# Lenses, Apertures and Resolution

## Lecture 3

# Lens, apertures & resolution

## Review of optics

- Ray diagrams, optical elements, lens equation, magnification, demagnification, focus

## Electron lenses

- How they work, how electrons travel through them

## Apertures & diaphragms

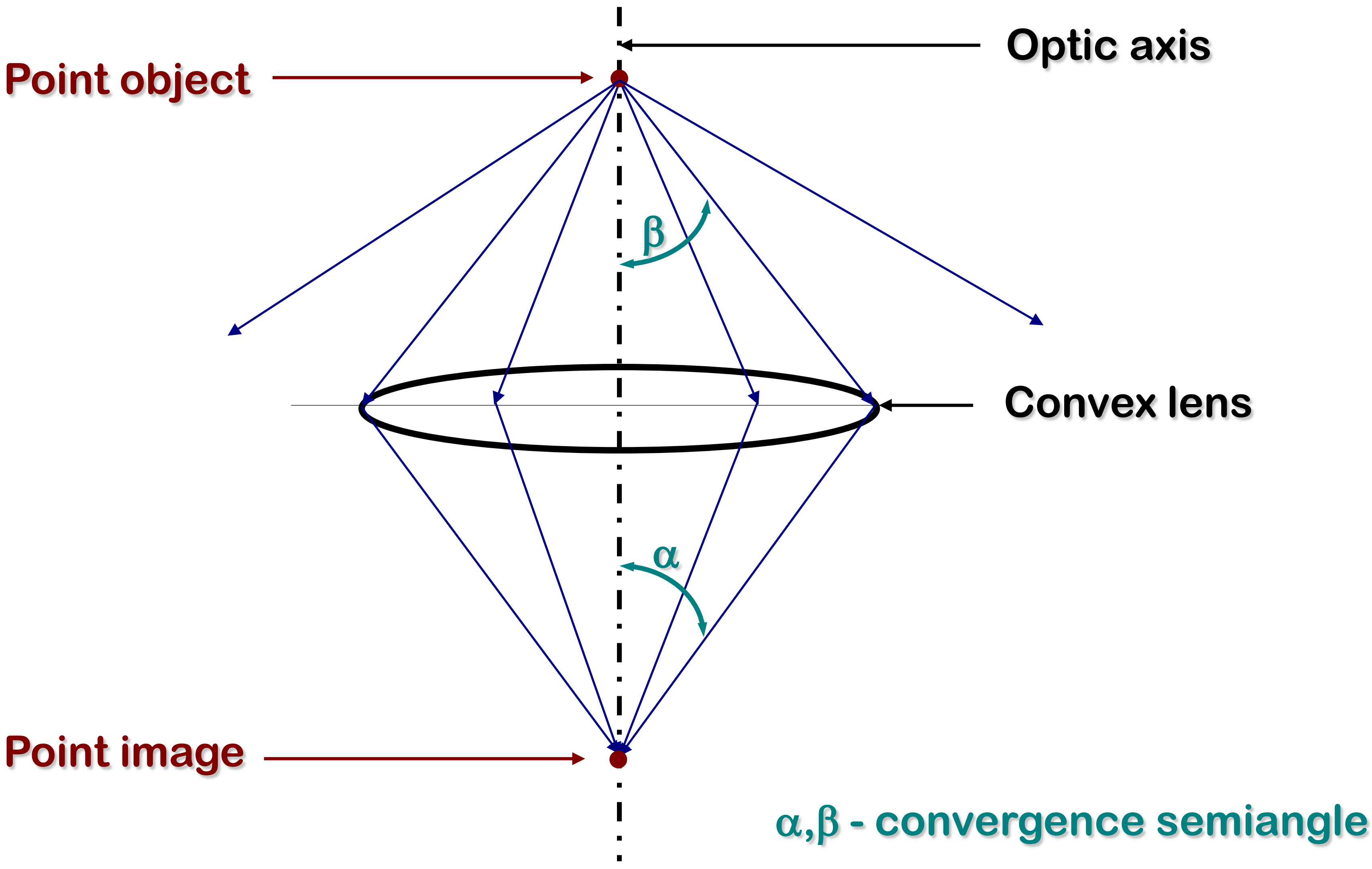
## (Primary) aberrations

## Resolution

## Depth of focus / depth of field

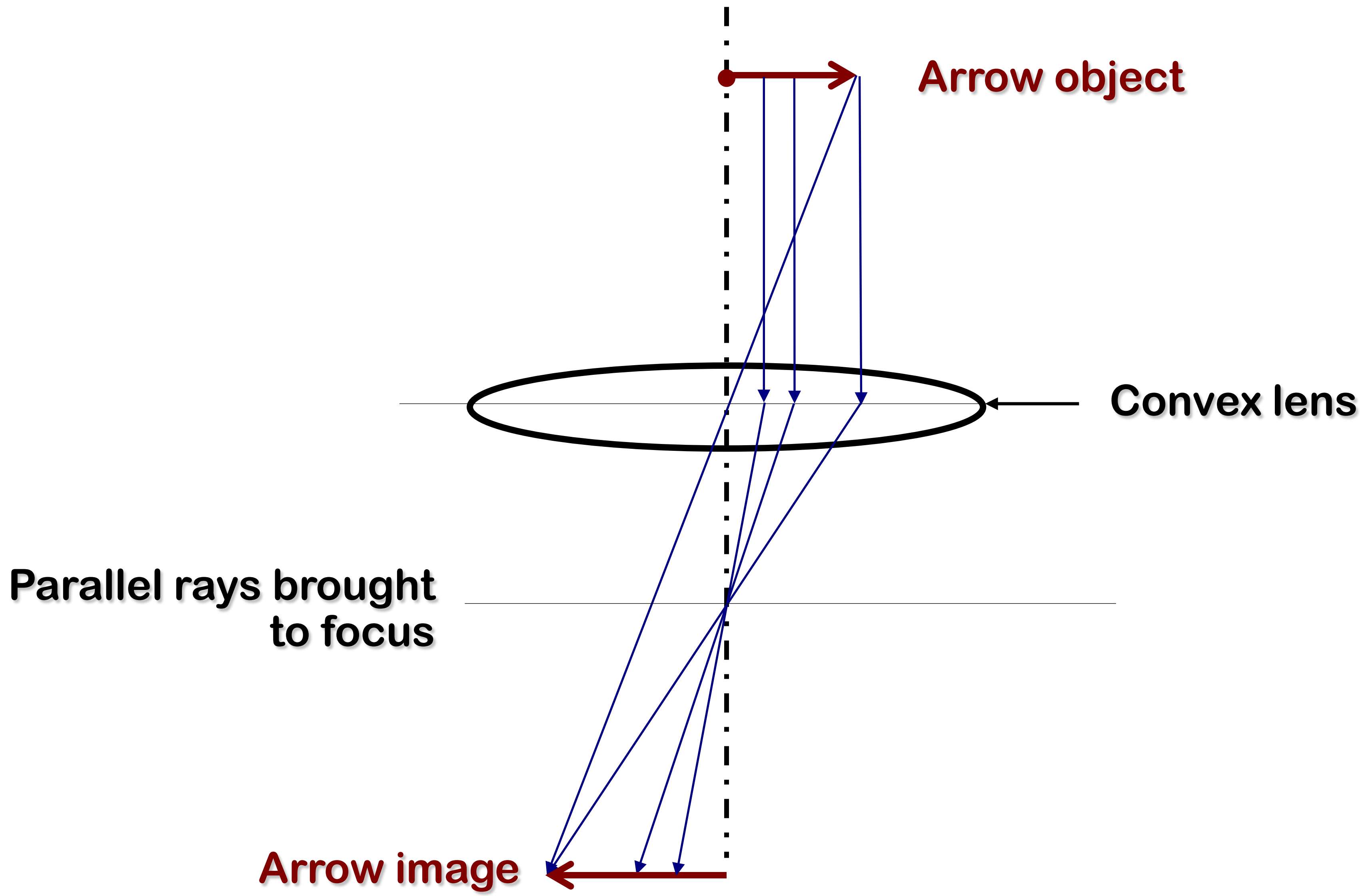
# Lenses

## Ray diagrams



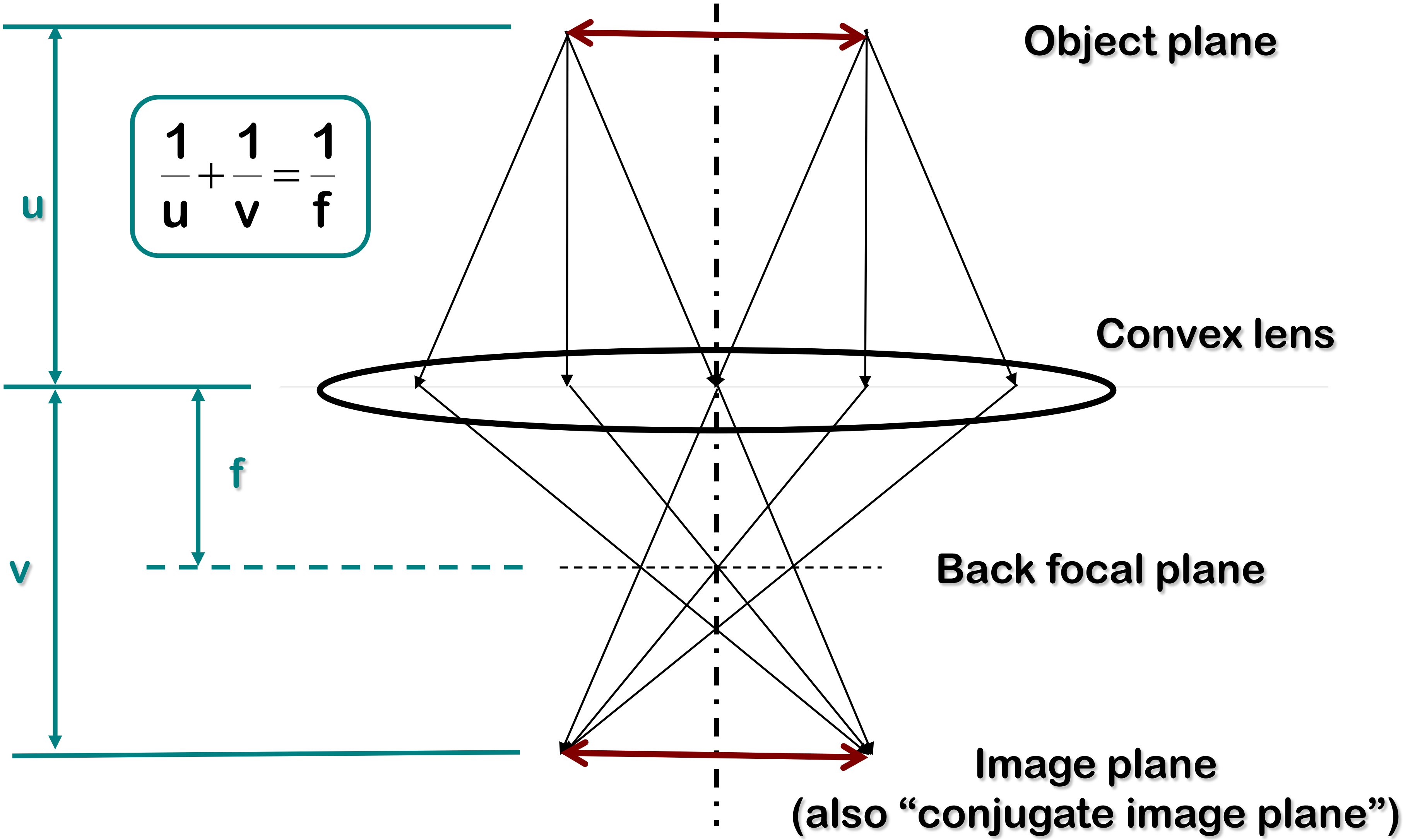
# Lenses

## Ray diagrams



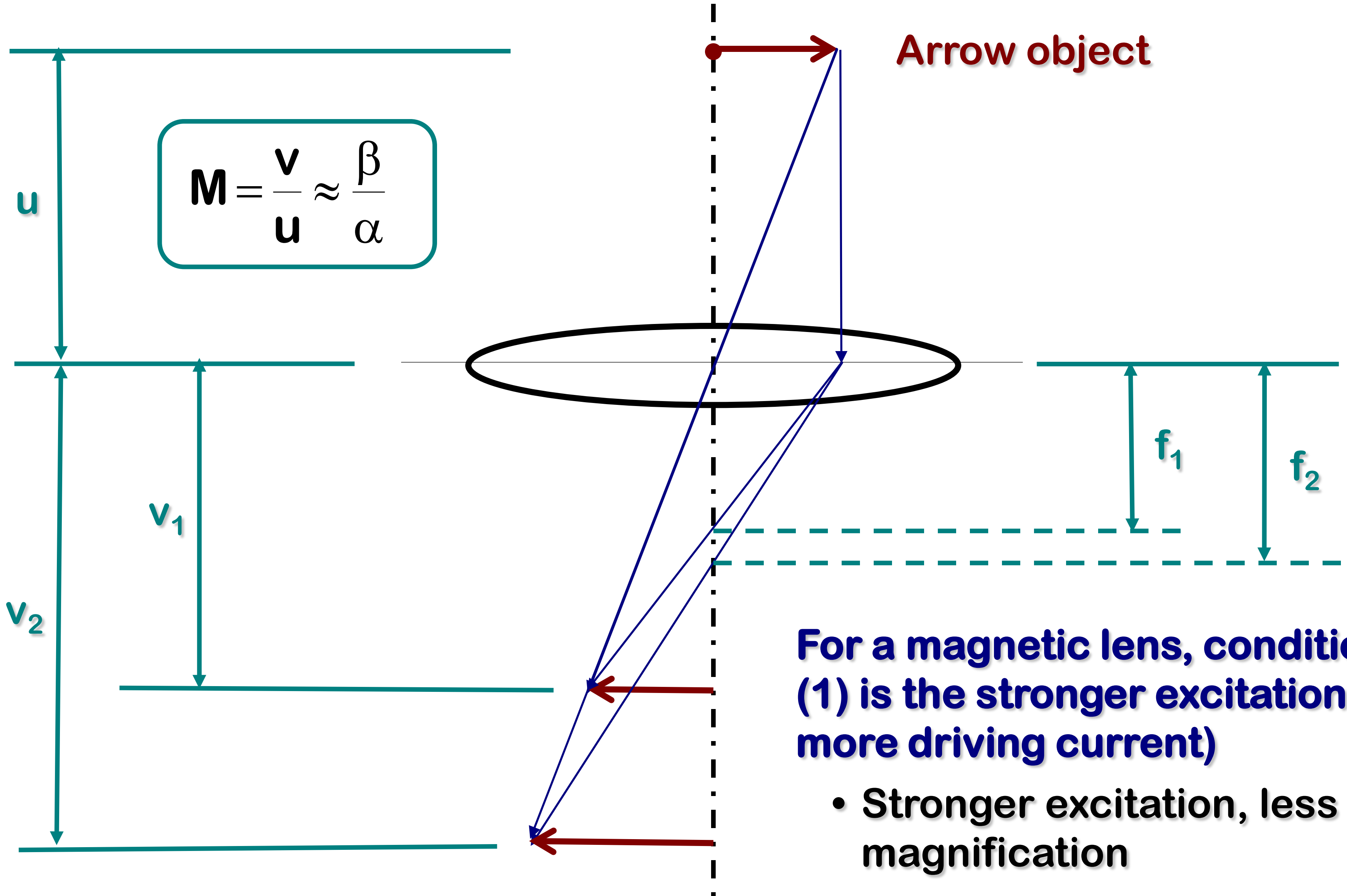
# Lenses

## Ray diagrams



# Lenses

## Ray diagrams



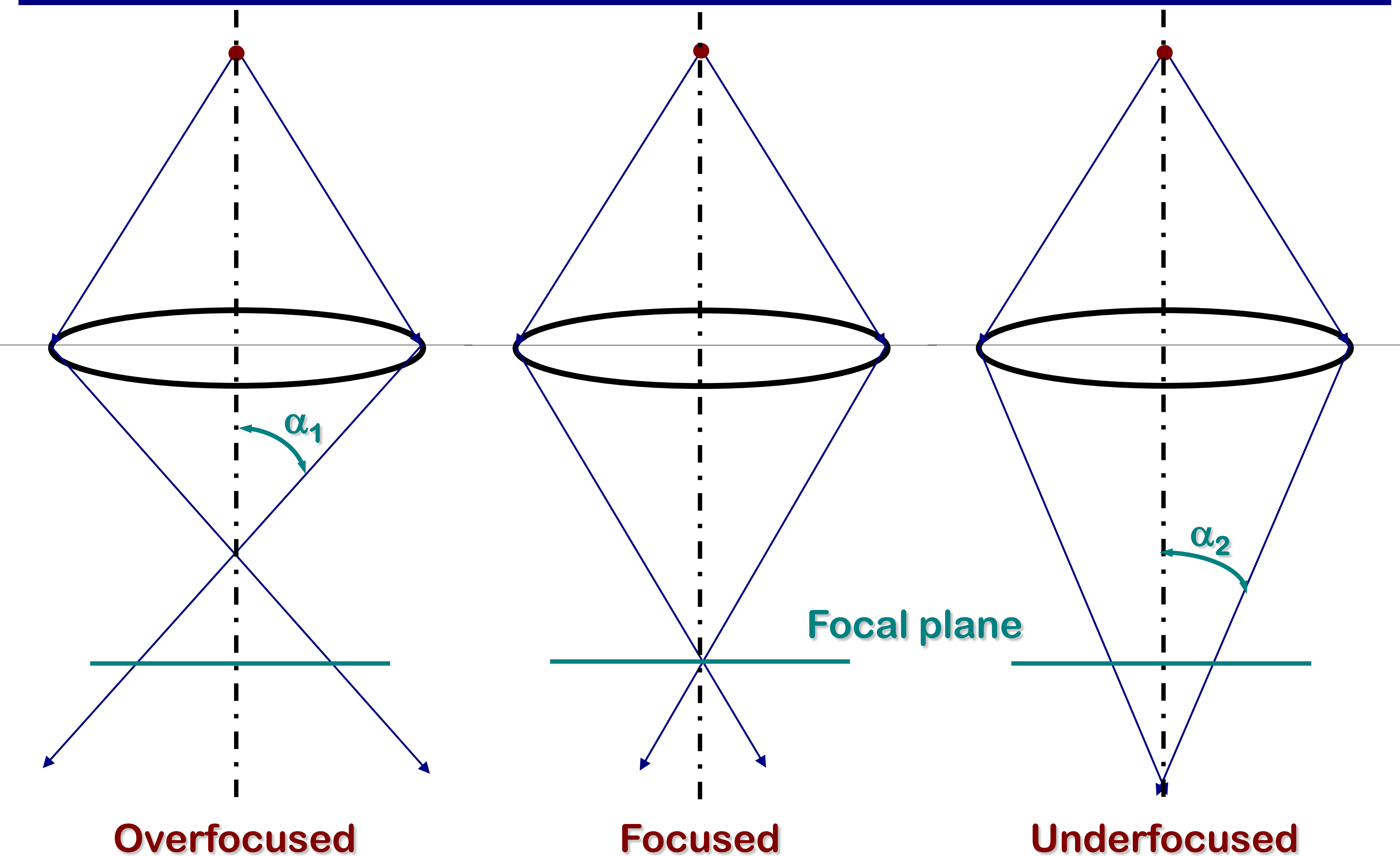

$$M = \frac{v}{u} \approx \frac{\beta}{\alpha}$$

**For a magnetic lens, condition (1) is the stronger excitation (i.e. more driving current)**

- Stronger excitation, less magnification

# Lenses

## Ray diagrams



# Lenses

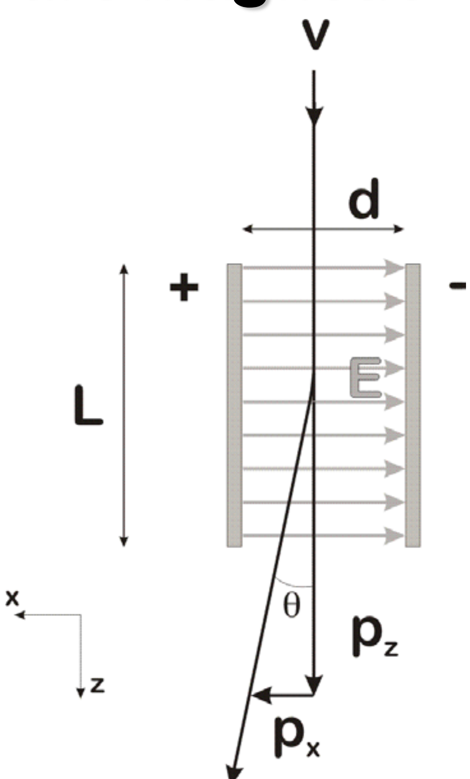
## Electric & magnetic fields

**Both electric and magnetic fields used to steer the electron beam**

- Scan coils are electrostatic
- Lenses are magnetic

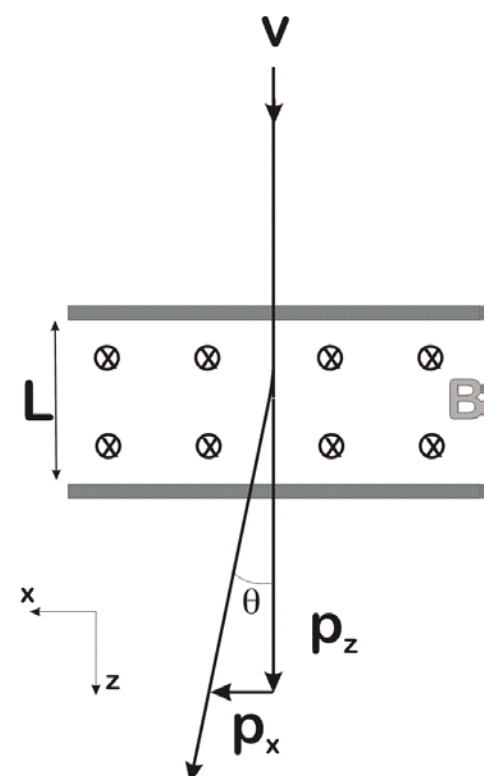
$$p_x = \frac{e|E|L}{v}$$

$$\theta = \frac{e|E|L}{mv^2}$$




$$p_x = e|B|L$$

$$\theta = \frac{e|B|L}{mv}$$



# Lenses

## Magnetic fields

### Lorentz force:

$$F = -e(E + v \wedge B) = e(v \times B)$$

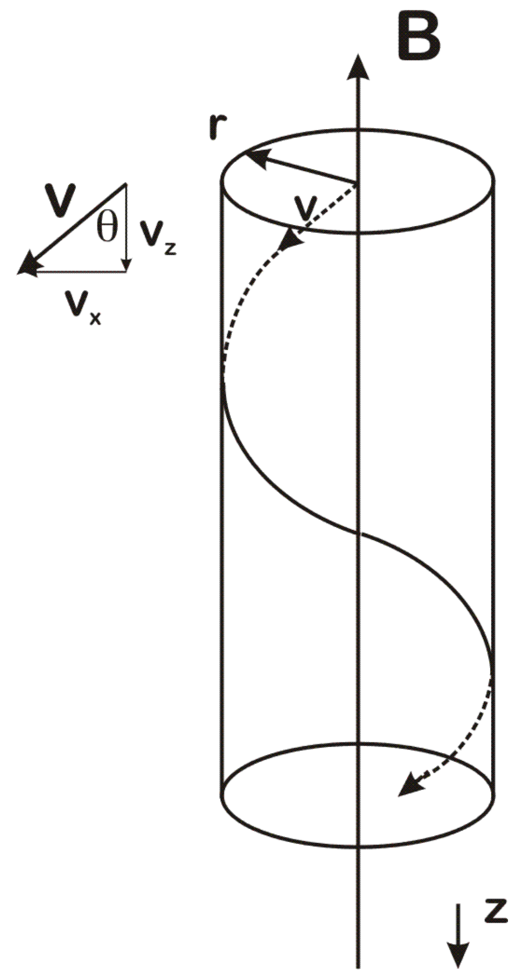
$$F = evB\sin\theta \approx evB = \frac{mv^2}{r}$$

$$r = \frac{mv}{eB}$$

Relativistic correction

$$r = \frac{\left[2m_oE\left(1 + \frac{E}{2E_o}\right)\right]^{1/2}}{B}$$

w/ B=1 Tesla, E = 100 kV $\Rightarrow r \approx 1$mm




### Cyclotron frequency:

$$\omega = \frac{2\pi}{T} = \frac{eB}{m}$$

# Lenses

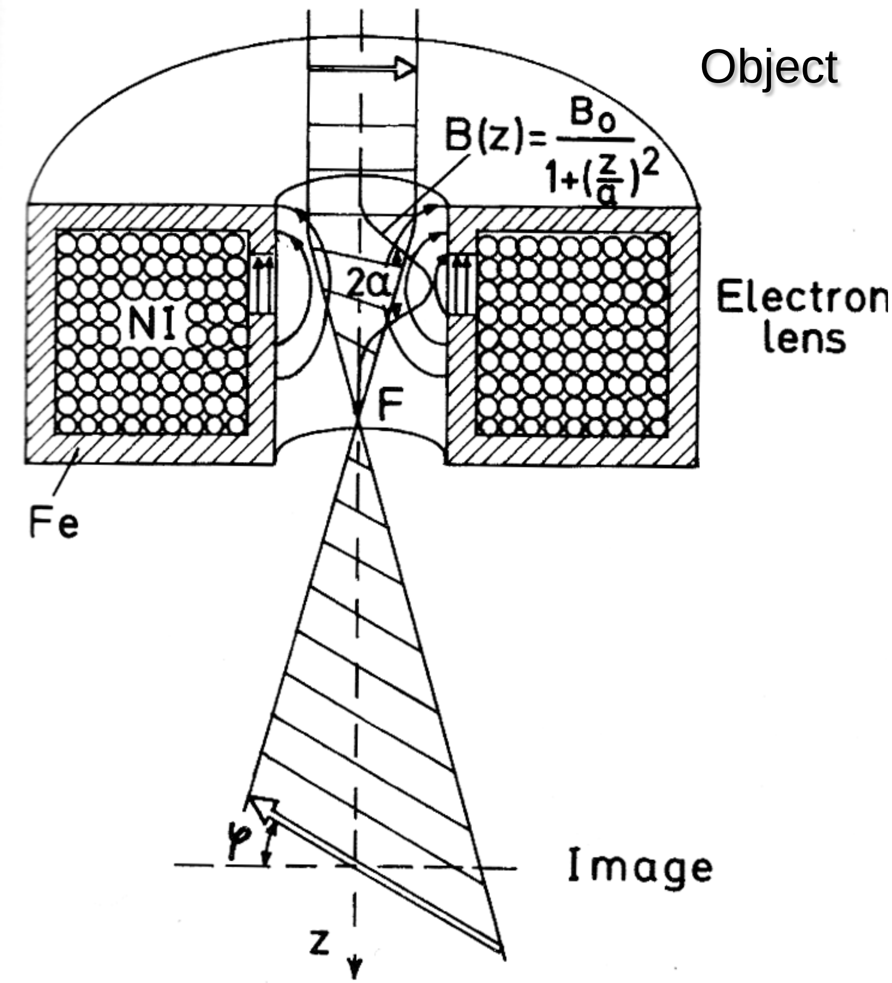
## Magnetic fields

Rotation of electron results in image rotation

Old microscopes must have this calibrated

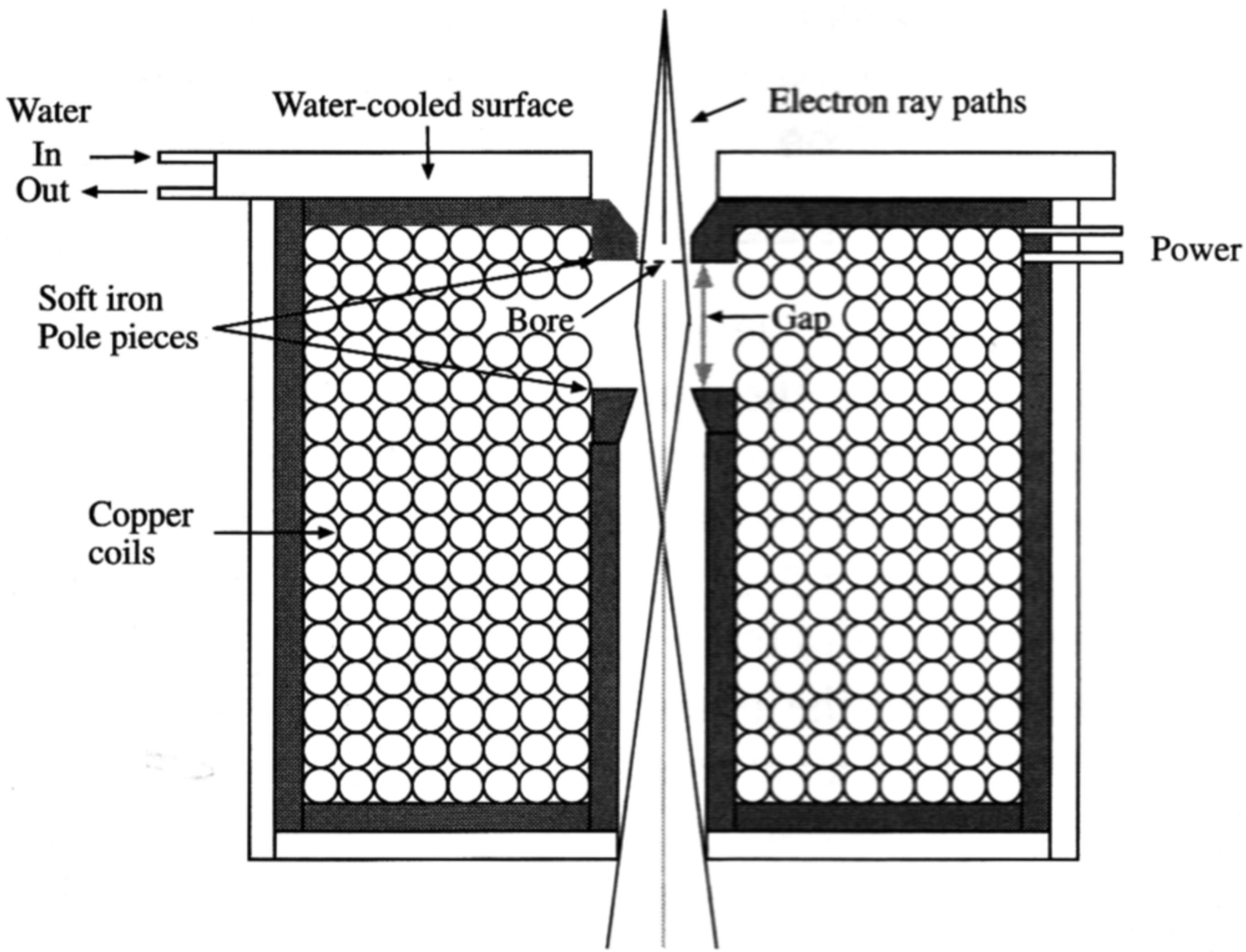
New microscopes add an extra projection lens

- Lens action coordinated to remove this rotation



# Lenses

## Electron lenses



# Lenses

## Electron lenses

## Objective lens must be strong

- Want specimen close to plane of objective lens (small u, large M)

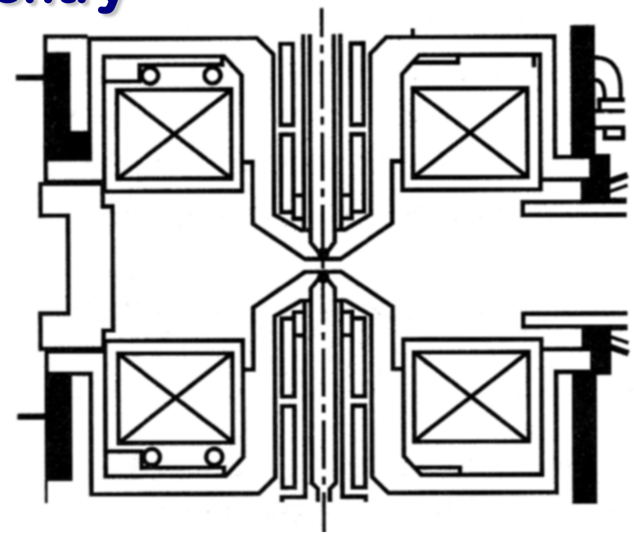
## Side entry

- Greater flexibility for sample rotation / probing

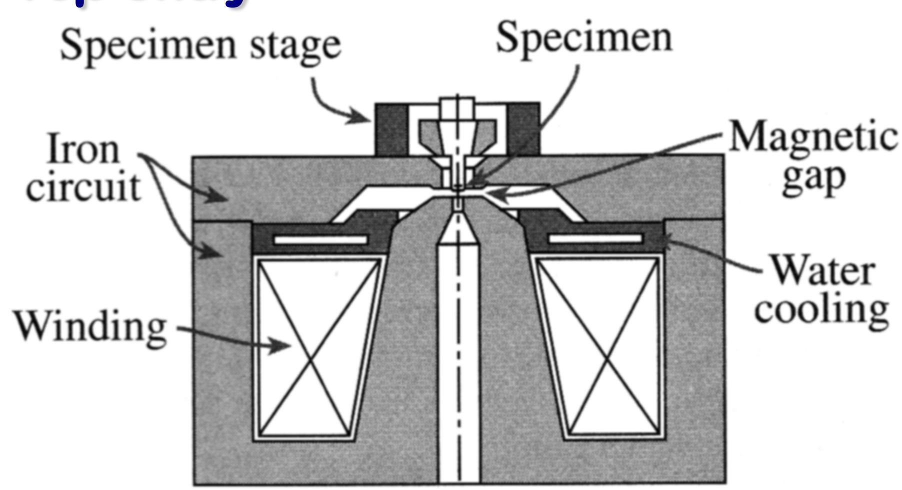
## Top entry

- Maximum resolution
  - Less aberration
  - Smaller u

Side entry



Top entry



# Lenses

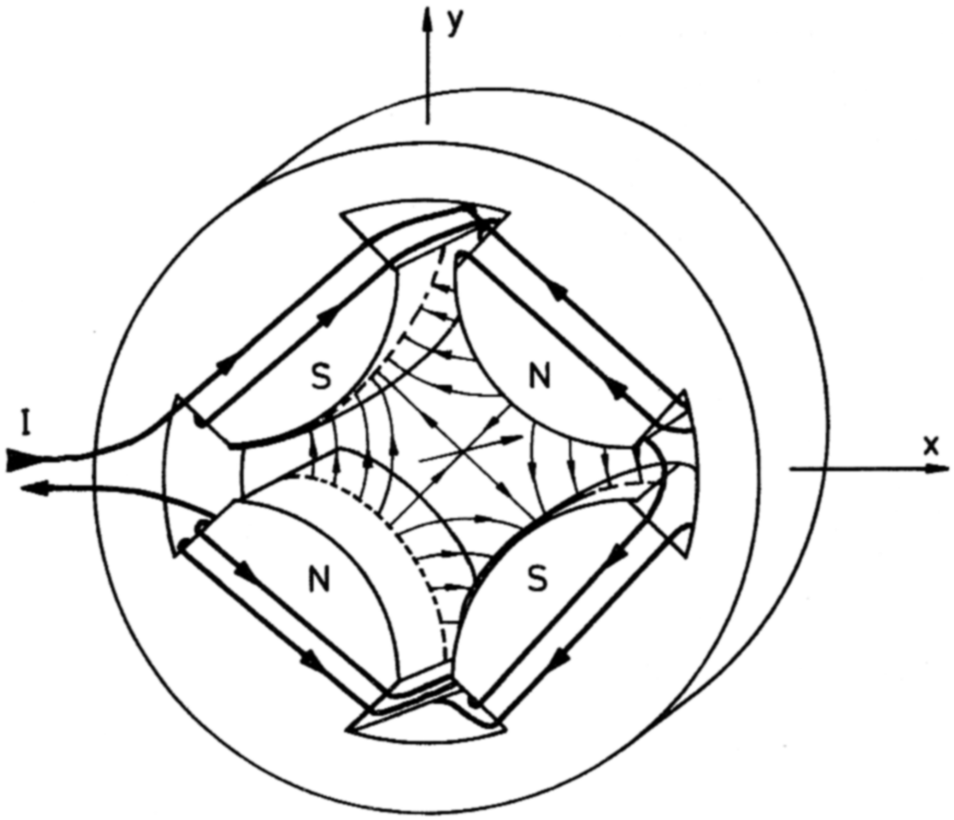
## Electron lenses

### Quadropole

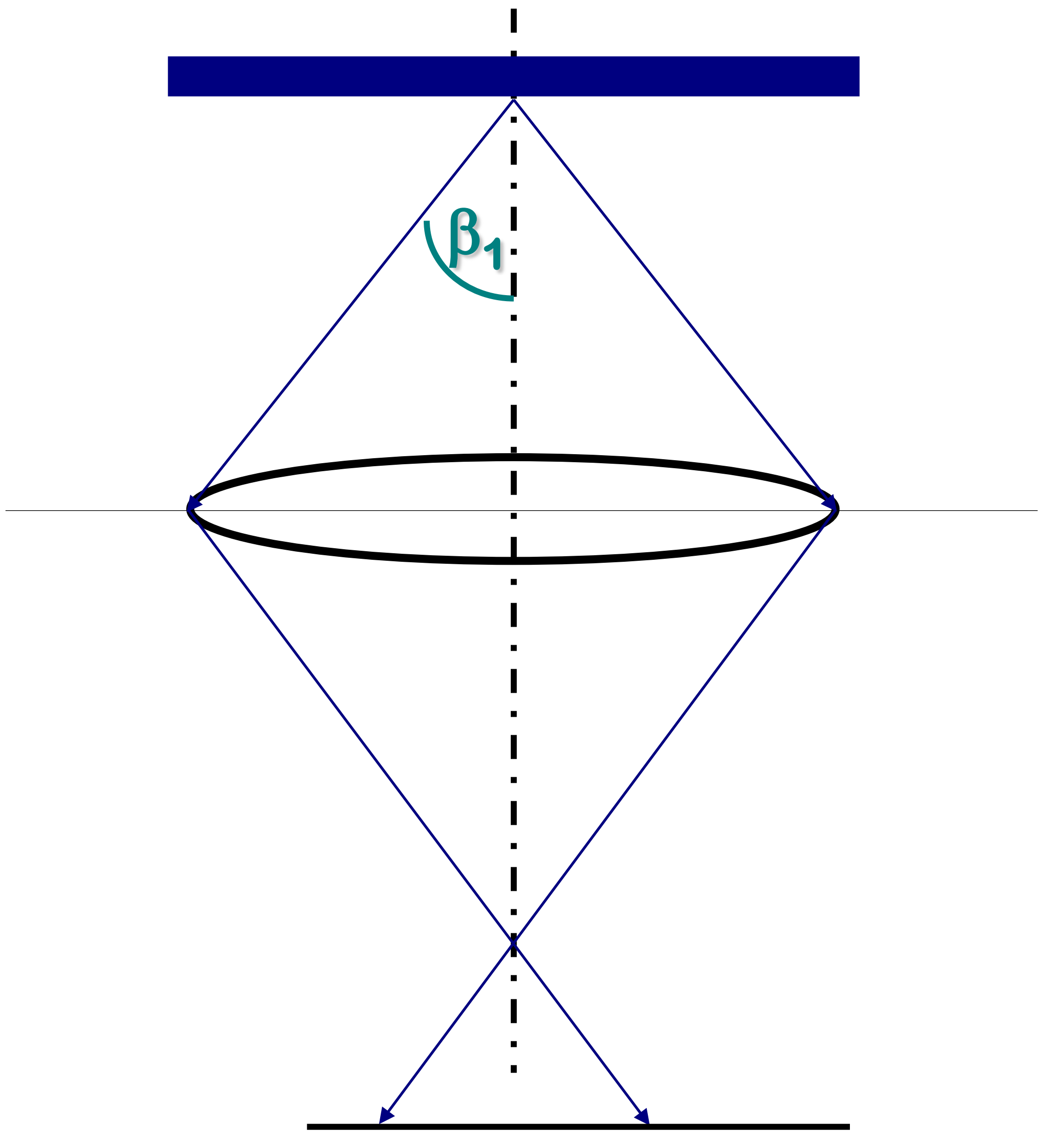
- Point object focused to a line image
- Used as stigmators

### Hexapole & Octupole

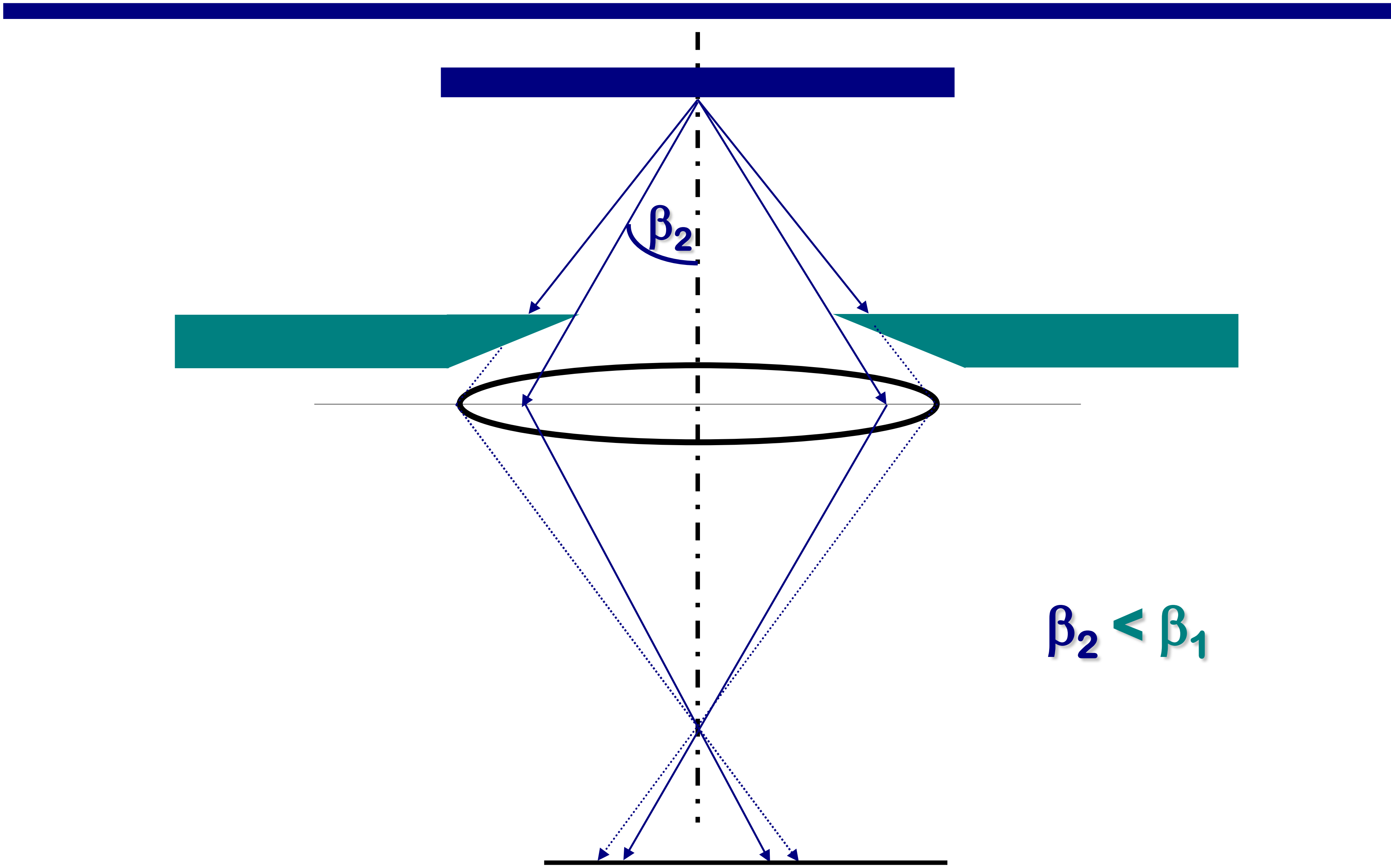
- Combinations for aberration correction



# Apertures



# Apertures



# Aberrations

## Magnetic lenses are far from perfect

## Suffer from a host of aberrations

**"Third order isotropic" aberrations:**

- Spherical
- Astigmatism
- Field curvature
- Distortion
- Coma

**Chromatic aberration**

**Astigmatism (first order)**

**Can be extended even further:**

- Third order anisotropic
- Fifth order aberrations

# Aberrations

## Spherical aberration

**Off-axis rays focused more strongly than on-axis rays**

**Disk of least confusion:**

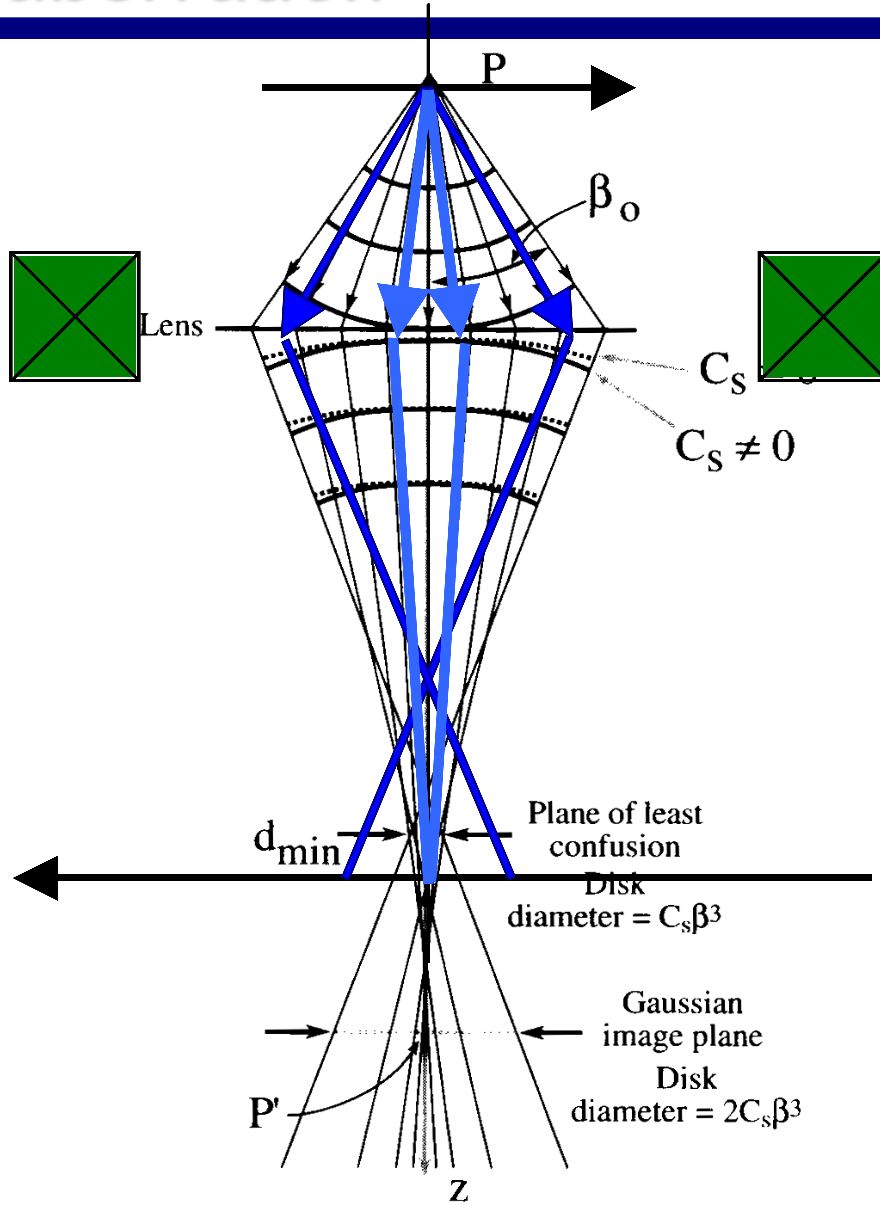
$$d'_{s,min} = C_s\beta^3$$

**Larger in image:**

$$d_{s,min} = 2C_s\beta^3$$

**$C_s$ usually 0.5 to 2 mm**

- **About equal to focal length**



# Aberrations

## Chromatic aberration

**Not from differences in ΔE from the HV Tank& source per se.**

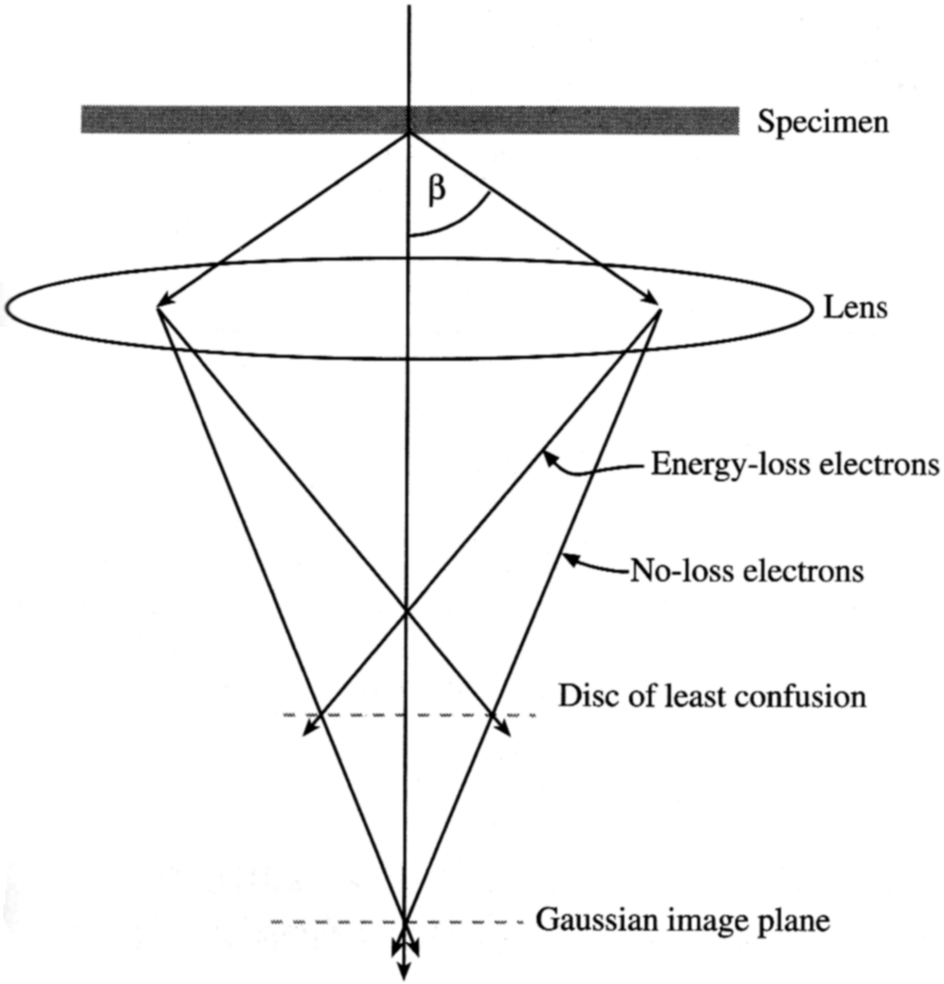
- HV ripple is 1 part in $10^6$
  ⇑ 0.1eV
- ΔE is source dependent

**ΔE arises from inelastic scattering**

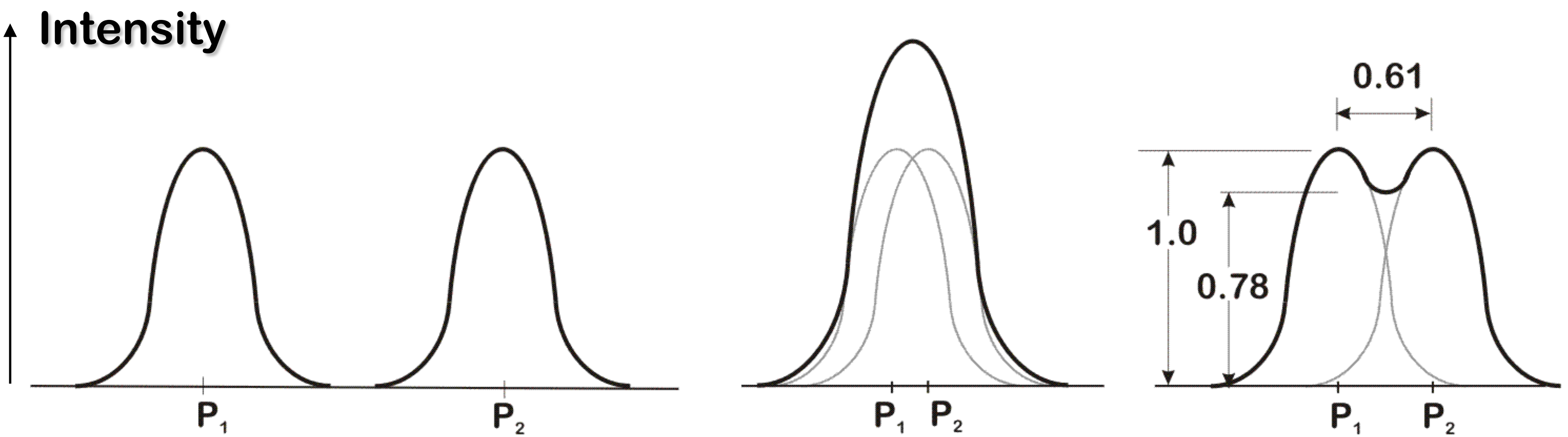
- Up to 2keV difference
- Most between 15-25 eV

**Disk of least confusion**

$$r_{chr} = C_c \frac{\Delta E}{E_o} \beta$$



# Resolution
## Theoretical




**Theoretical resolution given by Rayleigh criterion:**

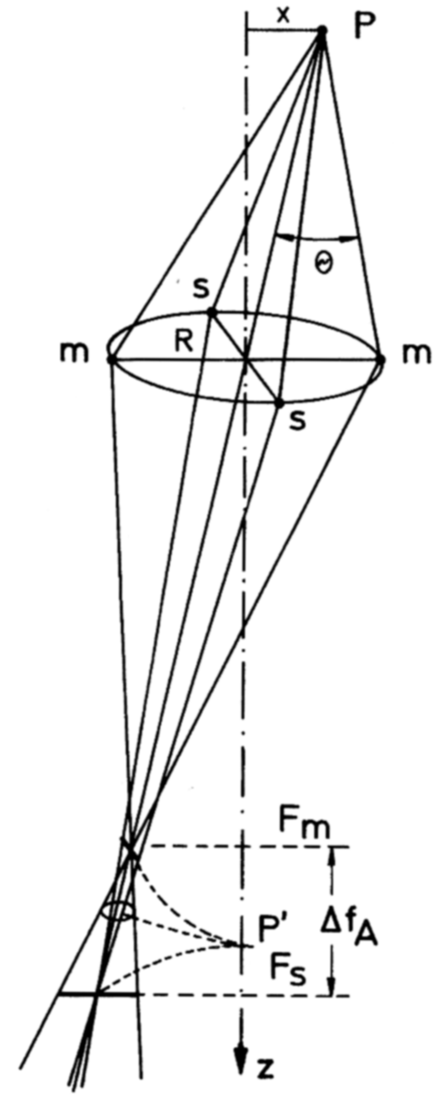
$$r_{th} = 0.61\frac{\lambda}{\beta}$$

# Aberrations
## Astigmatism

- Caused by inhomogeneities in the lens, aperture defects and aperture centering problems

- Fortunately, can be corrected
  - Stigmator octupoles

- Learning how is a big part of initial labs



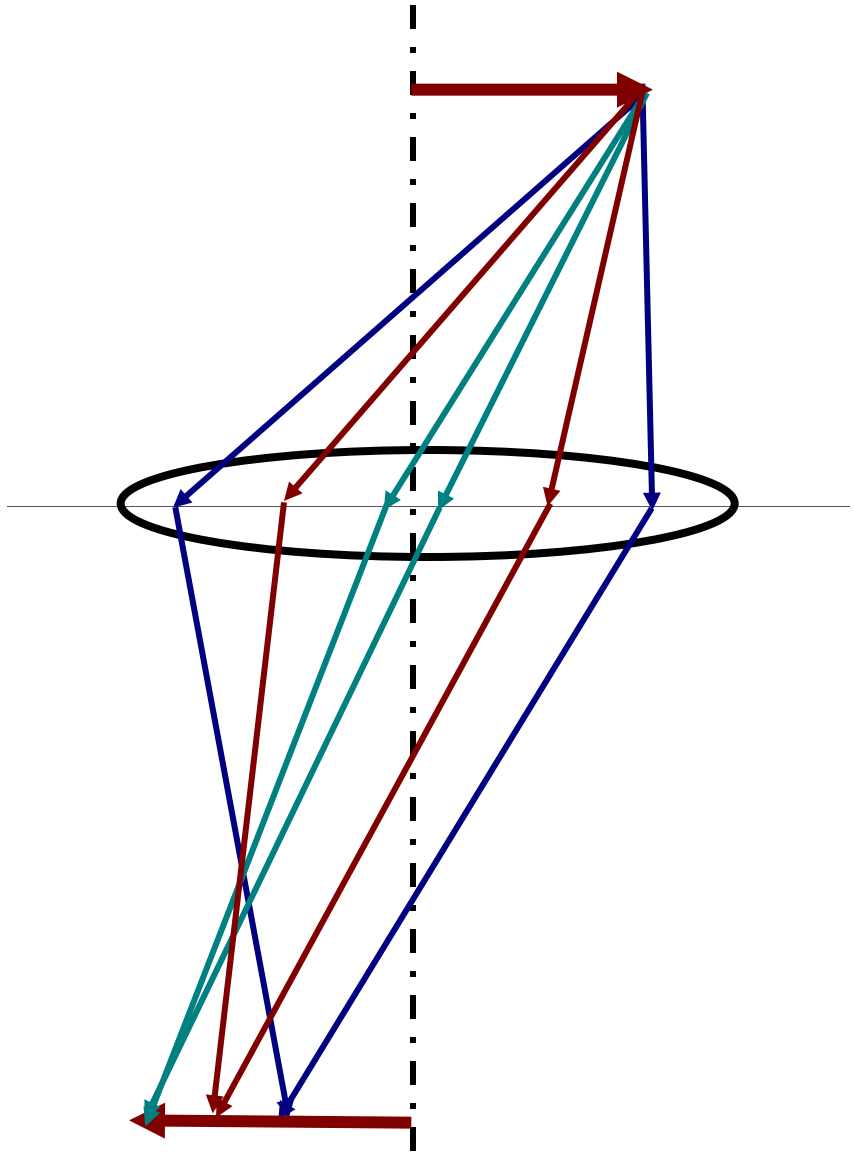
# Aberrations
## Coma

**Oblique, off-axis rays focused at different magnifications**

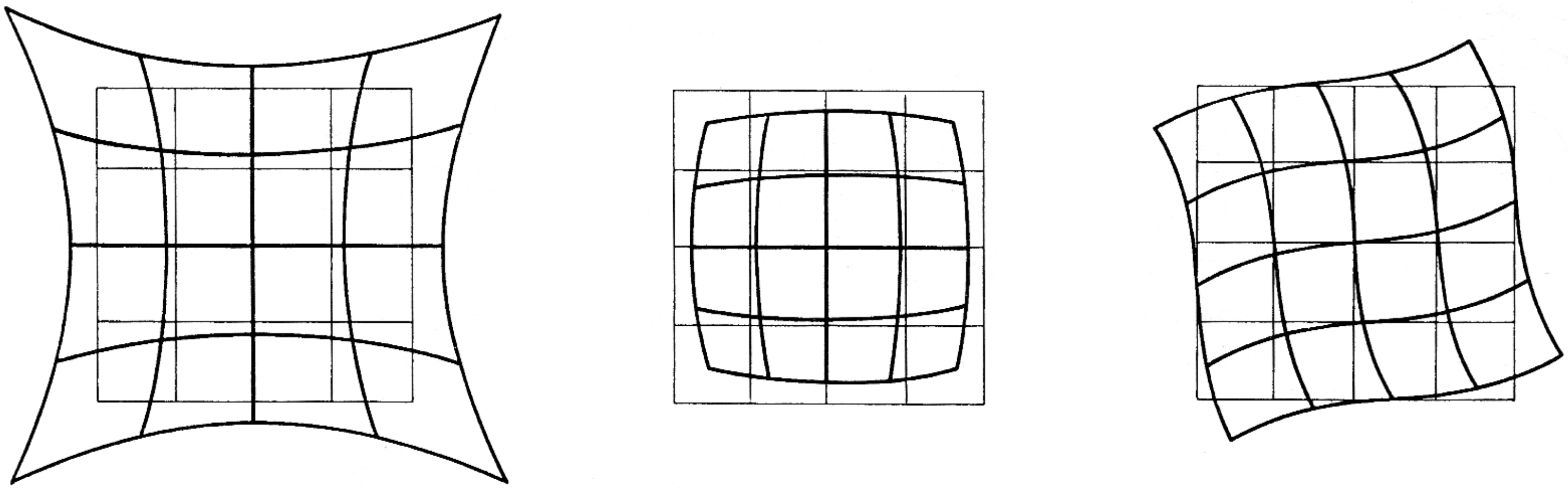
**This can be corrected through ‘coma-free’ alignment**

**Necessary for high resolution imaging**

- **Not as important in other work**



# Aberrations
## Distortions



Pincushion Barrel Spiral

**Only a worry in low magnification modes (Lorentz imaging)**

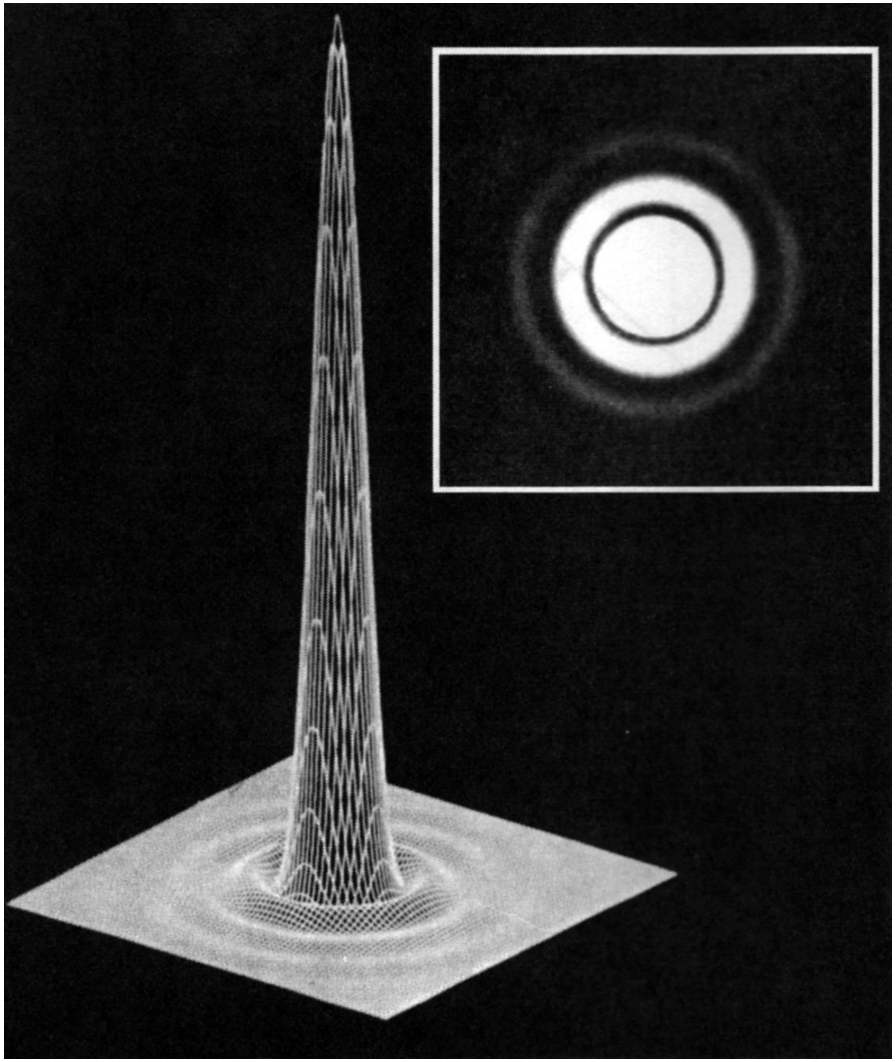
# Resolution

## Airy disc □

Presence of any aperture causes diffraction

At minimum, the main tube that runs down the column provides to provide vacuum acts as an aperture

Diffraction from a circular aperture yield an intensity known as an “Airy disc”



# Resolution

## Spherical aberration limited

**Recall:** $r_{sph} = C_s \beta^3$

**Add in quadrature (arbitrary):**

$$r = \left[ r_{th}^2 + r_{sph}^2 \right]^{1/2}$$

**Variation w/ β:**

$$r(\beta) = \left[ \left( 0.61 \frac{\lambda}{\beta} \right)^2 + \left( C_s \beta^3 \right)^2 \right]^{1/2}$$

**Find minimum:**

$$\frac{dr(\beta)}{d\beta} = 0 = -2 \frac{(0.61\lambda)^2}{\beta^3} + 6 C_s{}^2 \beta^5 \quad \Rightarrow \quad \beta_{opt} = 0.77 \frac{\lambda^{1/4}}{C_S^{1/4}}$$

$$r_{min} = 0.91 \left( C_S \lambda^3 \right)^{1/4}$$

# Depth of field & depth of focus

## Depth of field:

- Depth of 'sharpness' in object space

$$D_{ob} = \frac{d_{ob}}{\beta_{ob}}$$

- 2Å detail ⇒ 20 nm thick
- 2 nm detail ⇒ 200 nm thick

## Depth of focus:

- Depth of 'sharpness' in image space

$$D_{im} = \frac{d_{ob}}{\beta_{ob}} M^2$$

- 2Å detail ⇒ 500 kX ⇒ 5 km
- 2 nm detail ⇒ 50 kX ⇒ 5 m