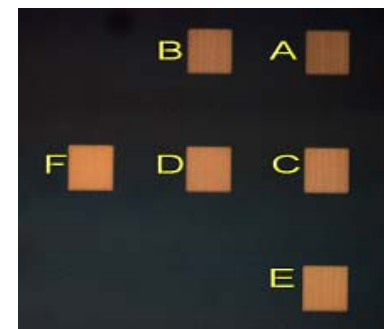
Non-resonant TE Transmission



Resonant TM Reflection



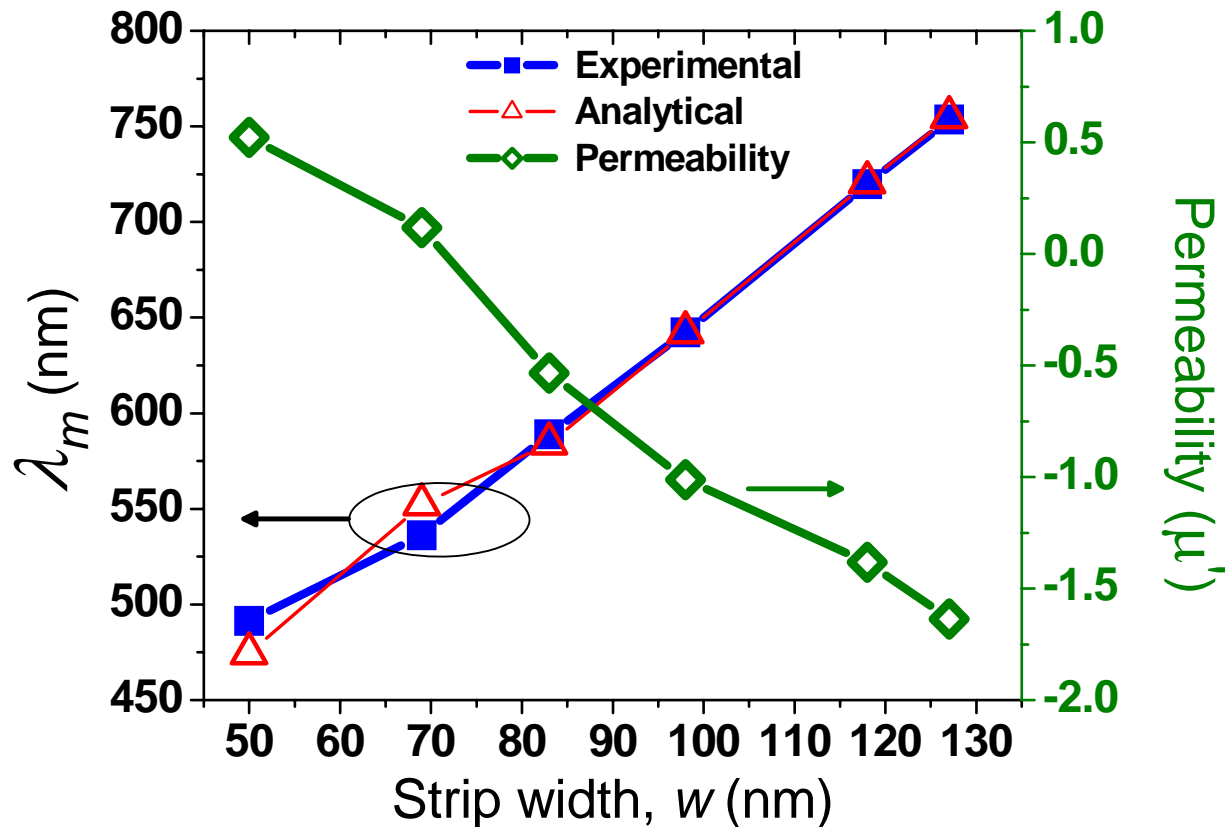
Non-resonant TE Reflection



Sample #	A	B	C	D	E	F
Width w (nm)	50	69	83	98	118	127

Cai, et al., Opt. Expr., 15, 3333 (2007)

Magnetism across the whole visible



λ_m as a function of strip width "w": experiment vs. theory

Negligible saturation effect on size-scaling (as opposed to SRRs)

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