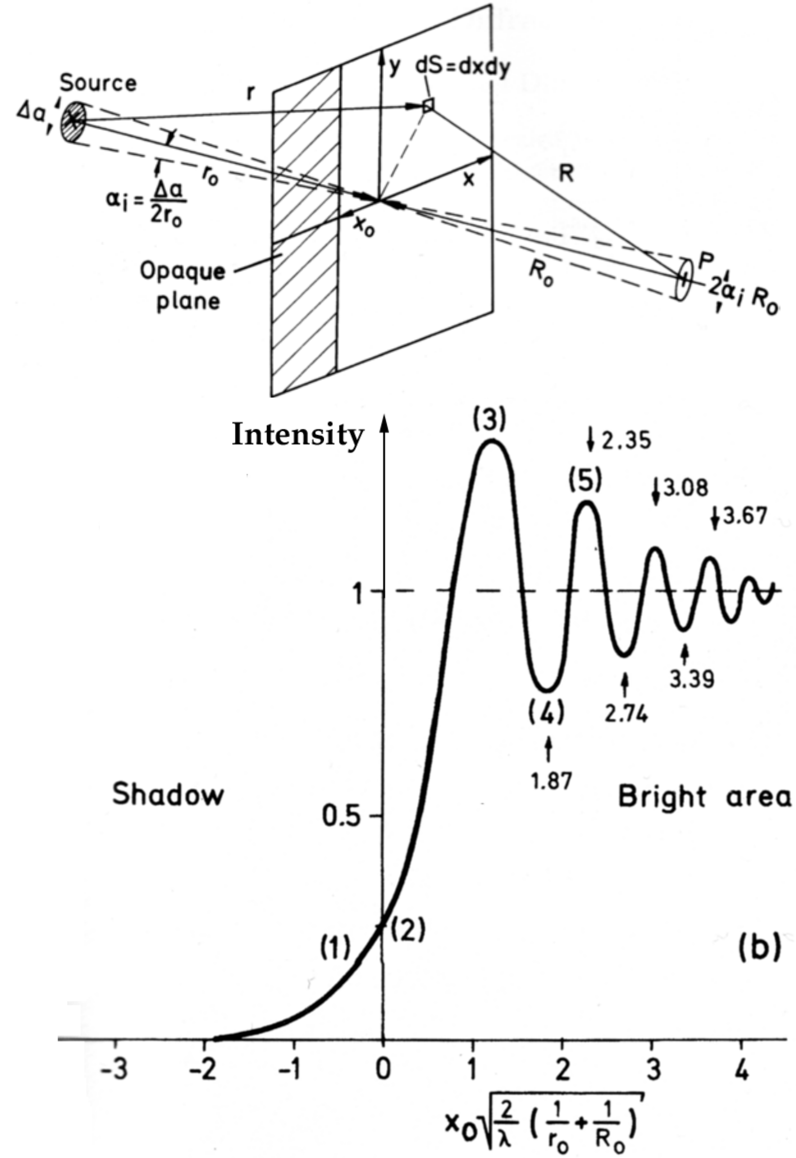
(an embarrassingly short discussion of)

Diffraction

Lecture 8

Fresnel diffraction at an edge

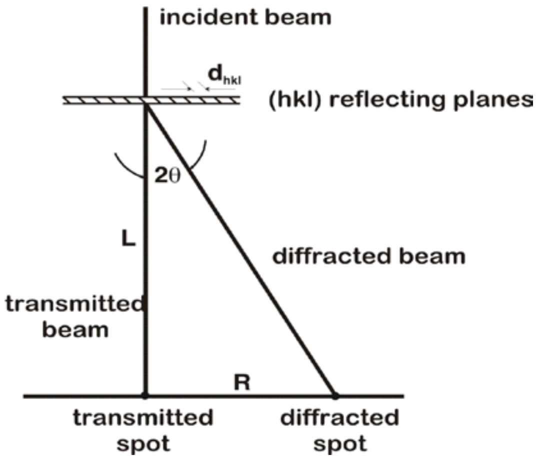
- Fresnel diffraction is 'near field' diffraction
- See the effects in images
- In LaB_6, number of visible images diminished due to:
  - Lack of coherence / finite source size
  - Incident convergence angle
- In FEG, multiple Fresnel fringes observed
  - Greater source coherence



Fraunhofer Diffraction

'Far field' diffraction

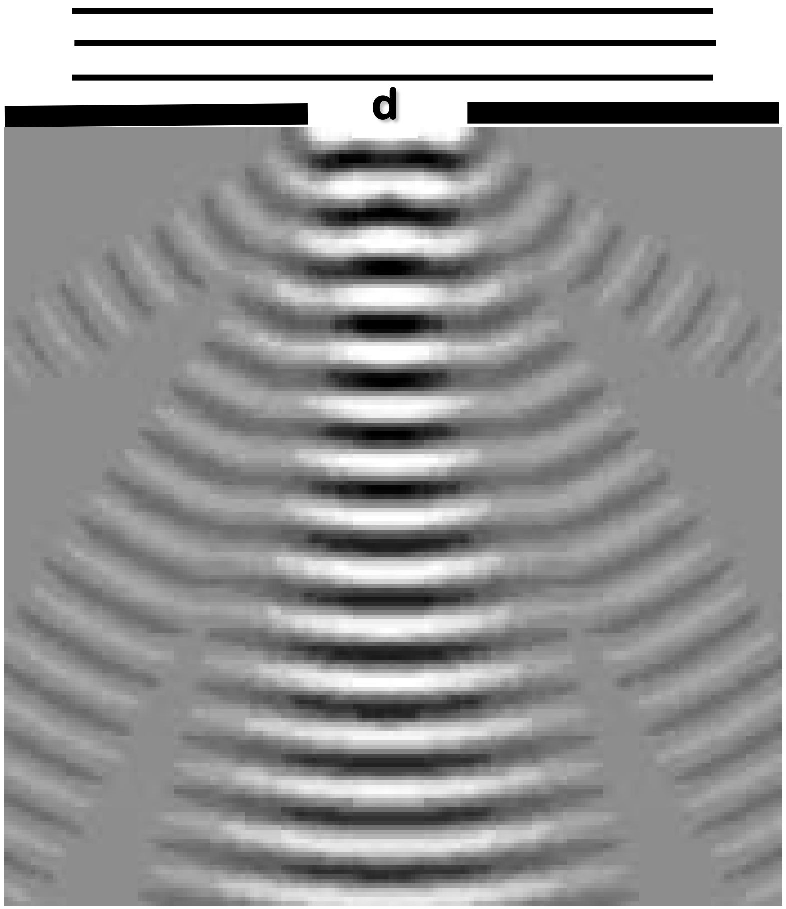
- Looking at the same event from an infinite distance
- i.e. the resultant pattern is independent of distance from screen

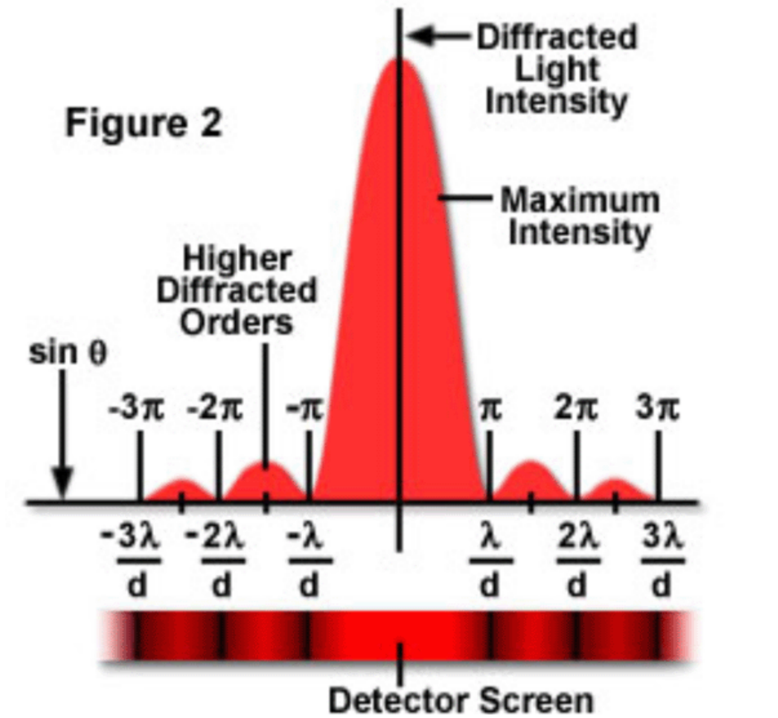


This is also the pattern observed in the back focal plane of the lens

- The back focal plane is effectively at infinite distance
  - Wavelength is 10's of picometers, image planes are on order of millimeters

Diffraction - one slit

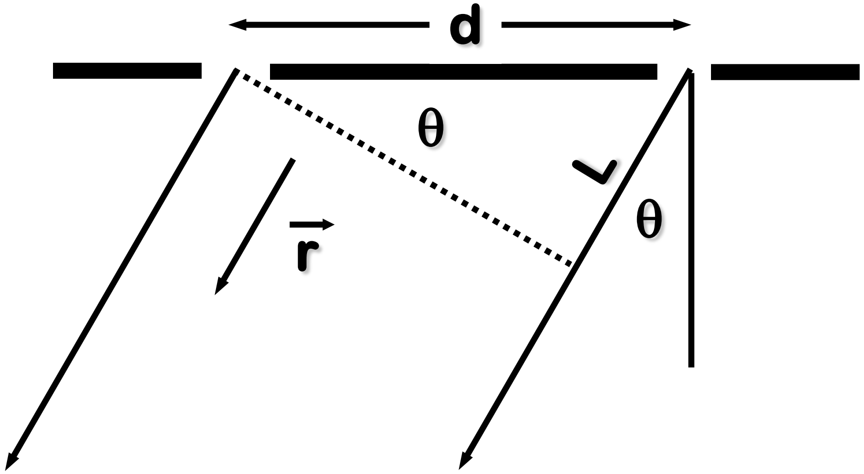


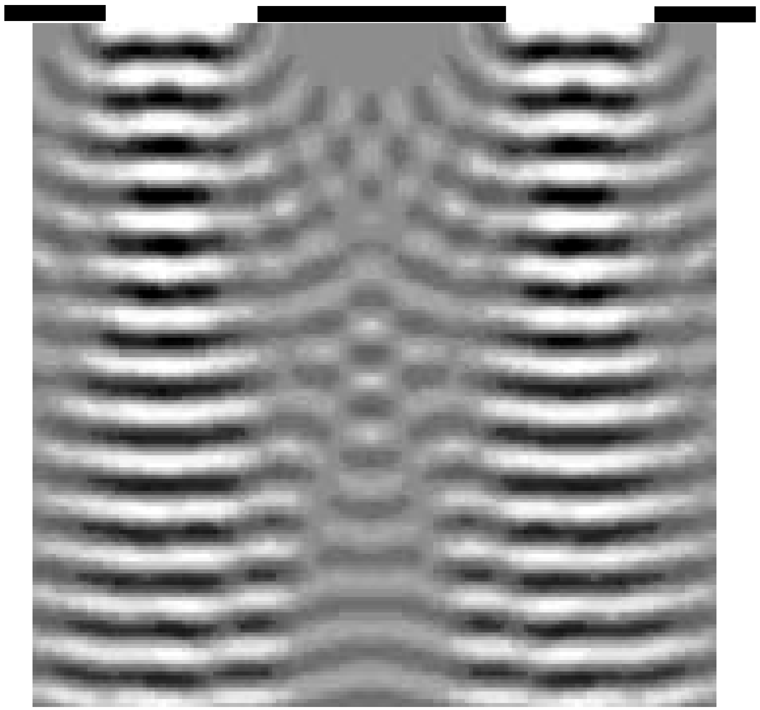


$$I = I_o\left(\frac{\sin\beta}{\beta}\right)^2 \qquad \beta = \frac{\pi d \sin\theta}{\lambda}$$

http://www.micro.magnet.fsu.edu/primer/java/diffraction/basicdiffraction/index.html

Diffraction from two slits





Very narrow slits select just two of the Huygen's wavelets

- These must have the same phase at the slits

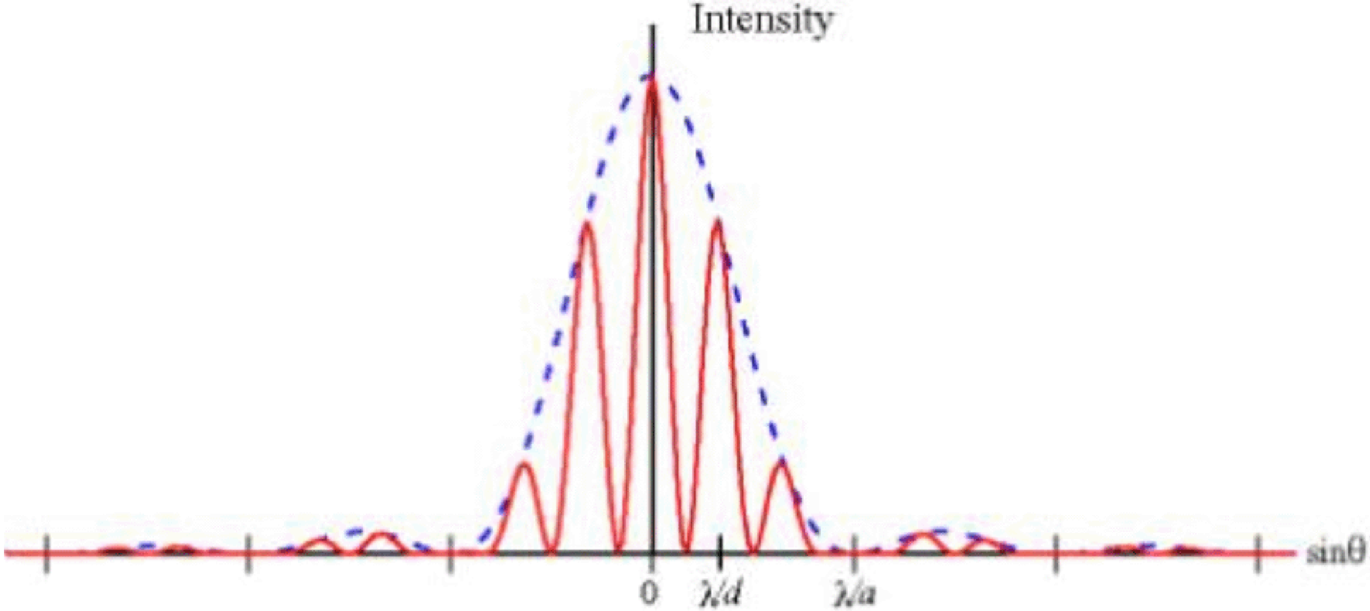
Path difference $L = d \sin\theta$

For an arbitrary direction $\vec{r}$, phase difference $\Delta\phi = 2\pi L/\lambda$

Constructive interference when $d \sin(\theta) = n\lambda$

Diffraction and interference combine

Diffraction from two slits



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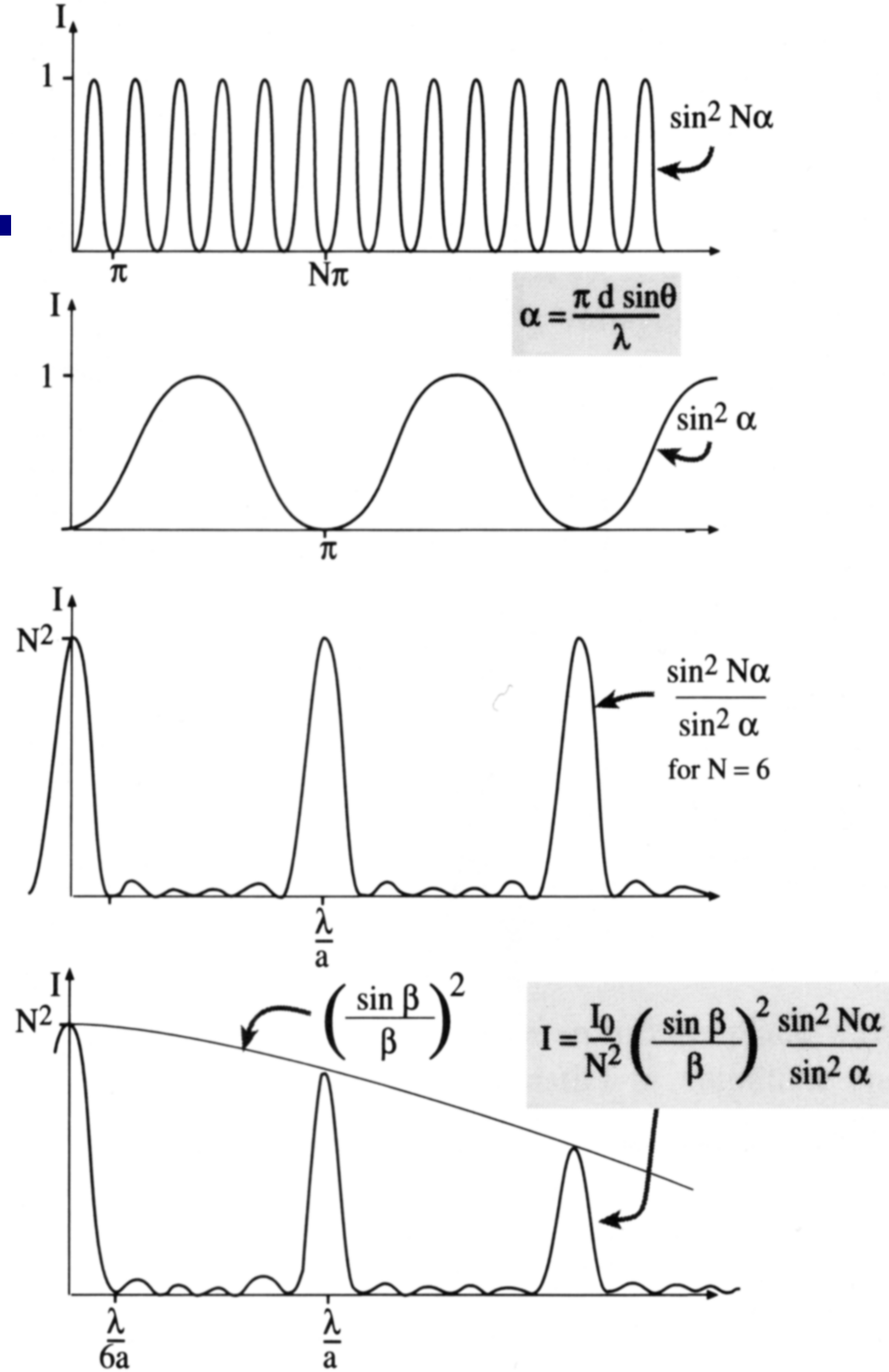
Path difference $L = d \sin\theta$

For an arbitrary direction r, phase difference $\Delta\phi = 2\pi L/\lambda$

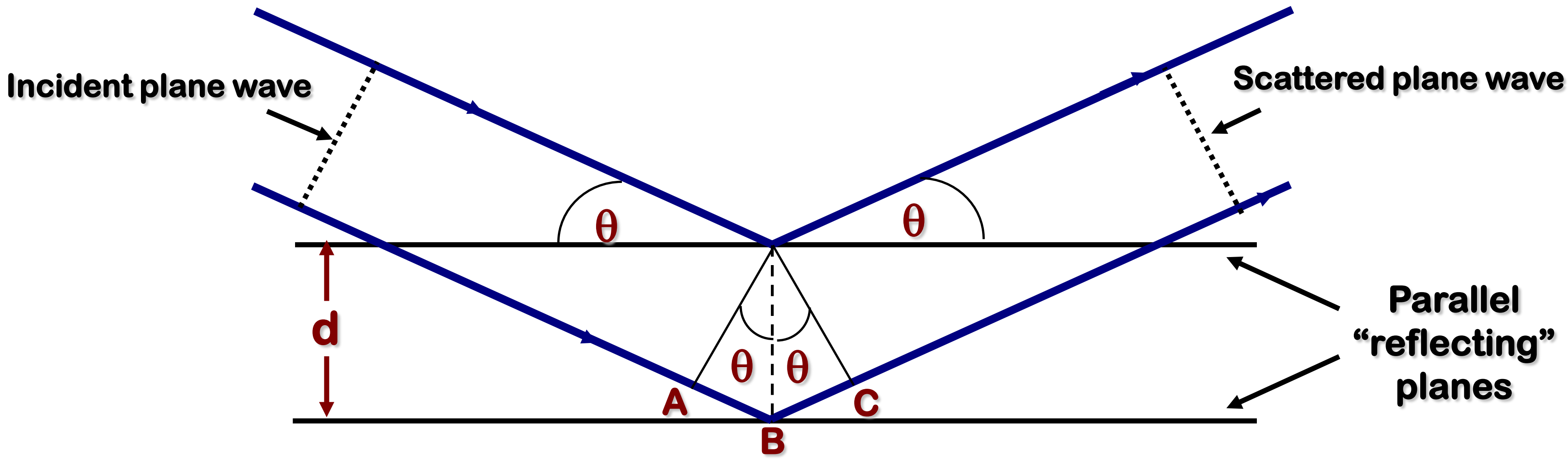
Constructive interference when $d \sin(\theta) = n\lambda$

Diffraction from multiple slits

$$I = \frac{I_o}{N}\left(\frac{\sin\beta}{\beta}\right)^2 \frac{\sin^2 N\alpha}{\sin^2\alpha}$$



Bragg's Law



Here path difference is $\overline{AB} + \overline{BC} = 2 \cdot d \cdot \sin\theta$

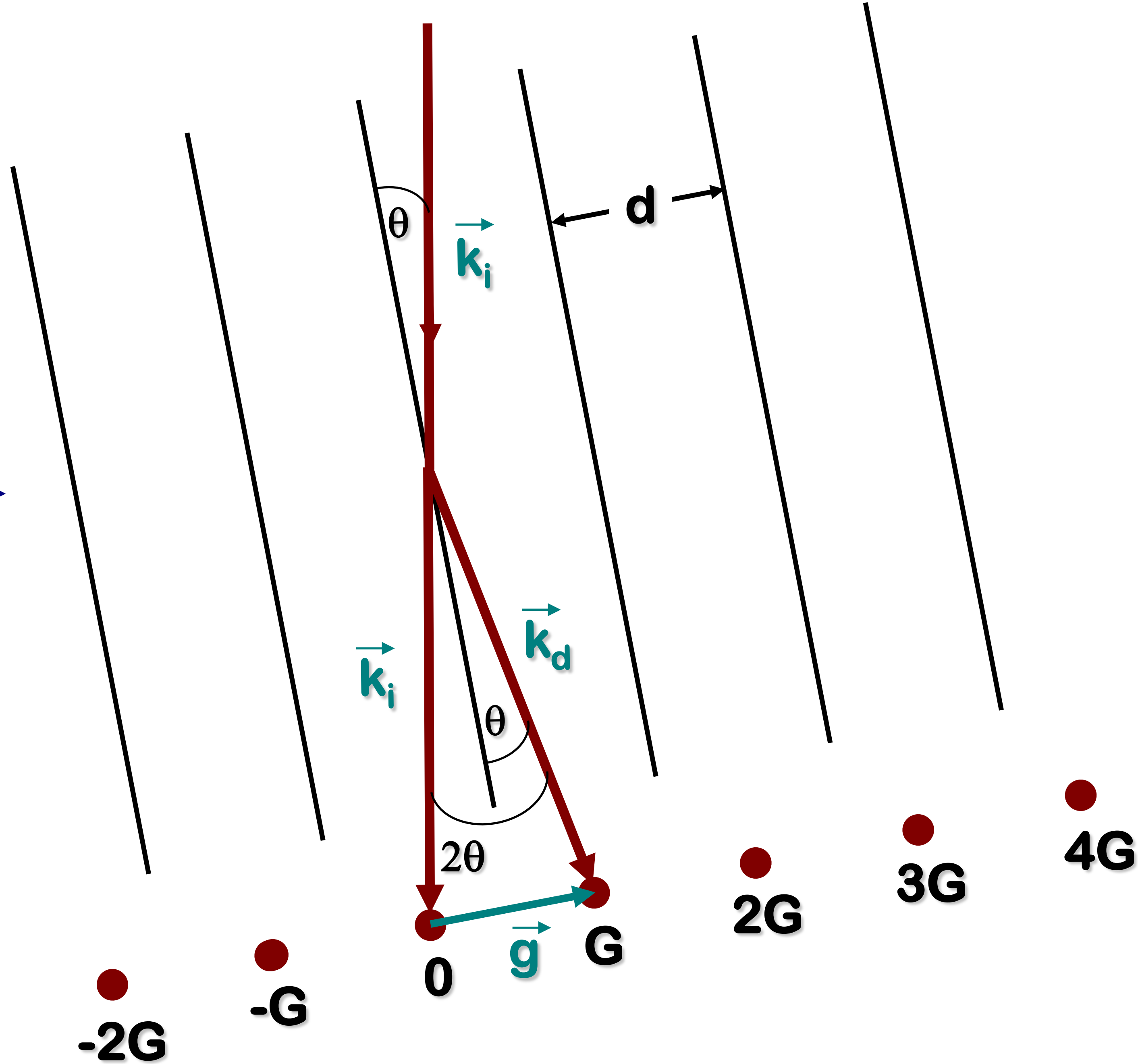
Constructive interference when:

$$n\lambda = 2d\sin\theta$$

$$\vec{k}_d - \vec{k}_i = \vec{K}$$

$$\vec{k}_d - \vec{k}_i = \vec{g}$$

$$\vec{K} = \vec{g}$$



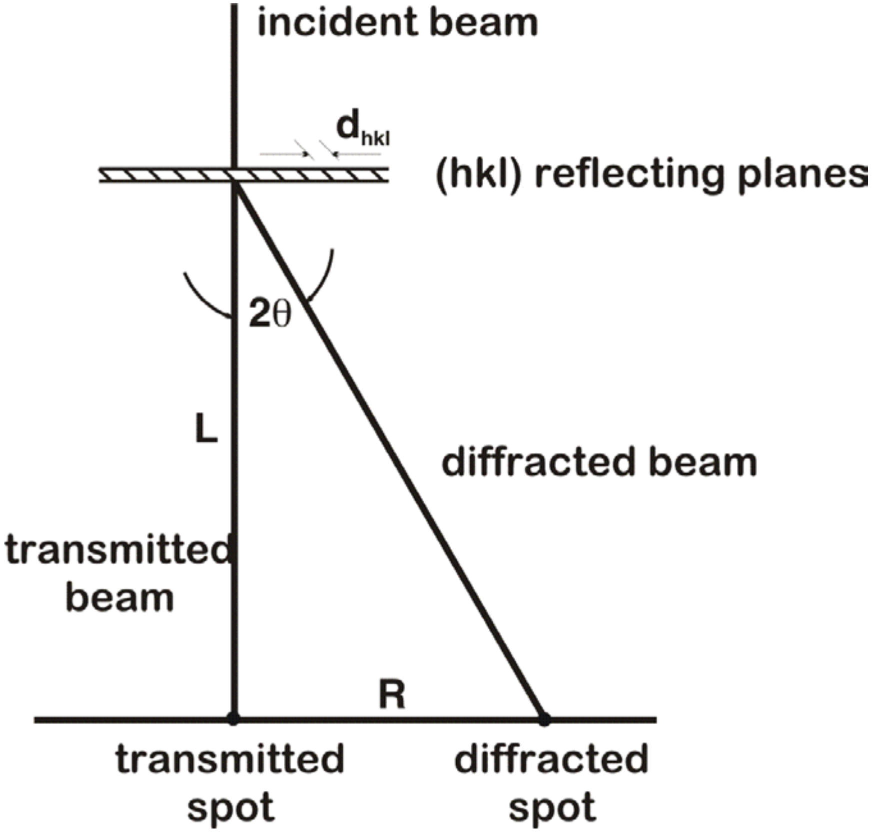
Geometry of diffraction patterns

For small angles (such as we have in the TEM), Bragg Law approximates as:

$$\lambda = 2d\theta$$

Geometry of diffraction shown to right indicates that:

$$R / L = 2\theta$$



Manipulate to get:

$$Rd = \lambda L$$

Key equation

Diffraction pattern types & uses

Diffraction pattern types:

- Spot (Selected area diffraction)
- Ring (multiple crystal - polycrystals)
- Amorphous (Diffuse rings)
- CBED (Convergent beam)
- Kikuchi (Inelastic scattering)
- HOLZ (High order Laue Zones)

From diffraction patterns we can:

- measure the average spacing between layers or rows of atoms
- determine the orientation of a single crystal or grain
- find the crystal structure of an unknown material
- measure the size, shape and internal stress of small crystalline regions

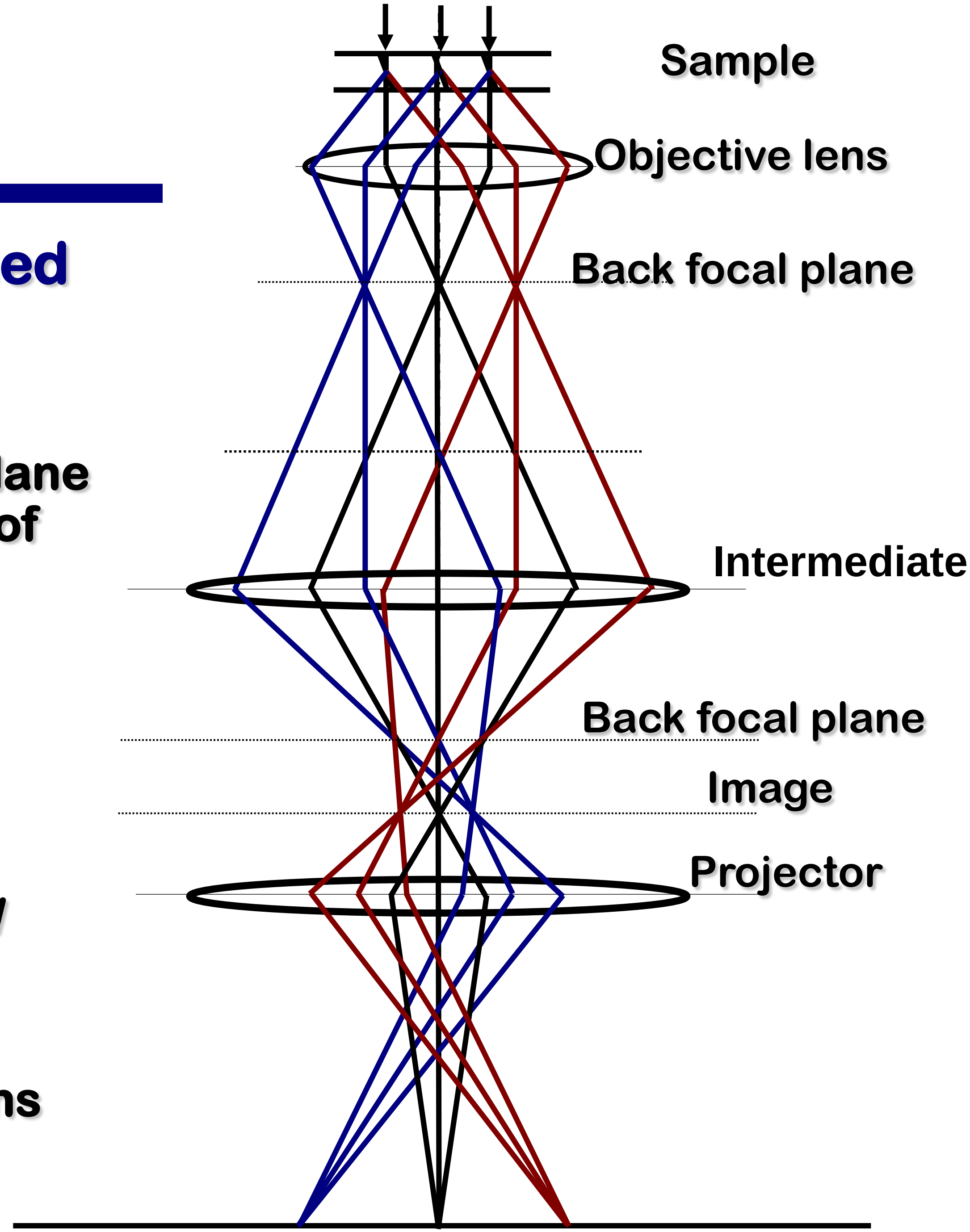
SAD pattern formation

Diffraction pattern formed in back focal plane of objective lens

- Location of back focal plane determined by strength of objective lens (i.e. lens current)

Intermediate lens must focus at this point

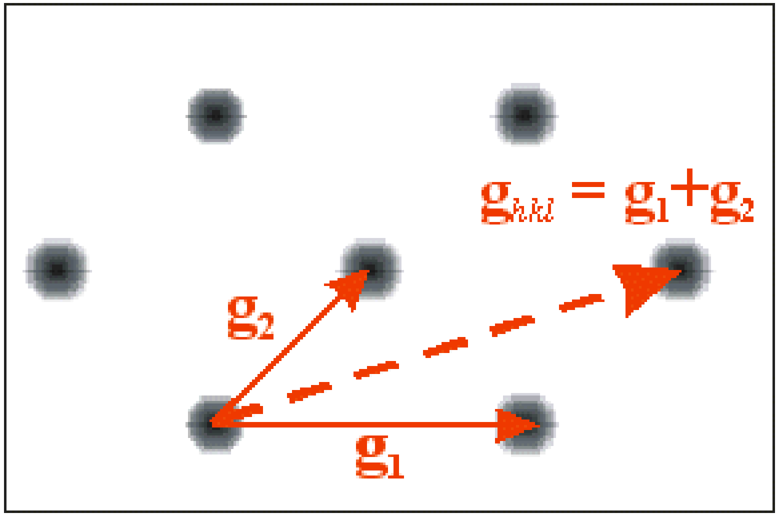
- Need to adjust exact intermediate lens focus / current
- DP magnification thus depends on objective lens current

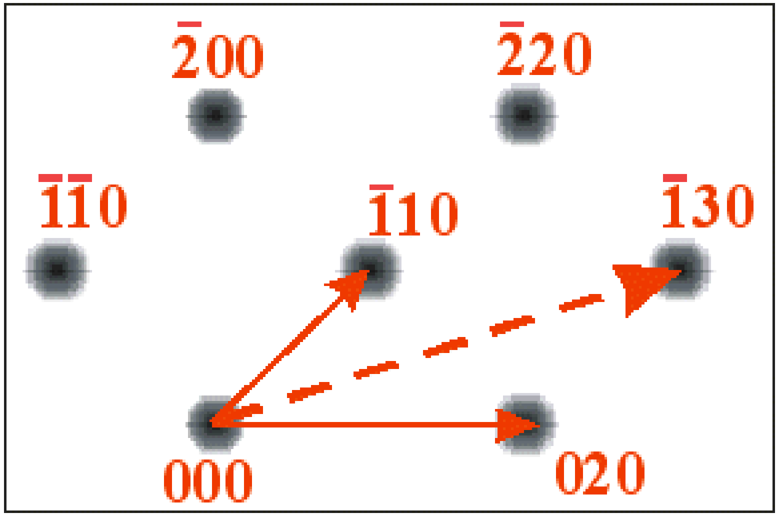


Single crystal materials

If we know the index for closest set of diffraction spots it is possible to index the rest of the spots by using vector addition

Every diffraction spot can be reached by a combination of these two vectors





Indexing single crystal patterns

Tilt sample until in zone axis

- i.e. until crystal is tilted so that the beam is parallel to a zone of planes

Zone axis:

- "Weiss Zone Law"

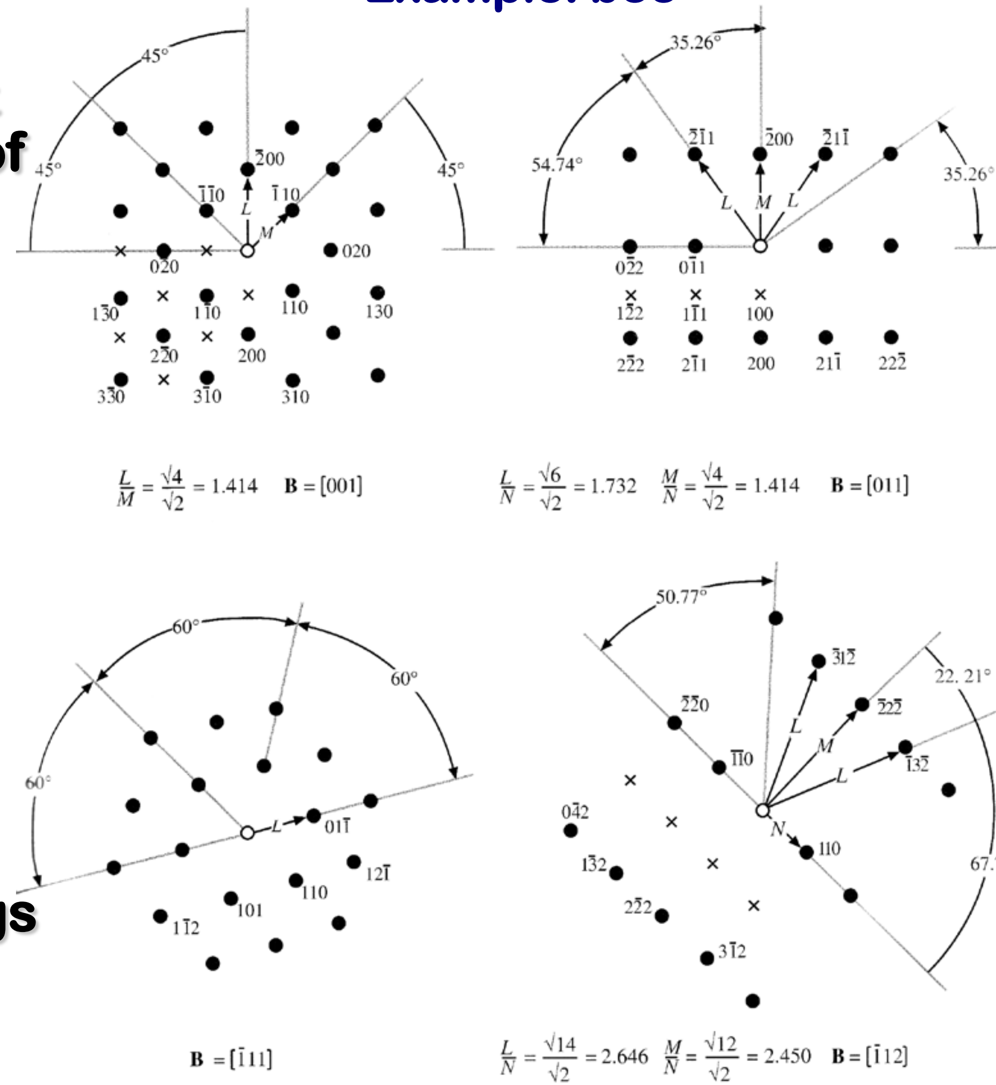
$$hU + kV + lW = 0$$

- Thus (hkl) // to <UVW>

Use $Rd = \lambda L$ = constant

- $R_1 d_1 = R_2 d_2 = R_3 d_3 = \ldots$
- Ratio of d_2 / d_1, d_3 / d_1, etc. characteristic of the zone
- Compare with knowns (e.g. Figs 18.17 - 18.19, or computed patterns)

Example: bcc



Polycrystalline materials

In small grained samples, random distribution results in rings in DP

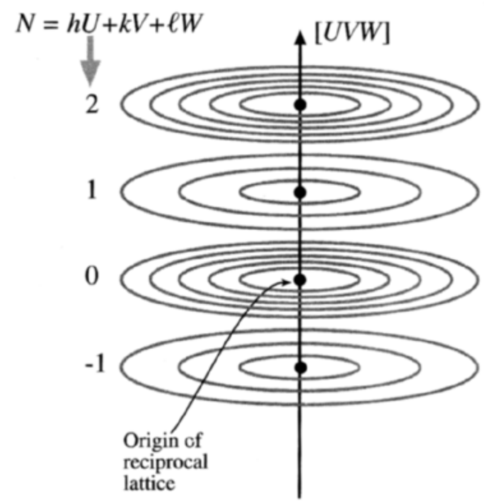
Indexing is simple:

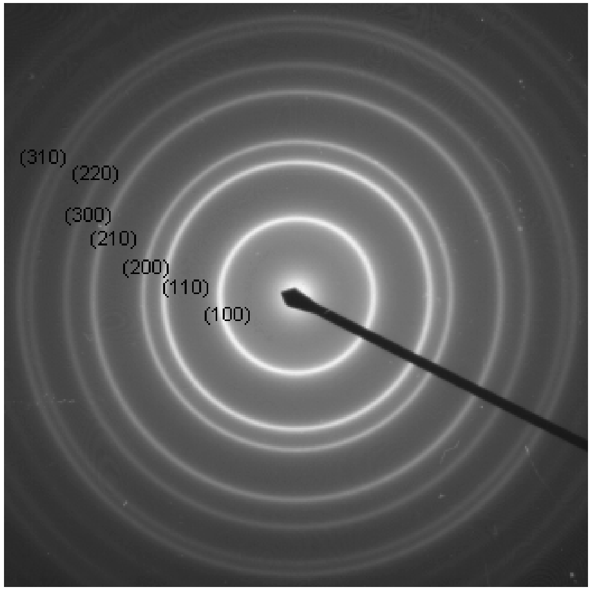
$$Rd = \lambda L$$

Measure R on pattern

Camera contrast λL is calibrated

- Strongly dependent on objective lens current
- Must be certain you use the same current when calibrating with respect to a known standard

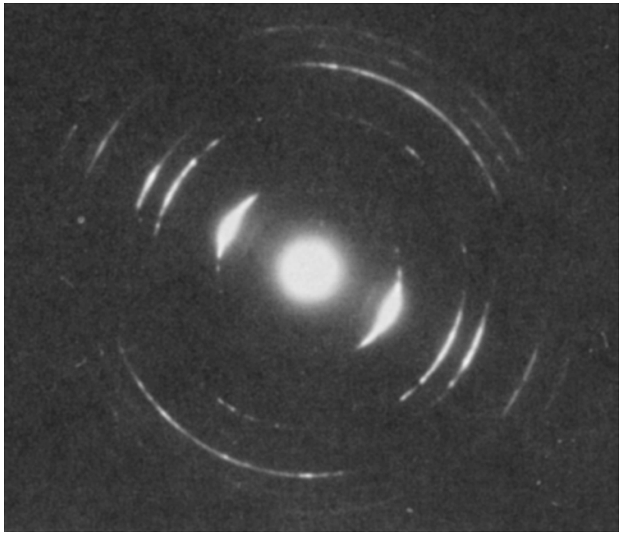


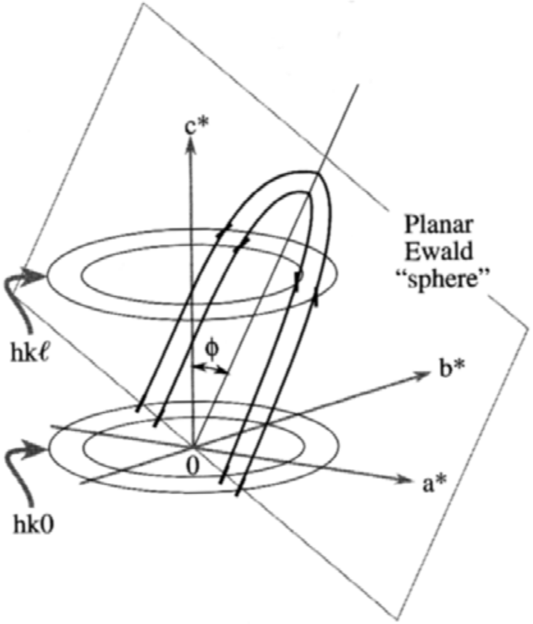


Texture in ring patterns

Texture: Distribution of orientations is not random, but preferred

Readily visible in ring patterns as arcs of intensity





Amorphous diffraction patterns

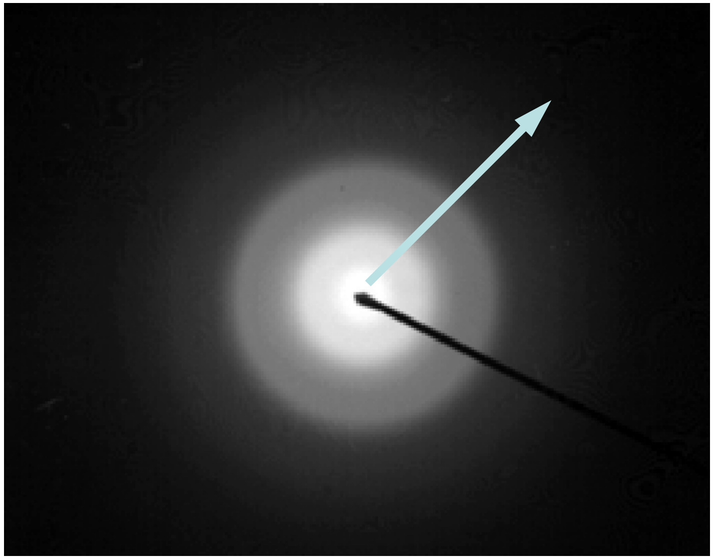
Amorphous materials do not have ‘random’ placement of atoms

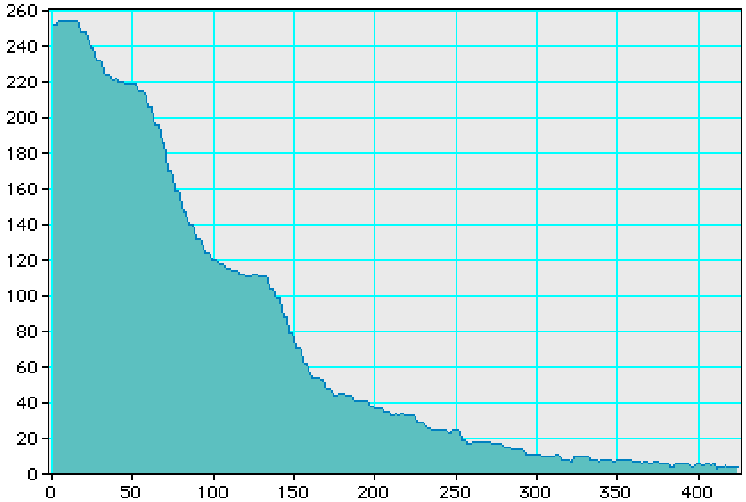
Instead distance between neighbors follows a probability function

- Radial distribution function
- Can be recorded and measured

New technique - “Fluctuation Microscopy”

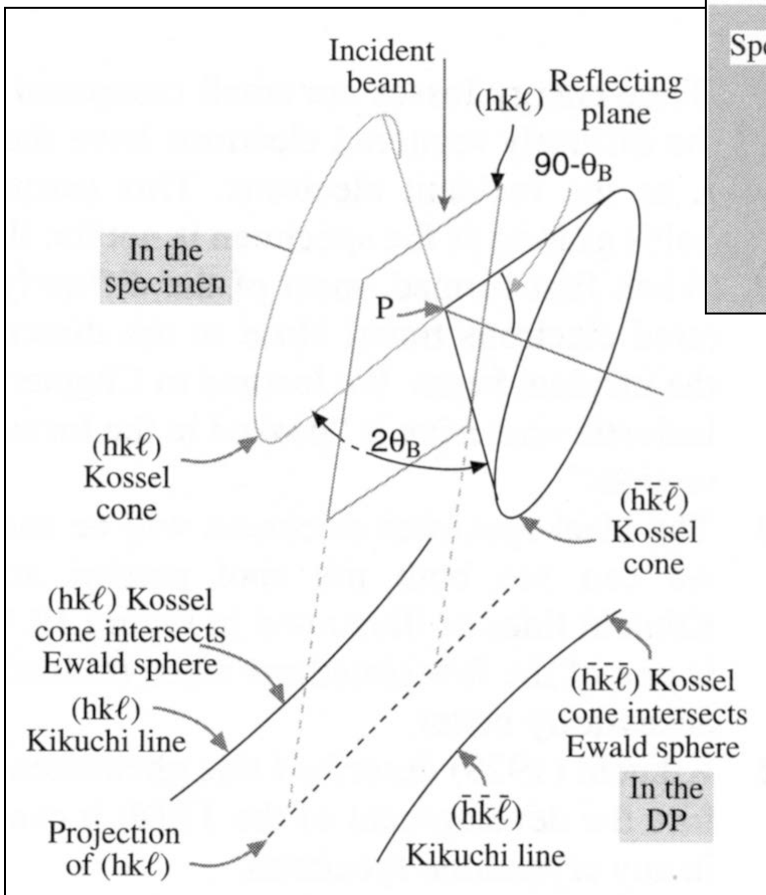
- Second order correlations
- “Medium range order”

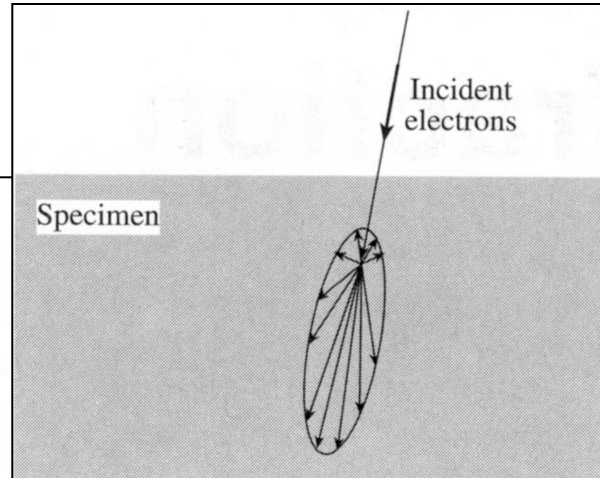


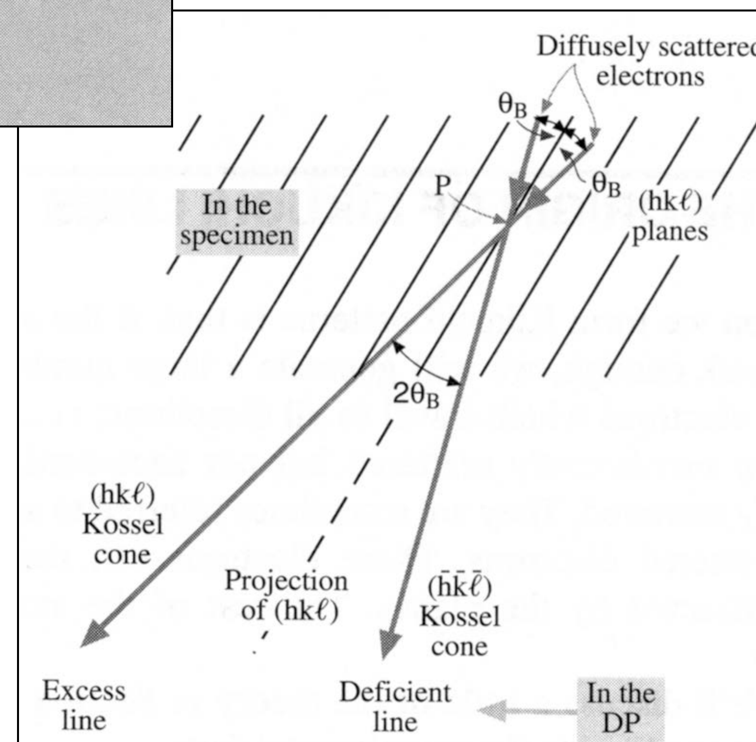


DP from amorphous TiO_2 and measured RDF

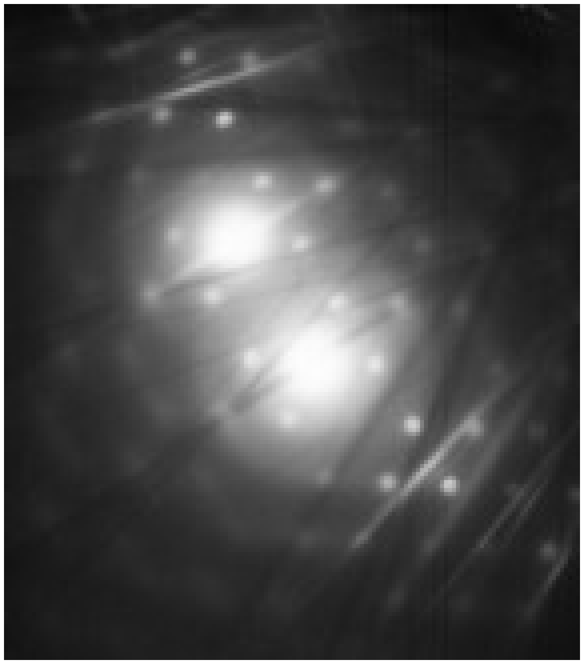
Kikuchi diffraction





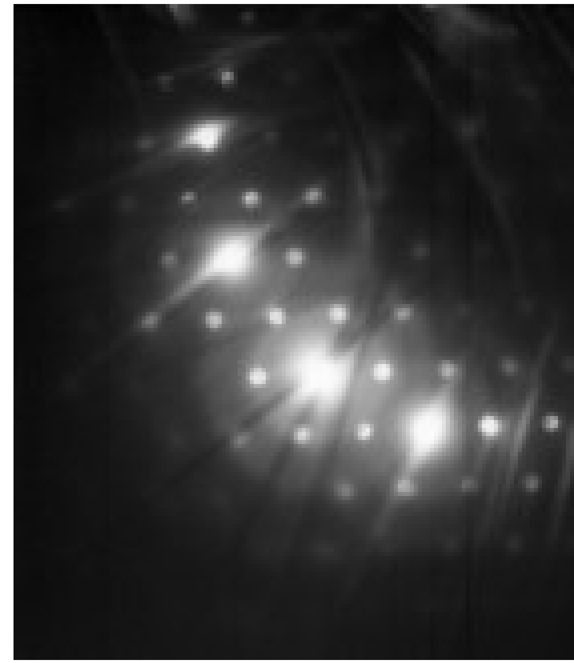


Kikuchi diffraction



The Kikuchi lines pass straight through the transmitted and diffracted spots. The diffracting planes are therefore tilted at exactly the Bragg angle to the optic axis

The crystal has now been titled slightly away from the Bragg angle, so that the Kikuchi lines no longer pass through the transmitted and diffracted spots.



Here the crystal is tilted so that more that one set of planes are diffracting. Each set of diffracting planes has its own pair of Kikuchi lines.