Basic Properties of Electrons and Electron Sources

Lecture 2

Basic properties of electrons

a reminder …

Basic properties of electrons

wave-particle duality

Wave - particle duality of electrons is a manifested routinely in the electron microscope.

Let's think about this:

- **Electron accelerated at 100kV:**

$$v \approx 0.5c \approx 1.6 \cdot 10^8 \ \text{m/s}$$

- **Electron current ≈ 1.0 μA ⮝ 10^{12} electrons / sec**
- **Implication: each electron separated by ≈ 1.6 mm**

This implies that there is only one electron that is interacting with the specimen at any given time.

Basic properties of electrons

wave-particle duality

Despite this both particle & wave phenomena are observed:

- Electron diffraction - wave phenomenon
- Electron interference - interference between different waves
- Photoelectric effect - particle phenomenon

Key point

- Get used to the idea that you consider the electron as *both* routinely when using the TEM
  - Diffraction - think wave
  - Scattering - think particle

Basic properties of electrons

wave-particle duality

A wave is a periodic disturbance in both space and time:

$$\Psi = \sin\left(\frac{2\pi x}{\lambda} + \omega t\right) = \sin(kx + \omega t) \quad \text{with } k = \frac{2\pi}{\lambda}$$

Consider two waves with slightly different frequencies:

$$\Psi_1 = \sin[kx + \omega t]$$

$$\Psi_2 = \sin[(k + \Delta k)x + (\omega + \Delta\omega)t]$$

Superimpose these two waves, use a bit of trigonometry:

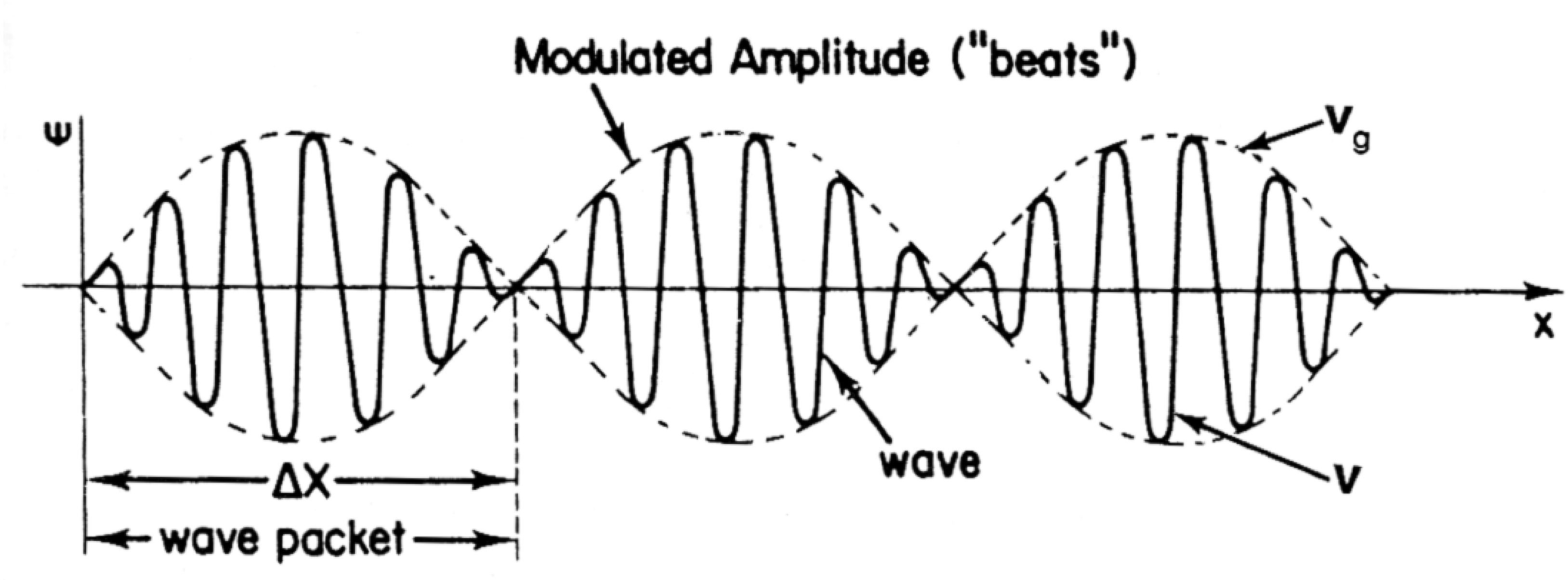
$$\Psi_1 + \Psi_2 = \Psi = \underbrace{2\cos\left(\frac{\Delta\omega}{2}t + \frac{\Delta k}{2}x\right)}_{\text{Modulated amplitude}} \cdot \underbrace{\sin\left[\left(k + \frac{\Delta k}{2}\right)x - \left(\omega + \frac{\Delta\omega}{2}\right)t\right]}_{\text{Sine wave}}$$

Modulated amplitude

Sine wave

Basic properties of electrons

wave-particle duality



An analogue to this is "beats" in music.

Let's look at this dynamically a bit:

http://galileo.phys.virginia.edu/classes/109N/more_stuff/Applets/sines/GroupVelocity.html

From Hummel

Basic properties of electrons

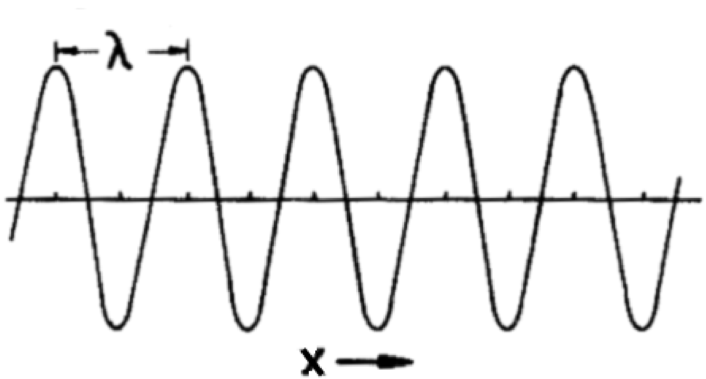
wave-particle duality

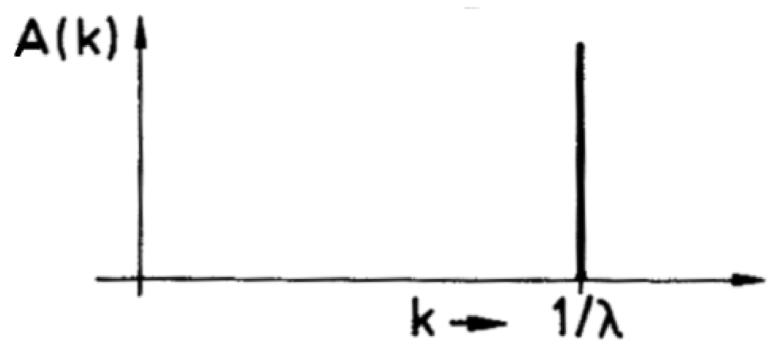
Two cases are illustrative:

$$\Delta\omega \to 0 \;;\; \Delta k \to 0$$

Infinitely long wave packet $\to$ monochromatic wave

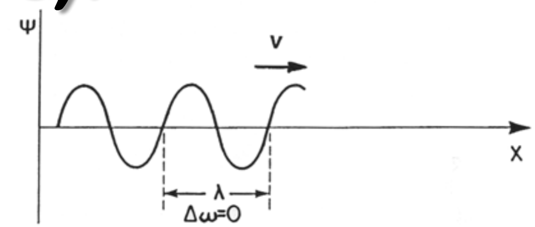
$\to$ plane wave





Phase velocity (velocity of the wave):

$$v = \frac{x}{t} = \frac{\omega + \Delta\omega}{k + \Delta k} = \frac{\omega'}{t'}$$



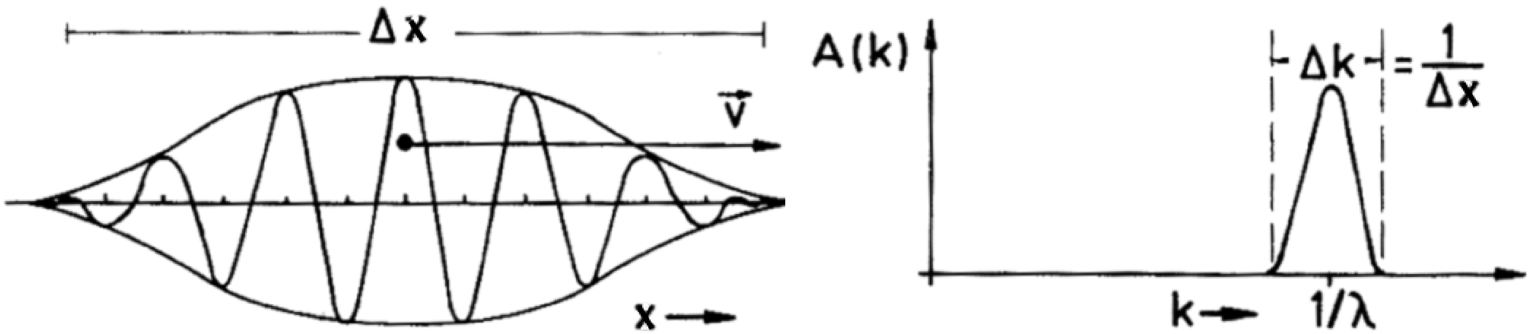
From Reimer

Basic properties of electrons

wave-particle duality

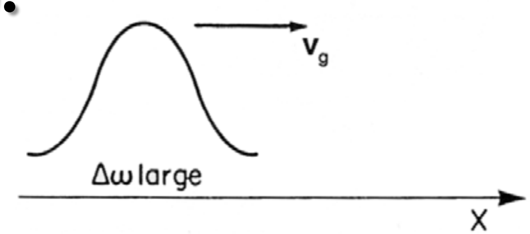
Consider instead that you have many waves superimposed (i.e. $\Psi_1, \Psi_2, \ldots \Psi_\infty$) which fill frequencies between ω and $\Delta\omega$, where $\Delta\omega$ is large:

- Reduces to one wave packet
- This wave packet can be considered as "the electron as particle"



Group velocity (velocity of the particle):

$$v_g = \frac{x}{t} = \frac{\Delta\omega}{\Delta k}$$



From Reimer

Basic properties of electrons

De Broglie Eqn.: $\lambda = h/p$

All energy is kinetic, i.e.: $eV = \frac{m_o v^2}{2}$

Momentum is: $p = m_o v = \left(2 m_o eV\right)^{1/2}$

Substituting:

$$\lambda = \frac{h}{(2 m_o eV)^{1/2}}$$

Key point: the higher the voltage, the smaller the wavelength

Relativistic correction needed

$$\lambda = \frac{h}{\left[2 m_o eV\left(1 + \frac{eV}{2 m_o c^2}\right)\right]^{1/2}}$$

Basic properties of electrons

Accelerating Voltage (kV)	Non-relativistic wavelength (Å)	Relativistic wavelength (Å)	Mass ($x\ m_o$)	Velocity ($x\ 10^8$ m/s)	Velocity ($x\ c$)
100	0.0386	0.0370	1.196	1.644	0.54
200	0.0273	0.0251	1.391	2.086	0.69
300	0.0223	0.0197	1.587	2.330	0.78
400	0.0193	0.0164	1.783	2.484	0.83
1000	0.0122	0.0087	2.957	2.823	0.94

Electron wavelength & resolution

Resolution (Rayleigh criterion):

$$\delta = \frac{0.61\lambda}{\mu \sin\beta} \rightarrow \delta \approx \frac{0.61\lambda}{\beta}$$

So - even at 100 kV, we have λ = 0.04Å

- More than sufficiently small to image atoms (diameter ≈ 0.3Å)
- Lens imperfections (largely spherical aberration, chromatic aberration) limit resolution

Angles & Distances …

Some things worth remembering …

We deal often with very small angles:

- 1° = 17.5 milliradians (mrad) ≈ 15 mrad

We use very high magnifications:

- At 1000 X, 1 cm = 10 μm
- At 10,000X, 1 cm = 1 μm ←
- At 50,000X, 1 cm = 200 nm — I just memorize these two
- At 100,000X, 1 cm = 100 nm ←
- At 500,000X, 1 cm = 20 nm = 200Å

Electron sources

Lecture 2

Electron sources

What are source characteristics?

– Brightness, Temporal (ΔE) coherency and Spatial coherency / source size

Source types?

– Thermionic, Schottky field emission, Cold field emission

How do electron guns work?

– Thermionic, Field emission (both types)

How / why do you measure gun properties

– Beam current, convergence angle, beam diameter, energy spread, spatial coherency

Source characteristics

Source characteristics

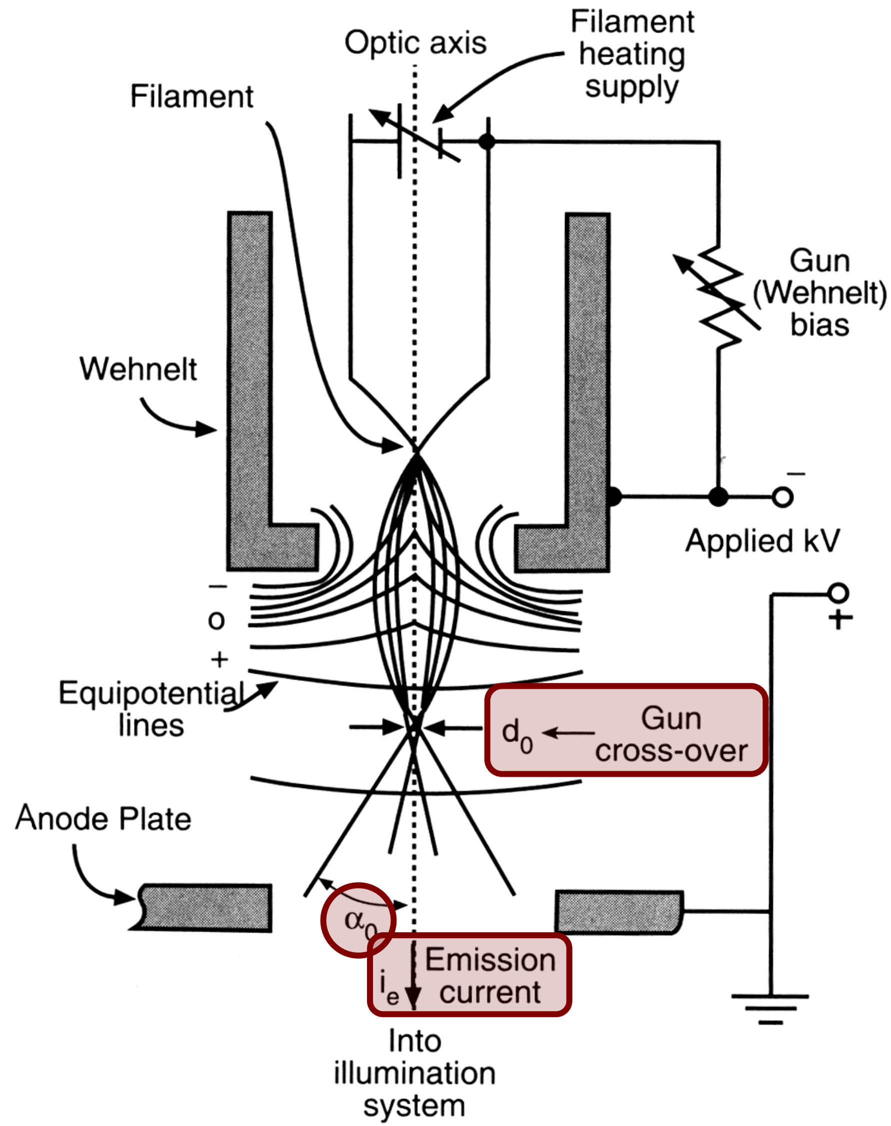
Brightness

Definition:

- **<u>Brightness:</u>** Current density per unit solid angle
- Current density is: # electrons per unit area per unit time

Some properties:

- Beam diameter: d_o
- Cathode emission current: i_e
- Semi-angle of divergence from source: α_o



Source characteristics
Brightness

Brightness: $$\beta = \frac{i_e}{(\pi d_o \alpha_o)^2}$$

Brightness (A/m²·sr)

Thermionic	10^9
Schottky	$5 \cdot 10^{10}$
Cold field emission	10^{13}

This is a key parameter:

- impacts exposure times
- analytical work

Note: text has an error in def^n of solid angle

Source characteristics

Temporal coherency & energy spread

Temporal coherency refers to the energy spread of the source

- Analogue to light optics is “color”
- Coherence length:

$$\lambda_c = \frac{vh}{\Delta E}$$

Typical ΔE

- Tungsten thermionic: 3 eV
- LaB_6 thermionic: 1 eV
- Schottky field emission: ≈ 0.8 eV
- Cold field emission: 0.3 eV
  - Note this is on top of 200 to 300 keV

Important with respect to EEL spectroscopy

Source characteristics

Spatial coherency & source size

Spatial coherency is associated with the physical 'point of origin' of the electrons

Related to the "effective source size" (d_c):

$$d_c << \frac{\lambda}{2\alpha}$$

To improve spatial coherency:

$d_c \Downarrow$ (FEG); $\lambda \Uparrow$ ($\Uparrow$ kV); $\alpha \Downarrow$ ($\Downarrow$ aperture)

Improved spatial coherency:

- Helps with high resolution imaging
- Gives sharper diffraction patterns
- Gives better diffraction contrast images

The answer is clear => FEG!? Not necessarily …

- Expensive (+ $700k), fringes in HREM images, less intense when beam is spread

Source characteristics

Stability

Stability is important:

- HREM imaging
- Microanalysis

Typical stabilities in modern TEM's are:

- "Ripple": 1 ppm RMS (latest are 0.1 ppm RMS)
- "Drift": 2 ppm over 10 minute periods

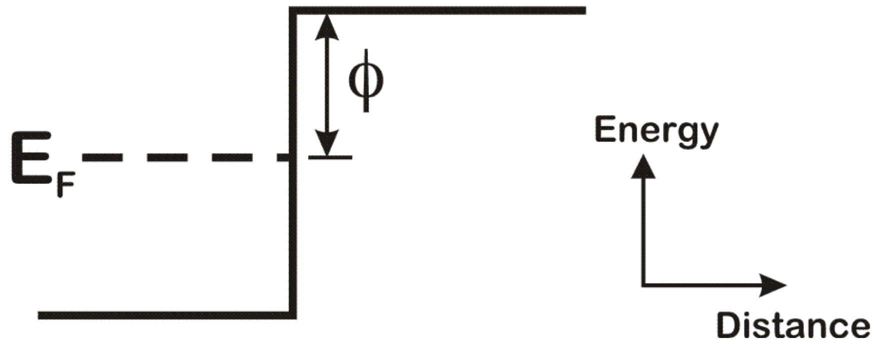
Thermionic > Schottky FEG > Cold FEG

Emission physics

Emission physics
thermionic emission

Recall: “work function”

- Energy required to remove an electron from a material

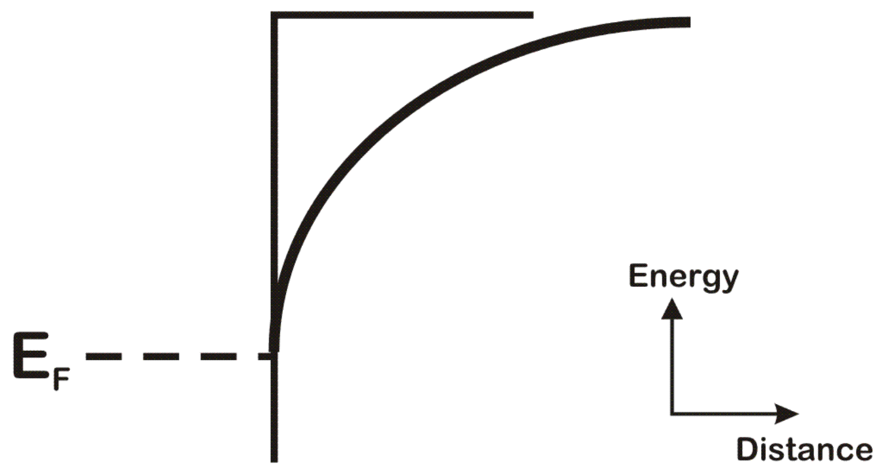


Schottky effect

- Includes ‘image field’

Thermionic emission

- Energy supplied by heat alone



Emission physics

thermionic emission

Richardson Law:

$$j_c = AT_c^2 \exp\left(\frac{-\phi}{kT}\right)$$

- Exponential dependence means 10% change in ϕ, T yields factor of 8 increase in emission
- Function of T, surface condition & crystallography
- Lifetime: W $\approx$ 200 hr; LaB6 $\approx$ 1000 hr ; $CeB_6 \approx$ 1500 hr
  - Sputtering

Material	ϕ (eV)	T_m (K)
Cs	1.9	301
Cu	4.45	1358
Co	4.4	1768
W	4.5	3695
LaB_6	2.7	≈2800
CeB_6	2.5	high ...

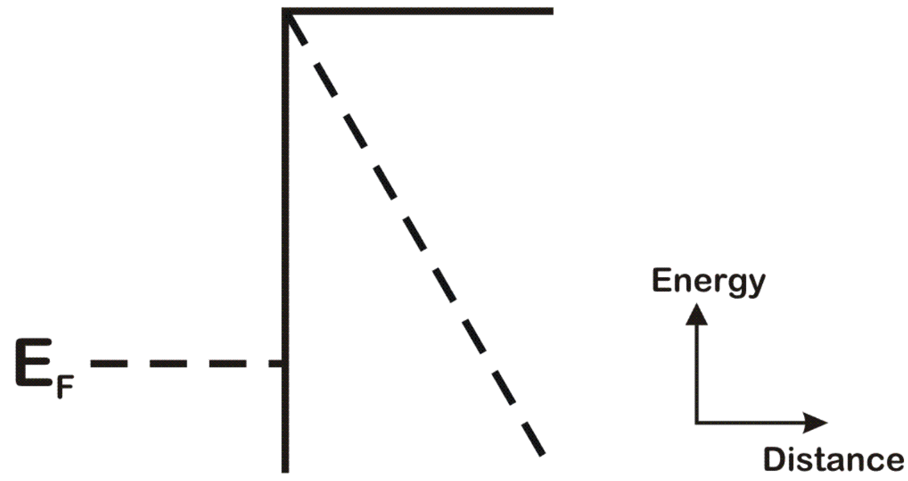
Emission physics

Field emission

Additional electric field lowers barrier

$$V = eE(x)$$

If field is sufficiently strong, electrons can tunnel out



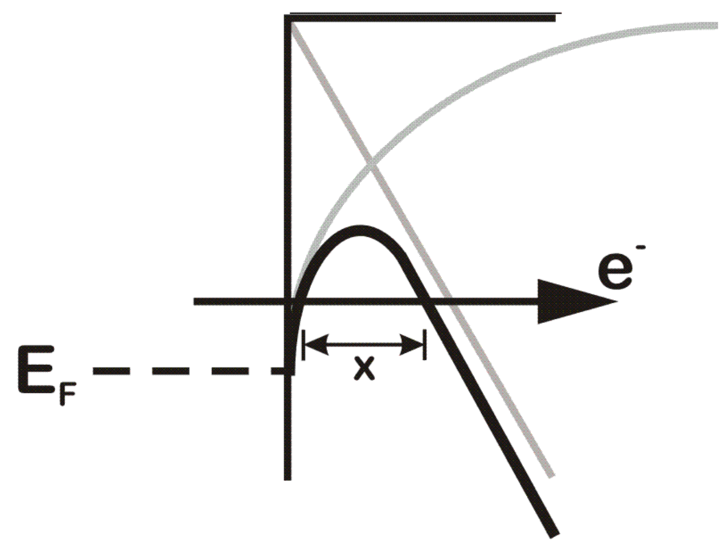
Schottky FEG

- Temp & field
- ZrO/W @1800k)

Cold FEG

- Field only

UHV necessary

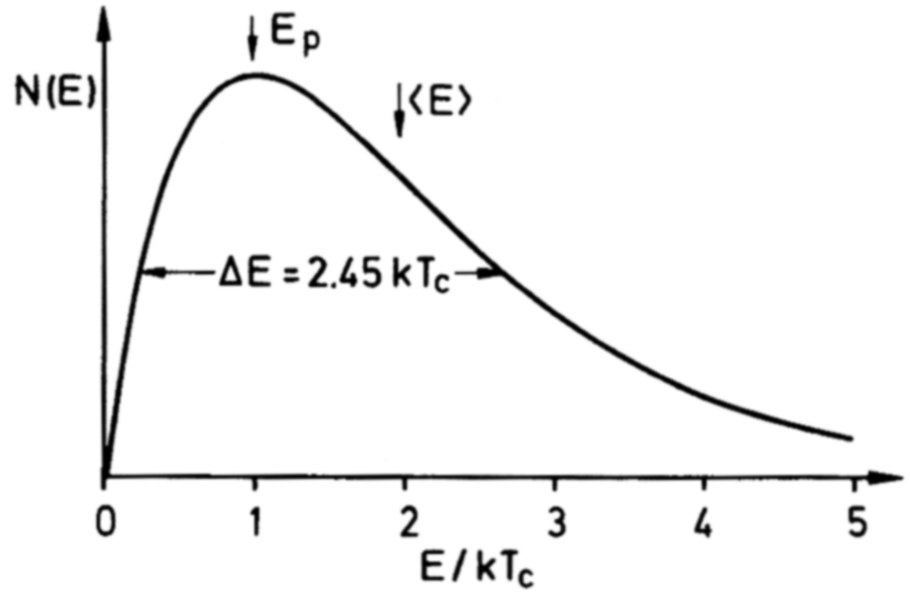


Emission physics

For field emission use instead Fowler - Nordhiem Eqn:

$$j_c = \frac{k_1 |E|^2}{\phi} \exp\left(\frac{k_2 \phi^{3/2}}{|E|} \right)$$

Energy distribution for all cases (thermionic, Schottky, cold FEG) is Maxwell-Boltzmann



Emission physics

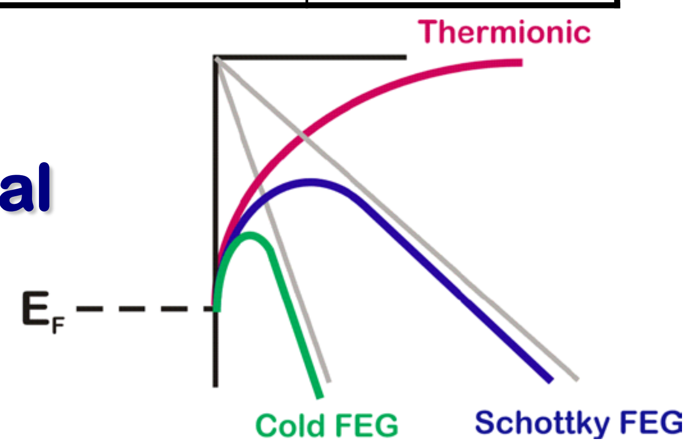
summary

	β (A/m²sr)	ΔE (eV)	d	Vacuum (Pa)
W	10^9	1.5 - 3	20 - 50 µm	10^{-3}
LaB_6	$5 \cdot 10^9$	1 - 2	10 - 20 µm	10^{-4}
Schottky FEG	$5 \cdot 10^{10}$	0.7	15 nm	10^{-6}
Cold FEG	10^{13}	0.3	2.5 nm	10^{-8}

Conventional TEM - LaB_6 / CeB_6

Schottky FEG - Conventional analytical

Cold FEB - Very high end analytical



Probe comparison

Radiation	Source Brightness (particles/$cm^2 \cdot sr \cdot eV$)	Elastic Mean Free Path (Å)	Absorption Length (Å)	Minimum Probe Size (Å)
Neutrons	10^{14}	10^{8}	10^{9}	10^{7}
X-rays	10^{26}	10^{4}	10^{6}	10^{3}
Electrons	10^{29}	10^{2}	10^{3}	1

Cold FEG is the brightest continuous radiation source known in the universe.

Because of high spatial & temporal coherency can be focused to the smallest probe available (0.78Å!)

Lots of energy, lots of potential for specimen damage

Electron Guns

How do they work?

Electron guns

thermionic

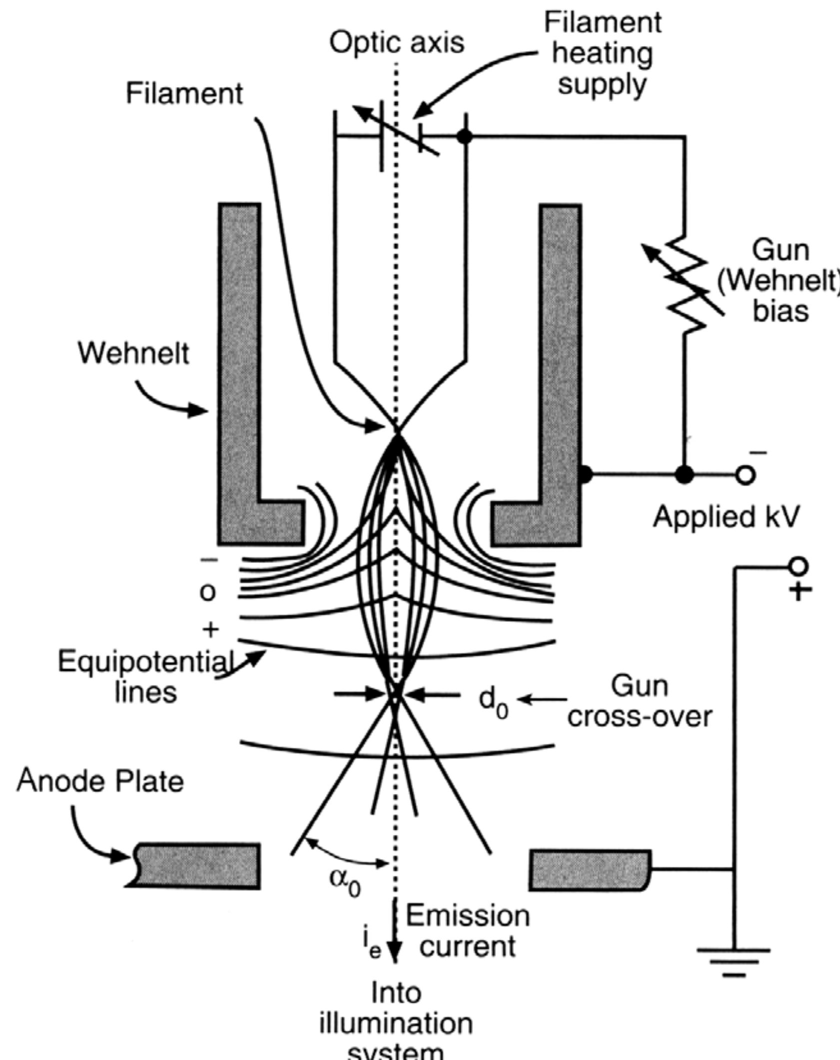
Filament heated to give thermionic emission

- Directly (W) or indirectly (LaB_6)

Filament at negative potential to ground

Wehnelt produces a small negative bias

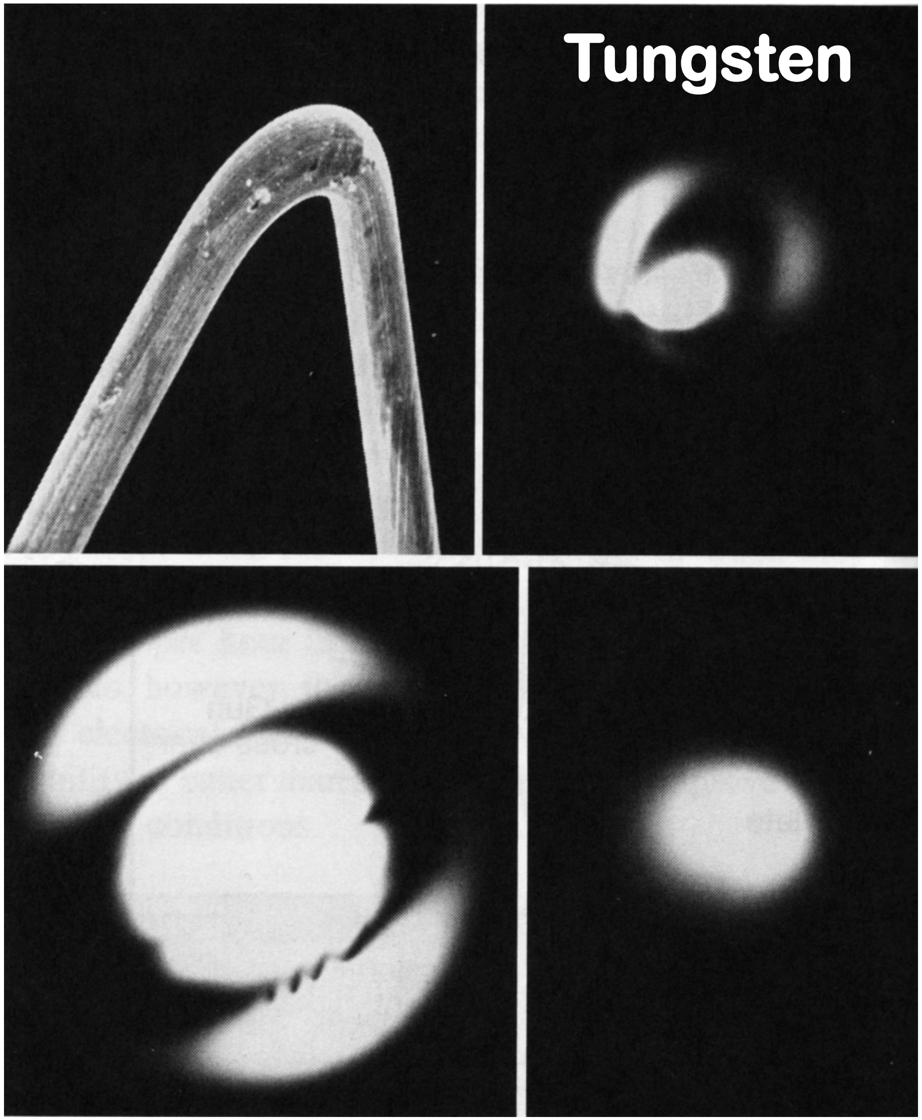
- Brings electrons to crossover

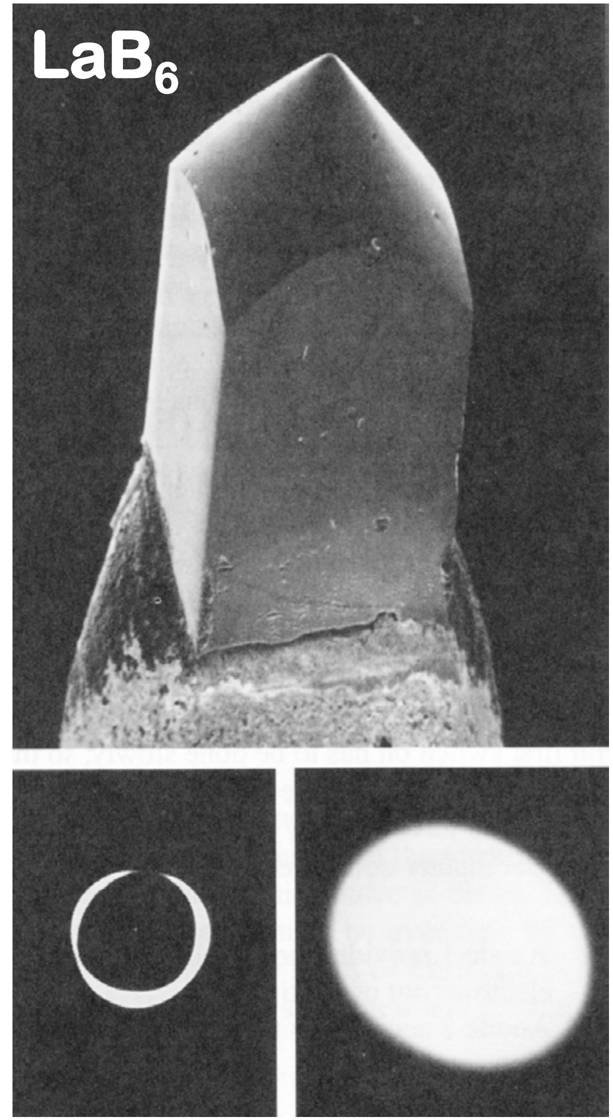


http://www.matter.org.uk/tem/electron_gun/electron_gun_simulation.htm

Electron guns

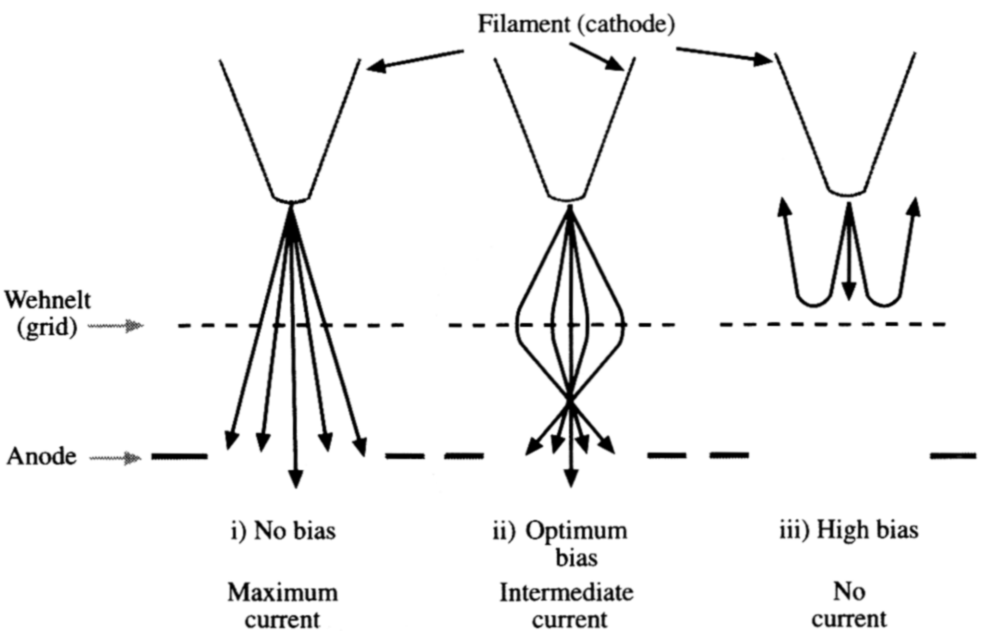
thermionic

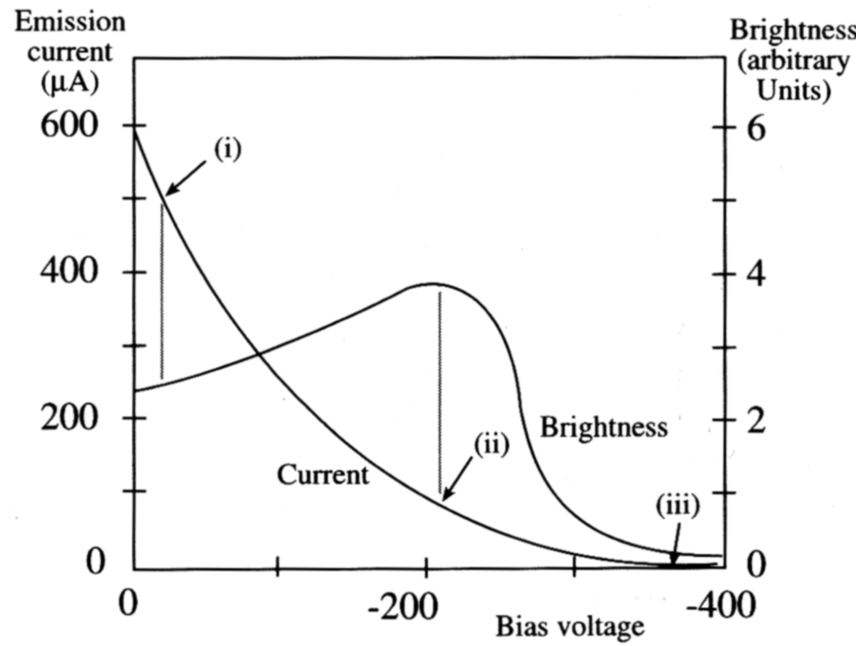




Electron guns

thermionic





Perfect saturation / bias aids filament lifetime

Also yields smallest source size, best coherency, best images

Electron guns
field emission

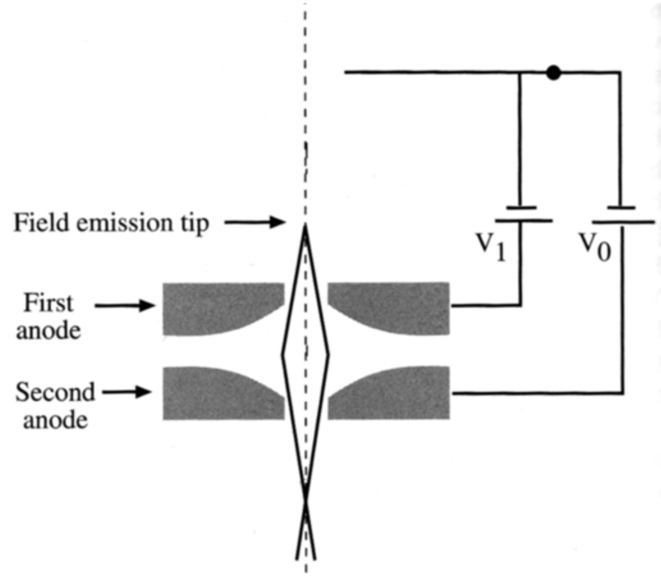
First anode (V_1) is extraction voltage

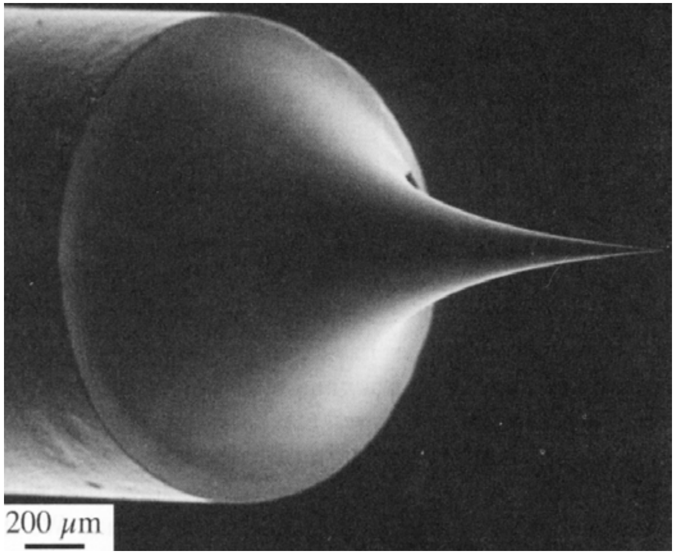
Second anode (V_2) acts as an electrostatic lens

As an opertator, you slowly increase V_1

- That's pretty much it.
- Automated in latest machines

Different extraction voltages for different operation modes





Electron guns

summary

TABLE 5.1. Characteristics of the Three Principal Sources Operating at 100 kV

	Units	Tungsten	LaB_6	Field Emission
Work function, Φ	eV	4.5	2.4	4.5
Richardson's constant	A/m^2K^2	6×10^5	4×10^5	
Operating temperature	K	2700	1700	300
Current density	A/m^2	5×10^4	10^6	10^{10}
Crossover size	μm	50	10	<0.01
Brightness	A/m^2sr	10^9	5×10^{10}	10^{13}
Energy spread	eV	3	1.5	0.3
Emission current stability	%/hr	<1	<1	5
Vacuum	Pa	10^{-2}	10^{-4}	10^{-8}
Lifetime	hr	100	500	>1000